A-LEVEL
MATHEMATICS

MFP2 Further Pure 2
Report on the Examination

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General

The early questions were answered well by most students, allowing them to demonstrate their understanding of particular topics such as summation of series, multiplication of complex numbers, properties of roots of cubic equations, basic loci of complex numbers and hyperbolic functions. The questions without structure caused major problems to those who could not recall and apply basic techniques from A-Level Mathematics. Proofs were very rarely set out in a logical manner with a concluding statement. The presentation of solutions from a significant number of students was very disappointing and fell short of the standard expected for this level of examination. There is a growing tendency for students to use advanced calculators to find exact values of integrals and other expressions and then try to fool the examiner by working back from this answer. Unless the necessary steps of working were shown students were penalised.

Question 1

Part (a) was an easy starter and should have filled students with confidence. Occasionally, the poor use of brackets or careless algebra resulted in an incorrect value of A being obtained.

In part (b), the difference method was widely understood, and most students obtained a telescopic series that cancelled to $\frac{1}{3}-\frac{1}{203}$. After combining these terms as a single fraction, a few forgot to divide by 4, failing to see the link between parts (a) and (b). Some students obtained $\frac{50}{609}$ by typing the expression for the series into their calculator, but no marks were earned as they gave no evidence of using the method of differences.

Question 2

The structure in part (a) resulted in a high success rate with almost every student writing down the correct complex conjugate β , but a few weaker students were unable to simplify $(1+2\mathrm{i})(1-2\mathrm{i})$ correctly.

Those who lost marks in part (b) usually failed to consider the coefficient of z^3 when finding $\sum\alpha\beta$, but it was encouraging to see many correct answers for the third root.

In part (c), if students did not write $\alpha+\beta+\gamma=-\frac{p}{3}$ or $\alpha\beta\gamma=-\frac{q}{3}$, they could not obtain the correct values of p and q, but it was pleasing to see many fully correct answers for this part and for the question as a whole.

Question 3

In part (a), although the majority of students found the correct value of $\frac{\mathrm{d}y}{\mathrm{d}x}$, some made a sign error. Those with the correct derivative usually were able to proceed to the printed answer for the arc length, although some omitted the $\mathrm{d}x$ from the integral and lost a precious mark.

In part (b), this integral caused major difficulties for a large number of students and many scored zero. Too many split the integrand into $\frac{1}{1-x^2}+\frac{x^2}{1-x^2}$ but were then unable to integrate the second expression. The best students wrote the integrand as $\frac{2}{1-x^2}-1$ and obtained the correct answer in a couple of lines. Others used partial fractions to good effect writing the integrand as $\frac{1}{1-x}+\frac{1}{1+x}-1$ and then they too had a quick route to the correct value for s. Unfortunately, a large number of students carried out several pages of working without making any progress whatsoever, trying various rearrangements or substitutions to no avail. Some made the substitution $x=\sin u$ and rearranged the resulting integrand to $2\sec u-\cos u$; others let $x=\tanh y$, thus obtaining $2-\operatorname{sech}^2 y$ before using standard integrals to obtain the correct answer. There was an increase this year in students using calculators with an integration feature who were clearly working back from the answer of $\ln 7-\frac{3}{4}$, but students were penalised heavily if the correct working was not evident.

Question 4

In part (a), it was disappointing to see almost half of the students being unable to use the chain rule correctly. Many students simply wrote the derivative as $\frac{1}{1+3x}$, or equivalent. Since the formulae booklet was available, there was no excuse for errors such as $\frac{\mathrm{d}(\tan^{-1}u)}{\mathrm{d}x}=\frac{1}{1-u^2}$.

In part (b), most students who had a multiple of the correct answer in part (a) were able to obtain half the marks in this part by using antidifferentiation. There were a lot of correct answers to the integral but some made errors by not noticing that the vinculum was only above the 3.

Once again, many students tried to fool examiners by obtaining the exact value of the integral from their calculators and then worked backwards, fudging factors as they did so; this practice is something that should be strongly discouraged, even though using a calculator to check the final answer is a very good idea.

Question 5

In part (a), it was encouraging to see almost all the students finding the correct value of the modulus with the majority showing their working.

In part (b)(i), the vast majority of students drew a circle, most of which were drawn in the correct quadrant and touched the negative imaginary axis, though one or two circles were drawn in the third or fourth quadrant. It was necessary to identify the centre of the circle, and most indicated this by marking 4 on the real axis and $-4\sqrt{3}$ at the lowest point of the circle.

In part (b)(ii), no marks were given for simply writing $\frac{2\pi}{3}$ without supporting calculations; this usually involved considering appropriate right angled triangles with hypotenuse 8 and another side of length equal to the radius of the circle. Weaker students made no attempt at this part but many correct solutions were seen.

In part (c), far too many students were unable to find the correct argument of $-4\sqrt{3}+4\mathrm{i}$ and this resulted in a large number of students losing more than half the marks when finding the cube roots of this complex number. Those with the correct argument were usually successful in finding the three roots in the required form.

Question 6

It had been expected that all students would have seen the standard proof in part (a) during the teaching of hyperbolic functions, but the reality was that many students made little or no progress in this part of the question. The expected approach was via a quadratic equation in e^x but far too many students simply replaced the $\pm$ by $+$ when applying the quadratic equation formula, with only the very best students justifying the rejection of the negative sign at a later stage in their working. No doubt the printed answer prompted many to disregard the minus sign from the start. Quite a few used verification, substituting $\cosh x$ for $\sqrt{1+y^2}$ when $y=\sinh x$, but it was difficult to earn full marks using this approach.

In part (b)(i), most students found $\frac{\mathrm{d}y}{\mathrm{d}x}$ and extracted the common factor $\cosh x$. In order to score full marks, it was necessary to explain that no stationary points existed when $\cosh x = 0$, not simply cancelling $\cosh x$ from the equation, and then to show that the only stationary point occurred when $x=\ln\left(\frac{2}{3}\right)$. Those who converted their equation into exponentials from the start had a daunting task solving a quartic in order to find the stationary point; others converted to exponential functions after obtaining $\sinh x = -\frac{5}{12}$, instead of using the result in part (a), and so lost precious time and often failed to find the correct value of x in the logarithmic form requested.

Question 7

This was a challenging variation on previous proof-by-induction questions since it involved an inequality. A few students thought they needed to substitute $p=-1$ for the initial case. Other students missed out on the easy mark for verifying the inequality when $n=1$; they showed that the left hand side and right hand side were each equal to $1+p$ but failed to write a statement such as "the inequality is true when $n=1$". The crux of the proof lay in assuming that the result was true when $n=k$ and then **multiplying** both sides of the inequality by $1+p$. The resulting expression on the right hand side was then $(1+kp)(1+p)=1+kp+p+kp^2 \geq 1+(k+1)p$ since $kp^2 \geqslant 0$. Those students who tried to add terms to both sides of the inequality made little progress. Another approach was to consider $(1+p)^{k+1}=(1+p)^k+p(1+p)^k$, but many students fudged the result at this point since it required them to prove the result $p(1+p)^k \geqslant p$ rather than simply stating it as a fact. The most able students proved this result by considering positive, zero and negative values of p within the given interval for p.

Question 8

It was good to see not just the very able students scoring marks on this last question.

In part (a), a mark was available for all students who wrote down de Moivre's theorem for $n = 4$. Those students who also used $(\cos\theta - \mathrm{i}\sin\theta)^4 = \cos 4\theta - \mathrm{i}\sin 4\theta$ arrived at the printed result in just a few lines of working. Others took over a page or more to obtain an identity for $\cos 4\theta$ in terms of $\tan\theta$ and then attempted to establish the printed result. Some did this successfully but others simply tried to deceive the examiners that they had completed a proof.

In part (b), it was not enough to write something like "$\theta = \frac{\pi}{8}$, $\cos 4\theta = 0$ so satisfies equation".

Clear mention needed to be made of the root $\mathrm{i}\tan\frac{\pi}{8}$ and which equation was being satisfied.

Many students gave three other roots, but some of these were not in the required form and lost a mark.

The main problem in part (c) was that students did not answer the question but chose instead to use an equation in $\tan\theta$ found in their working for part (a). They needed to use their complex roots from part (b) and to simplify the equation in z, expressing it in the form $z^4 + 6z^2 + 1 = 0$. It was also necessary to state that $\tan\frac{7\pi}{8} = -\tan\frac{\pi}{8}$ and $\tan\frac{5\pi}{8} = -\tan\frac{3\pi}{8}$. Using the quartic equation, it was then fairly easy to find the product of the roots that gave the answer to part (c)(i). Using the sum of the squares of the four roots from part (b) or the six terms in the sum of the product of pairs of roots, it was then possible to find the answer to part (c)(ii).

The question could be solved holistically by considering the quadratic equation formed from letting $y = z^2$ and then using the sum and product of the roots of this quadratic.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.

Converting Marks into UMS marks

Convert raw marks into Uniform Mark Scale (UMS) marks by using the link below.

UMS conversion calculator