Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

A-level
MATHEMATICS

Unit Mechanics 5

Friday 17 June 2016   Afternoon   Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A simple pendulum consists of a light inextensible string of length x metres and a small spherical bob. The pendulum is released from rest with the string taut. When it is released, the angle between the string and the vertical is $\frac{\pi}{20}$ radians. The period of the motion of the pendulum is $\frac{4\pi}{7}$ seconds.

(a) Find x.

[3 marks]

(b) Find the time, from release, that it takes for the pendulum to move through an angle of $\frac{3\pi}{40}$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 1

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QUESTION PART REFERENCE	Answer space for question 1

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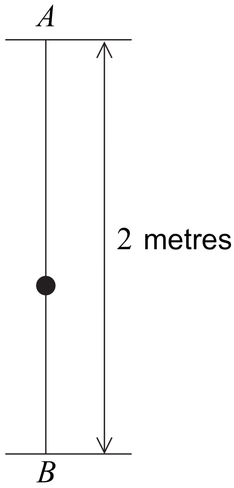
2 A particle of mass $2.6\,\text{kg}$ is attached to two elastic strings. The end of one string is attached to the point A, and the end of the other string is attached to the point B, which is 2 metres directly below A, as shown in the diagram.



The upper string has natural length 0.5 metres and modulus of elasticity $4g\,\text{N}$.
The lower string has natural length 0.4 metres and modulus of elasticity $3g\,\text{N}$.

The particle is in equilibrium at the point O, which is at a distance d metres below A.

(a) Find d.

[5 marks]

(b) The particle is pulled down and then released from rest 0.2 metres below O.

(i) Find an expression for the acceleration of the particle when it is x metres below O.

[5 marks]

(ii) Find the period of the motion.

[2 marks]

(iii) Find the maximum speed of the particle during the motion.

[2 marks]

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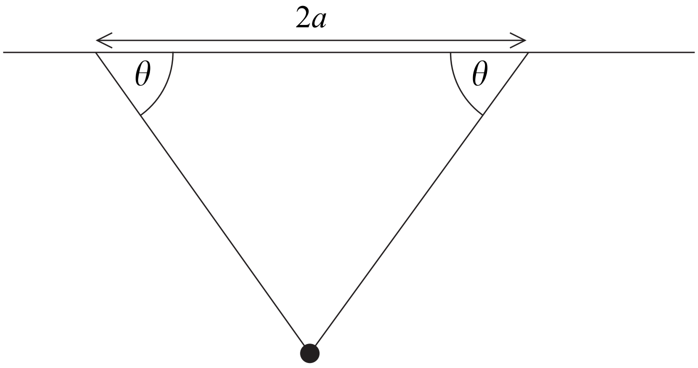
3 A particle of mass m kg is suspended by two elastic strings. One end of each string is attached to the particle and the other ends are attached to fixed points, at the same level, which are a distance of $2a$ metres apart. Both strings have natural length a metres and modulus of elasticity $2mg$ newtons. The angle between each string and the horizontal is θ radians. The diagram shows the strings and the particle.



Gravitational potential energy is taken as zero at the level of the tops of the strings.

(a) Find the total potential energy of the system, V, in terms of m, g, a and θ.

[3 marks]

(b) Use an energy method to show that there is a position of equilibrium for a value of θ that is between 0.7 and 0.8 .

[6 marks]

(c) Given that there is only one position of equilibrium, state, with a reason, whether this is a position of stable or unstable equilibrium.

[2 marks]

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4 A particle, of mass m, moves on a curve and at time t has polar coordinates $(r,\ \theta)$, with respect to an origin O. The particle is subjected to a single force directed towards O of magnitude $\frac{km}{r^2}$, where k is a constant.

(a) Show that $\dot{\theta}$ is proportional to $\frac{1}{r^2}$.

[2 marks]

(b) Given that $\ddot{r} = 0$ when $r = 2a$, find $\ddot{r}$ in terms of k, r and a.

[5 marks]

(c) Given that $\dot{r} = U$ when $r = a$, find the magnitude of $\dot{r}$ when $r = 2a$ in terms of U, k and a.

[5 marks]

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5 A uniform metal bar, of mass m, is held at rest in a horizontal position. The ends of the bar are attached to identical light elastic strings, which each have stiffness $2m$. The strings are also attached to fixed points that are directly above the ends of the bar. A damping device is also connected to the bar.

The bar is released from rest with the strings vertical and at their natural length. As the bar falls, it remains horizontal and the damping device exerts an upward force of magnitude cmv on the centre of the bar, where c is a constant, in appropriate units, and v is the speed of the bar.

The motion of the bar is critically damped.

At time t after the bar has been released, the displacement of the bar below its initial position is x.

(a) Show that $c = 4$.

[4 marks]

(b) Find an expression for x in terms of g and t.

[8 marks]

(c) Find the value of x as t tends to infinity.

[1 mark]

(d) Find the maximum speed of the bar.

[4 marks]

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6 A scientist projects a small sphere of ice vertically upwards into the air. As it rises, the mass of the sphere increases as the water vapour in the air condenses on the sphere.

Assume that there is no air resistance acting on the sphere.

At time t, the mass of the sphere is m and its velocity is v.

At time $t = 0$, $v = U$ and $m = M$, where M and U are constants. Assume that the mass of the sphere increases at a rate that is equal to λm, where λ is a constant.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\lambda v - g$$

[4 marks]

(b) Find, in terms of λ, g and U, the time when the sphere reaches its maximum height.

[6 marks]

(c) Find the mass of the sphere when it is at its maximum height in terms of M, U, g and λ.

[3 marks]

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END OF QUESTIONS