

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

A-level MATHEMATICS

Unit Further Pure 2

Friday 24 June 2016

Morning

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Given that $f(r) = \frac{1}{4r-1}$, show that

$$f(r) - f(r+1) = \frac{A}{(4r-1)(4r+3)}$$

where A is an integer.

[2 marks]

(b) Use the method of differences to find the value of $\sum_{r=1}^{50} \frac{1}{(4r-1)(4r+3)}$, giving your answer as a fraction in its simplest form.

[4 marks]

QUESTION
PART
REFERENCE

Answer space for question 1



- 2** The cubic equation $3z^3 + pz^2 + 17z + q = 0$, where p and q are real, has a root $\alpha = 1 + 2i$.
- (a) (i)** Write down the value of another non-real root, β , of this equation. **[1 mark]**
- (ii)** Hence find the value of $\alpha\beta$. **[1 mark]**
- (b)** Find the value of the third root, γ , of this equation. **[3 marks]**
- (c)** Find the values of p and q . **[3 marks]**

QUESTION
PART
REFERENCE**Answer space for question 2**

3 The arc of the curve with equation $y = 4 - \ln(1 - x^2)$ from $x = 0$ to $x = \frac{3}{4}$ has length s .

(a) Show that $s = \int_0^{\frac{3}{4}} \left(\frac{1 + x^2}{1 - x^2} \right) dx$.

[4 marks]

(b) Find the value of s , giving your answer in the form $p + \ln N$, where p is a rational number and N is an integer.

[6 marks]

QUESTION
PART
REFERENCE

Answer space for question 3



4 (a) Given that $y = \tan^{-1} \sqrt{3x}$, find $\frac{dy}{dx}$, giving your answer in terms of x .

[2 marks]

(b) Hence, or otherwise, show that $\int_{\frac{1}{3}}^1 \frac{1}{(1+3x)\sqrt{x}} dx = \frac{\sqrt{3}\pi}{n}$, where n is an integer.

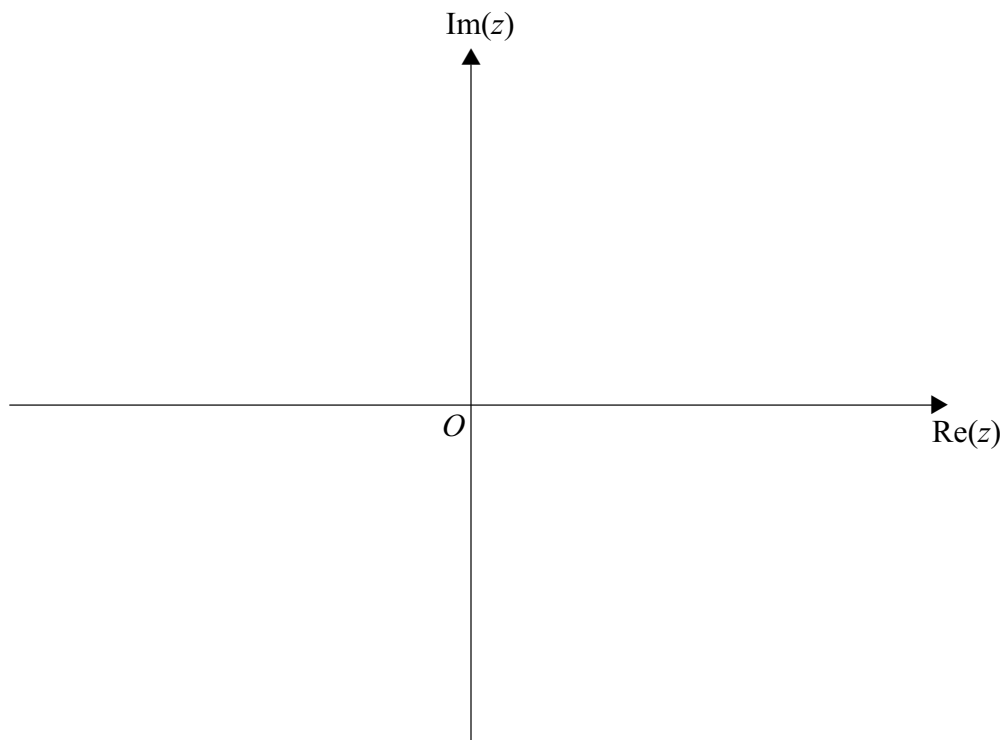
[4 marks]

QUESTION
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REFERENCE

Answer space for question 4



- 5 (a)** Find the modulus of the complex number $-4\sqrt{3} + 4i$, giving your answer as an integer. **[2 marks]**
- (b)** The locus of points, L , satisfies the equation $|z + 4\sqrt{3} - 4i| = 4$.
- (i)** Sketch the locus L on the Argand diagram below. **[3 marks]**
- (ii)** The complex number w lies on L so that $-\pi < \arg w \leq \pi$.
Find the least possible value of $\arg w$, giving your answer in terms of π . **[2 marks]**
- (c)** Solve the equation $z^3 = -4\sqrt{3} + 4i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. **[5 marks]**

QUESTION
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REFERENCE**Answer space for question 5****(b)(i)**

QUESTION
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REFERENCE

Answer space for question 5

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6 (a) Given that $y = \sinh x$, use the definition of $\sinh x$ in terms of e^x and e^{-x} to show that $x = \ln(y + \sqrt{y^2 + 1})$.

[4 marks]

(b) A curve has equation $y = 6 \cosh^2 x + 5 \sinh x$.

(i) Show that the curve has a single stationary point and find its x -coordinate, giving your answer in the form $\ln p$, where p is a rational number.

[5 marks]

(ii) The curve lies entirely above the x -axis. The region bounded by the curve, the coordinate axes and the line $x = \cosh^{-1} 2$ has area A .

Show that

$$A = a \cosh^{-1} 2 + b\sqrt{3} + c$$

where a , b and c are integers.

[5 marks]

QUESTION
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REFERENCE

Answer space for question 6



QUESTION
PART
REFERENCE

Answer space for question 6

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7 Given that $p \geq -1$, prove by induction that, for all integers $n \geq 1$,

$$(1 + p)^n \geq 1 + np$$

[6 marks]

QUESTION
PART
REFERENCE

Answer space for question 7



8 (a) By applying de Moivre's theorem to $(\cos \theta + i \sin \theta)^4$, where $\cos \theta \neq 0$, show that

$$(1 + i \tan \theta)^4 + (1 - i \tan \theta)^4 = \frac{2 \cos 4\theta}{\cos^4 \theta}$$

[3 marks]

(b) Hence show that $z = i \tan \frac{\pi}{8}$ satisfies the equation $(1 + z)^4 + (1 - z)^4 = 0$, and express the three other roots of this equation in the form $i \tan \phi$, where $0 < \phi < \pi$.

[2 marks]

(c) Use the results from part **(b)** to find the values of:

(i) $\tan^2 \frac{\pi}{8} \tan^2 \frac{3\pi}{8}$;

[4 marks]

(ii) $\tan^2 \frac{\pi}{8} + \tan^2 \frac{3\pi}{8}$.

[4 marks]

QUESTION
PART
REFERENCE

Answer space for question 8



QUESTION PART REFERENCE	Answer space for question 8

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