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| For Examiner's Use | |
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| Examiner's Initials | |
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| 1 | |
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AQA

General Certificate of Education
Advanced Level Examination
June 2015

# Mathematics MM05

## Unit Mechanics 5

## Tuesday 9 June 2015 9.00 am to 10.30 am

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

### Time allowed
- 1 hour 30 minutes

### Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

### Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

### Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

**1** A particle moves with simple harmonic motion on a line between two points, $A$ and $B$, which are $0.4$ metres apart. The maximum speed of the particle is $0.8\,\mathrm{m\,s^{-1}}$. The particle passes through a point $C$ that is $0.1$ metres from $A$.

**(a)** Find the period of the motion.

**[3 marks]**

**(b)** Find the speed of the particle when it is at $C$.

**[3 marks]**

**(c)** Given that the particle is at rest at $A$ at time $t = 0$, find an expression for the displacement of the particle from $A$ at time $t$ seconds.

**[3 marks]**

**(d)** Find the time that it takes for the particle to move from $A$ to $C$.

**[3 marks]**

QUESTION PART REFERENCE

**Answer space for question 1**

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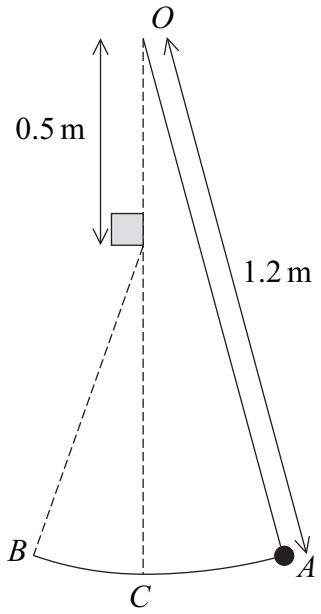
**2** A particle is attached to one end of a light inextensible string of length $1.2$ metres. The other end of the string is attached to a fixed point $O$. A square peg is fixed with its lowest edge at a distance $0.5$ metres directly below $O$. The particle is released from rest at the point $A$ and moves to the point $B$, where it comes to rest. During this motion the particle passes through the point $C$, which is vertically below $O$. These points are shown in the diagram. The path of the particle consists of two arcs, $AC$ and $CB$, of different radii.




Assume that the angle between the vertical and the string is always small and that there is no air resistance.

Find the time that it takes for the particle to move from $A$ to $B$.

**[4 marks]**

QUESTION PART REFERENCE

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**3** A particle moves on a curve and at time $t$ has polar coordinates $(r, \theta)$ with respect to an origin $O$. The curve is defined by $r = \frac{1}{2}e^{a\theta}$, where $a$ is a constant. At time $t = 0$, the polar coordinates of the particle are $\left(\frac{1}{2}, 0\right)$. A force acts on the particle so that it maintains a constant angular velocity, about $O$, of $2$.

**(a)** Find the radial and transverse components of the acceleration of the particle at time $t$, in terms of $a$ and $t$.

**[7 marks]**

**(b)** Given that the magnitude of the acceleration is $20$ when $t = 0$, find the possible values of $a$.

**[5 marks]**

QUESTION PART REFERENCE

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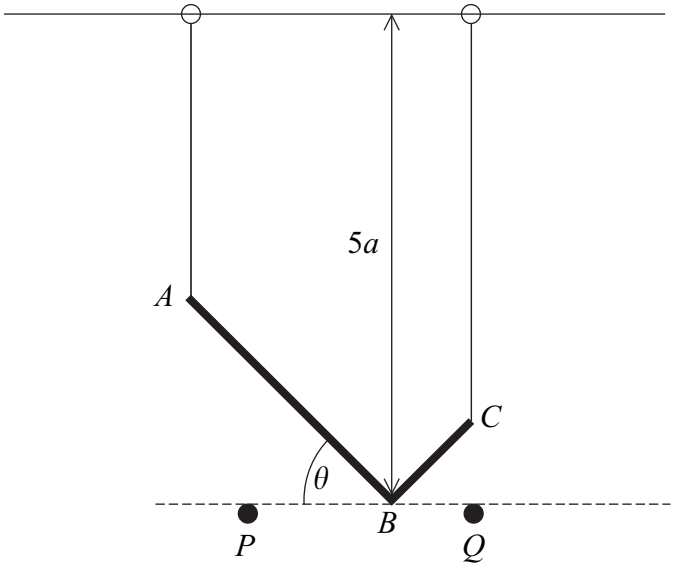
**4** Two uniform rigid rods, $AB$ and $BC$, are joined at right angles to make the rigid body shown in the diagram. The rod $AB$ has length $2a$ and mass $2m$. The rod $BC$ has length $a$ and mass $m$. The rigid body is pivoted at $B$ and is free to rotate about a horizontal axis through $B$ which is perpendicular to the two rods. Two pegs, $P$ and $Q$, are positioned as shown in the diagram to restrict the motion of the rigid body.

Two elastic strings are attached to the rigid body at $A$ and $C$. The other ends of the strings are attached to rings that move on a smooth horizontal wire at a distance of $5a$ above $B$. Assume that these strings remain vertical at all times. Both strings have natural length $a$. The string attached at $A$ has modulus of elasticity $2mg$ and the string attached at $C$ has modulus of elasticity $8mg$. The angle between $AB$ and the horizontal is $\theta$ radians.




Gravitational potential energy is taken to be zero at the level of $B$.

**(a)** Show that $V$, the total potential energy of the system, is given by

$$V = \frac{mga}{2}(168 - 28\sin\theta - 63\cos\theta) \quad \text{where} \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

**[7 marks]**

**(b)** Find the value of $\theta$ for which the rigid body is in equilibrium.

**[5 marks]**

**(c)** Confirm that the value of $\theta$ found in part **(b)** corresponds to a position of stable equilibrium.

**[3 marks]**

QUESTION PART REFERENCE

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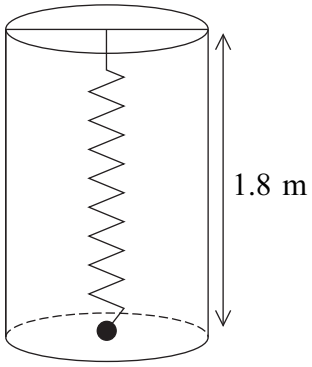
**5** A particle of mass $2\,\text{kg}$ is attached to a spring of stiffness $24\ \text{N}\,\text{m}^{-1}$ and natural length $0.35$ metres.

**(a)** Find the length of the spring when the mass hangs in equilibrium, giving your answer as a fraction.

**[3 marks]**

**(b)** The spring and particle are placed in a cylinder which is full of oil, with one end of the spring fixed at the top of the cylinder. The height of the cylinder is $1.8$ metres. The particle is released from rest at the base of the cylinder, as shown in the diagram.




The displacement of the particle from the centre of the top of the cylinder at time $t$ seconds is $x$ metres. As the particle moves, it experiences a resistance force of magnitude $14v$ N, where $v$ is the speed of the particle at time $t$.

**(i)** Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 7\frac{\mathrm{d}x}{\mathrm{d}t} + 12x = 14$$

**[4 marks]**

**(ii)** Find $x$ in terms of $t$.

**[10 marks]**

**(iii)** State the type of damping that is taking place in this situation.

**[1 mark]**

QUESTION PART REFERENCE

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**6** A block, of mass $M$ kg, is initially at rest on a smooth horizontal surface. A light inextensible string is attached to the block and passes over a smooth peg. A light cylinder full of water is attached to the other end of the string. Water escapes from the cylinder, through two holes located at the base and on opposite ends of a diameter. Relative to the cylinder the water moves horizontally as it leaves the cylinder. Assume that the water leaves the cylinder at a **constant rate** of $\lambda \,\mathrm{kg\,s^{-1}}$. The system is released from rest with the cylinder full, the string taut and the string above the cylinder vertical, as shown in the diagram.

At time $t$ seconds, the mass of the water in the cylinder is $m$ kg, and the cylinder and block both have speed $v\,\mathrm{m\,s^{-1}}$. When $t = 0$, $m = M$ and $v = 0$.

**(a)** Show that while the cylinder contains water

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{(M - \lambda t)g}{2M - \lambda t}$$

**[6 marks]**

**(b)** Find $v$ in terms of $M$, $g$, $\lambda$ and $t$.

**[5 marks]**

**(c)** Find the maximum speed of the block, in terms of $M$, $g$ and $\lambda$.

**[3 marks]**

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**END OF QUESTIONS**

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