# AS

# Mathematics

MPC2 – Pure Core 2
Mark scheme

6360
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Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| −x EE | deduct x marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.
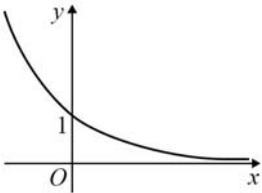
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

| Q1 | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **(a)** | $\dfrac{1}{x^2} = x^{-2}$ | B1 | | $\dfrac{1}{x^2} = x^{-2}$ seen, or used. PI by correct integration of $\dfrac{36}{x^2}$. |
| | $\displaystyle\int\left(\dfrac{36}{x^2} + ax\right)\,\mathrm{d}x = \left[-\dfrac{36}{x} + \dfrac{ax^2}{2}\right]\ (+k)$ | M1 | | Correct integration of either $\dfrac{36}{x^2}$ or $ax$. |
| | | A1 | 3 | Correct integration of both terms, ACF, accept unsimplified. Condone missing $+k$. |
| **(b)** | $\displaystyle\int_1^3\left(\dfrac{36}{x^2} + ax\right)\,\mathrm{d}x =$ $\left(-\dfrac{36}{3} + \dfrac{9a}{2}\right) - \left(-36 + \dfrac{a}{2}\right)$ | M1 | | Attempt to find $F(3) - F(1)$, following attempt at integration . (**M0** if $F(x) = \dfrac{36}{x^2} + ax$ or if uses an $F(x)$ which contains part of the given integrand) $-2$ |
| | $24 + 4a = 16$, $\quad (a =) -2$ | A1 | 2 | **NMS** scores 0/2 |
| | **Total** | | **5** | |
| **(b)** | Using the correct answer to (a), the example $-\dfrac{36}{3} + \dfrac{9a}{2} + 36 + \dfrac{a}{2}$ would get the M1 in (b) if no further evidence suggested the cand was working with $F(3)+F(1)$. | | | |

| Q2 | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **(a)** |  | B1 | | Only one $y$- intercept, marked as 1 or coordinates $(0,1)$ stated or '$y = 1$ when $x = 0$' |
| | | B1 | | Correct graph having no other 'crossing point' on either axis. |
| | | | 2 | |
| **(b)** | $x \log 0.2 = \log 4$ | M1 | | OE eg $(x =) \log_{0.2} 4$ |
| | $(x =) -0.861(35...) = -0.861$ (to 3sf) | A1 | 2 | Condone > 3sf, rounded or truncated. If use of logarithms not explicitly seen then score 0/2 |
| **(c)** | Reflection in the $y$-axis. | E1 | 1 | OE **E0** if more than one transformation |
| | **Total** | | **5** | |
| **(a)** | For large positive $x$-values, graph in $1^{st}$ quadrant, if not very close to the $x$-axis, must be approaching the horizontal which is clearly closer to the $x$-axis than through c's $y$-intercept | | | |
| **(b)** | OEs eg $-x \log 5 = \log 4$ ; $x \log 2 = \log 4 + x \log 10$ ; $x(1 - \log_2 10) = 2$ ; $(x - 2)\log 2 = x \log 10$ | | | |

| Q3 | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **(a)** | $(\dfrac{dy}{dx} =)\ \dfrac{6}{2}x^{-0.5} - 1\ = 3x^{-0.5} - 1$ | **B2,1** | **2** | ACF. If not B2, award B1 for correct differentiation of either $6x^{1/2}$ or $-x-3$ |
| **(b)** | $3x^{-0.5} - 1 = 0$ | **M1** | | Evidence of c's $\dfrac{dy}{dx}$ equated to 0 to form an equation in $x$. |
| | $3x^{-0.5} = 1,\quad x = 9$ | **A1F** | | Only ft if c's $\dfrac{dy}{dx} = ax^{-0.5} - 1$ ie $x_M = a^2$ |
| | ($y$-coordinate of $M$ is) 6 | **A1** | **3** | NMS scores 0/3 |
| **(c)** | At $P(4,5)$  $\dfrac{dy}{dx} = 3(4)^{-0.5} - 1$  (=0.5) | **M1** | | Attempt to find c's $\dfrac{dy}{dx}$ when $x = 4$. |
| | Gradient of normal $= -2$ | **m1** | | $m \times m' = -1$ used |
| | Eqn of normal  $y - 5 = -2(x - 4)$ | **A1** | **3** | ACF eg $y + 2x = 13$ |
| **(d)** | Translated normal: $y - 5 = -2(x - k - 4)$ | **M1** | | Either $x \rightarrow x - k$ or $x \rightarrow x + k$ with no change to $y$ in cand's eqn of normal seen or used |
| | Passes through $M$ (9, 6) so $6 - 5 = -2(9 - k - 4)$ | **m1** | | Subst of c's $M$ coordinates (both +'ve) into cand's eqn of normal with $x \rightarrow x - k$ and no change to $y$. |
| | $k = 5.5$ | **A1** | **3** | A correct value of $k$ with no errors seen. |
| **ALTn 1** | On normal, when $y=6$, $6 - 5 = -2(x - 4)$ | **(M1)** | | Sub answer (**b**) in answer (**c**): ie attempt to find $x_N$, the $x$-coordinate of point on c's normal in part (**c**) with same $y$-coord as c's part (**b**) answer. |
| | $x = 3.5;\ \ 3.5 + k = x_M = 9$ | **(m1)** | | (c's $x_N$) $+ k =$ (c's $x_M$) provided c's $x_M$ is $> 0$ |
| | $k = 5.5$ | **(A1)** | **(3)** | A correct value of $k$ with no errors seen. |
| **ALTn 2** | Line through $M$ parallel to normal at $P$ has equation  $y - 6 = -2(x - 9)$ | **(M1)** | | Correct ft eqn using c's $M$ coords, both>0 and c's normal at $P$ |
| | eg Using $y=0$, for normal at $P$, (6.5, 0) and for parallel line through $M$, (12,0) ($k =$) $12 - 6.5$ | **(m1)** | | For any single value of $y$, finding the $x$ coord of the point on both normal at $P$ and this parallel line through $M$ and then subtracting these $x$ coords in correct order |
| | $k = 5.5$ | **(A1)** | **(3)** | A correct value of $k$ with no errors seen. |
| | **Total** | | **11** | |

| Q4 | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **(a)** | $[S_{21}=]$ $\dfrac{21}{2}\big[2a + (21-1)d\big]$ | **M1** | | $\dfrac{21}{2}\big[2a + (21-1)d\big]$ OE |
| | $\dfrac{21}{2}\big[2a + 20d\big] = 168$ | **m1** | | Forming correct eqn |
| | $21(a+10d)=168 \implies a + 10d = 8$ | **A1** | **3** | AG $a + 10d = 8$ convincingly obtained with intermediate step shown eg $21(2a+20d)=168\times2$; $2a+20d=8\times2$ |
| **(b) (i)** | $a + d + a + 2d = 50 \quad (2a + 3d = 50)$ | **M1** | | $a + d + a + 2d = 50$ OE in terms of $a$ and $d$ |
| | $2(8 - 10d) + 3d = 50$ | **m1** | | Solving $a + 10d = 8$ OE simultaneously with c's $2a + 3d = 50$ OE as far as correctly eliminating either $a$ or $d$. PI by correct values for both $d$ and either $a$ or $u_{12}$. |
| | $d = -2$ ; $a = 28$ or $12^{\text{th}}$ term $= 8+d$ | **A1** | | $d = -2$ **and** either $a = 28$ or $8+d$ seen or used in part **(b)(i)**. |
| | $(u_{12} =)\ 6$ | **A1** | **4** | **NMS** scores 4/4 unless FIW |
| **(b)(ii)** | $\displaystyle\sum_{n=4}^{21} u_n = \sum_{n=1}^{21} u_n - \sum_{n=1}^{3} u_n$ | **M1** | | $\displaystyle\sum_{n=4}^{21} u_n = \sum_{n=1}^{21} u_n - \sum_{n=1}^{3} u_n$ OE eg $S_{21} - S_3$ stated or used |
| | $= 168 - (a + 50)$ or $168 - 1.5(2a + 2d)$ | **A1F** | | OE. If numerical form only then ft on c's non-zero values for $a$ and $d$. |
| | $= 90$ | **A1** | **3** | 90 NMS 90 scores 3/3 unless FIW. **SC** If 0/3 award 1 mark for answer 68 |
| | **Altn.** $A = a + 3d$ , $N=18$, $\displaystyle\sum_{n=4}^{21} u_n = \dfrac{18}{2}\big[2(a + 3d) + (18 - 1)d\big]$ | **(M1)** | | OE Seen or used. {OEs include $9(2a+23d) = 18a + 207d$} |
| | $= \dfrac{18}{2}\big[2(a + 3 \times (-2)) + (18 - 1)(-2)\big]$ | **(A1F)** | | Ft on c's $d \neq 0$ value in **(b)(i)**; if expression is entirely numerical, also ft on c's $a \neq 0$ value |
| | $\displaystyle\sum_{n=4}^{21} u_n = 90$ | **(A1)** | **(3)** | 90 |
| | **Total** | | **10** | |

| Q5 | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **(a)** | $h = 3$ | **B1** | | $h = 3$ OE stated or used. (PI by $x$-values 2, 5, 8, 11 provided no contradiction) |
| | $f(x) = \sqrt{x^2 + 9}$ <br><br> $I \approx \dfrac{h}{2}$ {f(2)+f(11)+2[f(5)+f(8)]} | **M1** | | $h/2${f(2)+f(11)+2[f(5)+f(8)]} seen or used OE summing of areas of the 'trapezia'.. ( **M0** if using an incorrect $f(x)$ ) |
| | $\dfrac{h}{2}$ with{} $= \sqrt{13} + \sqrt{130} + 2\left(\sqrt{34} + \sqrt{73}\right)$ <br><br> $= \dfrac{h}{2}$ { 3.60(5..)+11.4(0..)+ <br><br> $\qquad$ +2[5.83(09..)+8.54(4..)]} <br><br> $= \dfrac{h}{2}$ { 15.0(07..)+28.7(499..)]} | **A1** | | OE Accept 3sf or better evidence for surds. Can be implied by later <u>correct</u> work provided >1 term or a single term for I which rounds to 65.6 |
| | $(I \approx 1.5 \times 43.7(57..)) \quad (= 65.63(58\ldots)$ <br> $I = 65.6$ (to 1 dp) | **A1** | **4** | CAO Must be 65.6 <br> **SC** 4 strips used:**Max** B0M1A0; 65.5 A1 |
| **(b)(i)** | Translation $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ | **E2,1,0** | **2** | **E2**: 'translat…' and $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ ... If not **E2** award **E1** for 'translat… in $y$-dir' OE. <br> **E0** if more than one transformation |
| **(b)(ii)** | Stretch (**I**) in $x$-direction (**II**) <br> scale factor $\dfrac{1}{3}$ (**III**) | **E2,1,0** | **2** | Need (**I**) and (**II**) and (**III**) for **E2**. If not **E2** then award **E1** for seeing $3\sqrt{x^2 + 1} = \sqrt{(3x)^2 + 9}$ OE together with (**I**) and either (**II**) or (**III**) <br> **E0** if more than one transformation |
| | **Total** | | **8** | |
| **(a)** | For guidance, separate trapezia 14.1(5…)+21.5(6…)+29.9(18..) | | | |
| **(b)(i)** | NB For E2 the vector must be written in column format | | | |
| **(b)(ii)** | Accept 'horizontal…' OE for '$x$-direction' | | | |

| Q6 | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **(a)** | $5^2 = 8^2 + 9^2 - 2(8)(9)\cos\theta$ | **M1** | | Cosine rule used correctly. Accept eg $A$ for $\theta$ if intention is clear. PI by next line |
| | $\cos\theta = \dfrac{8^2 + 9^2 - 5^2}{2(8)(9)} \quad \left(= \dfrac{120}{144} = \dfrac{5}{6}\right)$ | **m1** | | Allow one sign slip in rearrangement from a correct M1 line |
| | $\theta = 0.5856(855..) = 0.586$ to 3sf | **A1** | **3** | AG Must see more than 3sf for the angle before seeing printed value 0.586 |
| **(b)** | $(\text{Area of triangle}) = \dfrac{1}{2} \times 9 \times 8 \sin\theta$ | **M1** | | OE Correct value or correct expression involving no unknown values. eg $\sqrt{11(11-9)(11-8)(11-5)}$ |
| | $= 36\sin\theta = 19.9 \ (\text{cm}^2)$ | **A1** | | If >3sf accept a value from 19.89 to 19.91 inclusive. |
| | | | **2** | NMS: Award 2 marks for 19.9 or 'better' |
| **(c)** | $(\text{Area of sector} =) \ \dfrac{1}{2}r^2\theta$ | **M1** | | $\dfrac{1}{2}r^2\theta$ seen, or used, for sector area |
| | $\dfrac{1}{2}r^2\theta = 36\sin\theta - \dfrac{1}{2}r^2\theta$ | **m1** | | OE Ft c's answer to **(b)** if incorrect eg $\dfrac{1}{2}r^2\theta = \dfrac{"19.9"}{2}$ |
| | $r^2 = \dfrac{36\sin\theta}{\theta} \approx 34, \quad r = 5.83$ | **A1** | | PI by later work. Accept 5.82 to 5.83 inclusive. |
| | $(\text{Arc length} =) \ r\theta$ Perimeter (of shaded shape) | **M1** | | $r\theta$ seen, or used, for arc length |
| | $= r\theta + 5 + 8 - r + 9 - r$ | **m1** | | Correct expression, seen or used, for the required perimeter. |
| | $= 22 + r(\theta - 2)$ $= 13.8 \ (\text{cm})$ to 3sf | **A1** | **6** | CAO must be 13.8 |
| | **Total** | | **11** | |
| **(a)** | Accept 0.5856 or 0.5857 or AWFW 0.5856 to 0.5857 as evidence for the A1 | | | |
| **(b)** | NMS: 'better' means 'value from 19.89 to 19.91 inclusive' | | | |

| Q7 | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **(a)** | $p = -10$;<br>$q = 40$;<br>$r = -80$ | **B1**<br>**B1**<br>**B1** | **3** | Accept correct embedded values for $p$, $q$ and $r$ within the expansion |
| **(b)** | $(2 + x)^7 = \ldots\ldots\ldots + mx^5 + nx^6 + x^7$ | **M1** | | Attempting to find **at least two of** $x^5$ term , $x^6$ term , $x^7$ term in the expansion of $(2 + x)^7$ |
| | $m = 84$ , $n = 14$ | **A1** | | Either correct. (M1 must be scored). PI by later correct work |
| | Coefficients of $x^{10}$ terms in expansion of $(1 - 2x)^5(2 + x)^7$ are $-32m + 80n + r$ | **m1** | | Identifying at least two of the three products $-32m$, $80n$, $r$ that give $x^{10}$ terms |
| | Coeff. of $x^{10} = (-32)(84) + (80)(14) + r$<br>$= -2688 + 1120 + r \quad = -1568 + r$ | **A1F** | | **Only** ft c's value of $r$ in **(a)**. If not shown in any of these forms, can be implied by final answer which matches correct evaluation of $(-1568 + c$'s $r)$ |
| | Coeff. of $x^{10} = -1648$ | **A1** | **5** | $-1648$ or left as ' $-1648\,x^{10}$ '.<br>Ignore other powers of $x$ terms |
| | **Total** | | **8** | |

| Q8 | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **(a) (i)** | $\dfrac{4\sin x}{\cos x} + \dfrac{5\cos x}{\cos x} = 0$ ; $4\tan x + 5 = 0$ | **M1** | | $\dfrac{\sin x}{\cos x} = \tan x$ clearly used to obtain a linear equation in $\tan x$. |
| | $\tan x = -\dfrac{5}{4} \quad (= -1.25)$ | **A1** | **2** | −1.25 OE<br>**NMS** mark as **B2** or **B0** |
| **(a)(ii)** | $\tan x = 1$, $\tan x = -1.25$ | **B1F** | | 1 and c's answer to **(a)(i)** vals for tan $(x)$<br>PI by a correct angle for both tan values |
| | $(x =)\ 45°,\ 225°,\ 129°,\ 309°$ | **B2, 1** | | **B2** 45, 225, AWRT 129, AWRT 309<br>If not **B2** award **B1** for at least two correct.<br>If more than four values in given interval, deduct 1 mark for each extra to min of B0. |
| | | | **3** | Ignore values outside $0° \leq x \leq 360°$ |
| **(b)** | $\dfrac{16 + 9\sin^2\theta}{5 - 3\cos\theta} = \dfrac{16 + 9(1 - \cos^2\theta)}{5 - 3\cos\theta}$ | **M1** | | Replacing $\sin^2\theta$ by $1 - \cos^2\theta$ in given expression or replacing $\cos^2\theta$ by $1 - \sin^2\theta$ in term $\pm 3q\cos^2\theta$. |
| | $= \dfrac{(5 - 3\cos\theta)(5 + 3\cos\theta)}{5 - 3\cos\theta}$ | **A1** | | Or any two of<br>$5p - 3q = 16$, $5q - 3p = 0$, $3q = 9$ |
| | $= 5 + 3\cos\theta$ | **A1** | | CSO. Or $q=3$, $p=5$ **and** checking remaining eqn is satisfied. |
| | Least possible value is 2 and occurs at $\theta = \pi$ | **B1F** | **4** | Ft on c's $p$ and $q$ non zero values.<br>{If $q>0$, least val=$p-q$  $\theta = \pi$}<br>{If $q<0$, least val=$p+q$ at $\theta = 0$}<br>Ignore values of $\theta$ outside given interval |
| | **Total** | | **9** | |

| | |
|---|---|
| **(a)(i)** | $\tan x = -1.25$ OE with no errors seen scores 2 marks. Methods involving squaring and $\tan x \neq -1.25$ OE, must give reasons for discounting certain solns before M1A0 scored |
| **(b)** | Multiplying the numerator and denominator of the given expression by $5 + 3\cos\theta$ does not score until the correct relevant identity has been used (M1); the 1st A1 will then be awarded when the rational expression has been written in a correct form with terms which can be cancelled legitimately e.g.<br>$\dfrac{(16 + 9\sin^2\theta)(5 + 3\cos\theta)}{(16 + 9\sin^2\theta)}$ |

| Q9 | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **(a)** | $c = 3^m, \quad d = 27^n$ | **M1** | | Either $c = 3^m$ or $d = 27^n$ seen or used |
| | $d = 3^{3n}, \quad d^2 = 3^{6n}$ | **A1** | | Either $d = 3^{3n}$ or $d^2 = 3^{6n}$ seen or used |
| | $\sqrt{c} = 3^{0.5m}$ | **A1** | | $\sqrt{c} = 3^{0.5m}$ seen or used |
| | $\dfrac{\sqrt{c}}{d^2} = 3^{\frac{m}{2} - 6n}$ | **A1** | **4** | $\dfrac{\sqrt{c}}{d^2} = 3^{\frac{m}{2} - 6n}$ OE expression for $y$ in terms of $m$ and $n$. |
| **Altn** | $\dfrac{1}{2}\log_3 c - 2\log_3 d = y\log_3 3$ | **(M1)** | | A correct expression in $y$ in terms of logs to base 3 or base 27 where no further log laws are required |
| | $\log_3 d = \dfrac{\log_{27} d}{\log_{27} 3}$ | **(A1)** | | $\log_3 d = \dfrac{\log_{27} d}{\log_{27} 3}$ or $\log_{27} c = \dfrac{\log_3 c}{\log_3 27}$ seen or used |
| | $\log_{27} 3 = \dfrac{1}{3}$ | **(A1)** | | $\log_{27} 3 = \dfrac{1}{3}$ or $\log_3 27 = 3$ seen or used |
| | $y = \dfrac{1}{2}m - 6n$ | **(A1)** | **(4)** | Correct expression for $y$ in terms of $m$ and $n$ OE eg $\dfrac{\sqrt{c}}{d^2} = 3^{\frac{m}{2} - 6n}$ |
| **(b)** | $1 = \log_4 4$ | **B1** | | $1 = \log_4 4$ seen or used at any stage. |
| | $\log_4 (2x+3)(2x+15) = 1 + \log_4 (14x+5)$ $\log_4 (2x+3)(2x+15) = \log_4 4(14x+5)$ | **M1** | | Applying a log law correctly to two correct log terms. [Condone missing base] NB: Lots of other possibilities after correct rearrangements! PI by $(2x+3)(2x+15) = 4(14x+5)$ OE with no errors seen |
| | $(2x+3)(2x+15) = 4(14x+5)$ | **A1** | | OE eqn with logs eliminated in a correct manner |
| | $4x^2 + 36x + 45 = 56x + 20$ $4x^2 - 20x + 25 = 0; \quad (2x-5)^2 = 0$ Only one solution  2.5 | **A1** | **4** | Must include statement and correct value |
| | **Total** | | **8** | |
| **(b)** | ......... $4x^2 - 20x + 25 = 0$, $b^2 - 4ac = 400 - 400 = 0$, only one soln  (which is $-\dfrac{b}{2a}$)  2.5 | | | |