AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Mechanics 4

Wednesday 29 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A circular flywheel of radius 0.25 metres is rotating freely with an angular speed of $10\,\text{rad}\,\text{s}^{-1}$ about a fixed axis through its centre and perpendicular to its plane. The moment of inertia of the flywheel about this axis is $I\,\text{kg}\,\text{m}^2$. A particle of mass $0.6\,\text{kg}$, initially held at rest, sticks to a point on the circumference of the flywheel. The angular speed of the flywheel and the particle immediately after this happens is $8\,\text{rad}\,\text{s}^{-1}$.

Find the value of I.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE	**Answer space for question 1**

2 A light rod has its ends at the points $P(1, -5, 4)$ and $Q(3, -1, -2)$. A force $\mathbf{F}$ acts at the midpoint, M, of PQ, where $\mathbf{F} = 4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

(a) (i) Find $\overrightarrow{PM}$.

[1 mark]

(ii) Show that the magnitude of the moment of $\mathbf{F}$ about the point P is $n\sqrt{35}$, where n is an integer to be found.

[5 marks]

(b) Hence find the acute angle between $\mathbf{F}$ and the rod, giving your answer to the nearest degree.

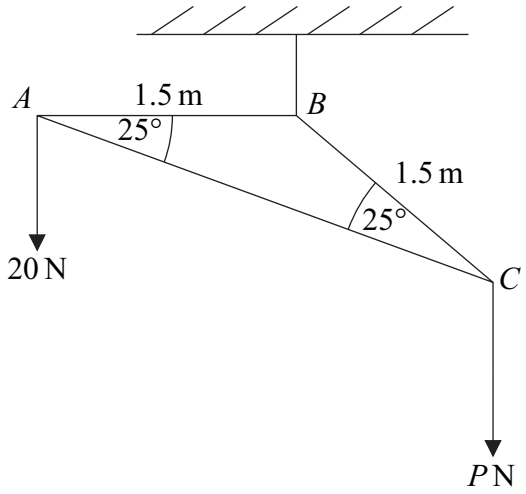
[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE	**Answer space for question 2**

3 A framework consists of three light inextensible smoothly jointed rods AB, BC and CA. Rods AB and BC each have length 1.5 metres and $\angle BAC = \angle ACB = 25°$. The framework is suspended from B by a vertical wire. A vertical force of magnitude 20 newtons acts on the framework at A and a vertical force of magnitude P newtons acts on the framework at C. The system is in equilibrium in a vertical plane, with AB horizontal, as shown in the diagram.



(a) By taking moments, find P.

[3 marks]

(b) Hence state the magnitude of the tension in the wire.

[1 mark]

(c) Find the magnitude of each of the forces in the rods AB, AC and BC, stating whether each rod is in tension or compression.

[7 marks]

QUESTION PART REFERENCE

Answer space for question 3

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4 The region bounded by the positive x-axis, the positive y-axis and the curve with equation $y = 1 - \frac{x^2}{4}$ is rotated through 2π radians about the x-axis to form a uniform solid.

(a) Show that the volume of the solid is $\frac{16\pi}{15}$ cubic units.

[4 marks]

(b) Find the x-coordinate of the centre of mass of the solid.

[5 marks]

(c) The solid is placed on a rough plane which is inclined at an angle θ to the horizontal. The solid rests in equilibrium with its flat circular face in contact with the plane.

Given that the solid is on the point of toppling, find the exact value of $\tan\theta$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 4

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5 A system of forces consists of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ which all act in the same plane.

The force $\mathbf{F}_1 = 2\mathbf{i} + \mathbf{j}$ and acts at the point with position vector $\mathbf{i} - \mathbf{j}$.

The force $\mathbf{F}_2 = 3\mathbf{i} - 2\mathbf{j}$ and acts at the point with position vector $4\mathbf{i} - 2\mathbf{j}$.

The force $\mathbf{F}_3 = -4\mathbf{i} + 5\mathbf{j}$ and acts at the point with position vector $-3\mathbf{i} + \mathbf{j}$.

The system is equivalent to a single force $\mathbf{F}$ acting at the point with position vector $-4\mathbf{i} + 6\mathbf{j}$ and a couple $\mathbf{G}$.

(a) Find $\mathbf{F}$.

[1 mark]

(b) Find the magnitude of $\mathbf{G}$.

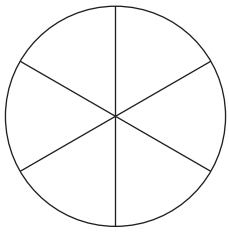
[8 marks]

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE	**Answer space for question 5**

6 A wheel can be modelled as a uniform hoop, of mass $6m$ and radius $2a$, which is attached to six equally spaced uniform rods, each of mass m and length $2a$, as shown in the diagram.



The wheel can rotate freely about a fixed axis perpendicular to its plane and through the centre of the wheel.

(a) Show that the moment of inertia of the wheel about its axis is $32ma^2$.

[3 marks]

(b) A light inextensible string is wrapped several times around the rim of the wheel, and a particle of mass $4m$ is attached to the free end of the string. The system is released from rest with the particle hanging freely. After time t, the wheel has turned through θ radians.

Assume that no slipping occurs during the motion.

(i) Find an expression for the angular acceleration of the wheel, in terms of a and g.

[6 marks]

(ii) Find an expression for the tension in the string, in terms of m and g.

[2 marks]

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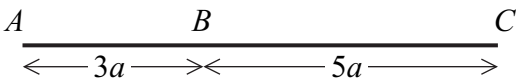
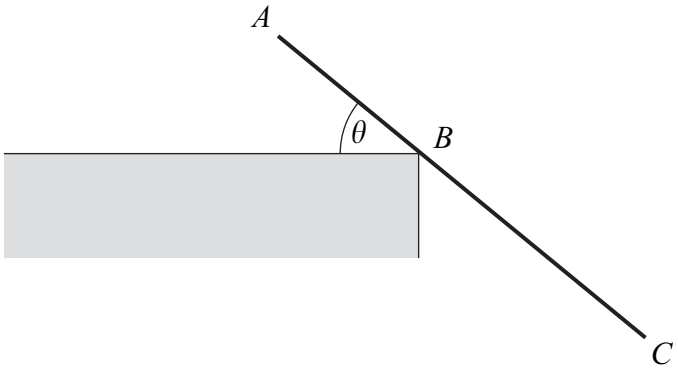
7 A uniform straight rod AC, of mass $2m$, has length $8a$. The point B lies on AC such that $AB = 3a$ and $BC = 5a$.



(a) Find the moment of inertia of the rod about an axis through the point B which is perpendicular to the rod.

[3 marks]

(b) The rod is held in equilibrium on a horizontal table with the rod perpendicular to the edge of the table. The section BC projects over the edge of the table. The rod is released from rest and begins to rotate in a vertical plane about the point B. The rod remains in contact with the edge of the table.



When the rod has turned through an angle θ, and has not yet begun to slip:

(i) show that the angular speed of the rod is $\sqrt{\dfrac{6g\sin\theta}{19a}}$;

[4 marks]

(ii) find an expression, in terms of a, g and θ, for the angular acceleration of the rod;

[2 marks]

(c) show that the rod begins to slip when $\tan\theta = \dfrac{16}{25}\mu$, where μ is the coefficient of friction between the table and the rod.

[8 marks]

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END OF QUESTIONS