AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Statistics 3

Monday 27 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 In advance of a referendum on independence, the regional assembly of an eastern province of a particular country carried out an opinion poll to assess the strength of the 'Yes' vote.

Of the 480 men polled, 264 indicated that they intended to vote 'Yes', and of the 500 women polled, 220 indicated that they intended to vote 'Yes'.

(a) Construct an approximate 95% confidence interval for the difference between the proportion of men who intend to vote 'Yes' and the proportion of women who intend to vote 'Yes'.

[6 marks]

(b) Comment on a claim that, in the forthcoming referendum, the percentage of men voting 'Yes' will exceed the percentage of women voting 'Yes' by at least 2.5 per cent. Justify your answer.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

QUESTION PART REFERENCE	**Answer space for question 1**

2 A plane flies regularly between airports D and T with an intermediate stop at airport M. The time of the plane's departure from, or arrival at, each airport is classified as either early, on time, or late.

On 90% of flights, the plane departs from D on time, and on 10% of flights, it departs from D late.

Of those flights that depart from D on time, 65% then depart from M on time and 35% depart from M late.

Of those flights that depart from D late, 15% then depart from M on time and 85% depart from M late.

Any flight that departs from M on time has probability 0.25 of arriving at T early, probability 0.60 of arriving at T on time and probability 0.15 of arriving at T late.

Any flight that departs from M late has probability 0.10 of arriving at T early, probability 0.20 of arriving at T on time and probability 0.70 of arriving at T late.

(a) Represent this information by a tree diagram on which labels and percentages or probabilities are shown.

[3 marks]

(b) Hence, or otherwise, calculate the probability that the plane:

(i) arrives at T on time;

(ii) arrives at T on time, given that it departed from D on time;

(iii) does not arrive at T late, given that it departed from D on time;

(iv) does not arrive at T late, given that it departed from M on time.

[8 marks]

(c) Three independent flights of the plane depart from D on time.

Calculate the probability that two flights arrive at T on time and that one flight arrives at T early.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

QUESTION PART REFERENCE	**Answer space for question 2**

QUESTION PART REFERENCE	**Answer space for question 2**

QUESTION PART REFERENCE	**Answer space for question 2**

3 Car parking in a market town's high street was, until 31 May 2014, limited to one hour free of charge between $8\,\text{am}$ and $6\,\text{pm}$. Records show that, during a period of 30 days prior to this date, a total of 315 penalty tickets were issued.

Car parking in the high street later became limited to thirty minutes free of charge between $8\,\text{am}$ and $6\,\text{pm}$. A subsequent investigation revealed that, during a period of 60 days from 1 October 2014, a total of 747 penalty tickets were issued.

The daily numbers of penalty tickets issued may be modelled by independent Poisson distributions with means λ_A until 31 May 2014 and λ_B from 1 October 2014.

Investigate, at the 1% level of significance, a claim by traders on the high street that $\lambda_B > \lambda_A$.

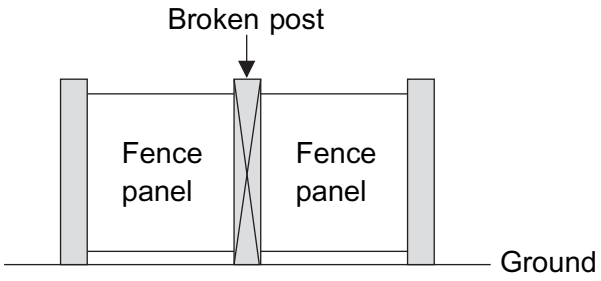
[7 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE	**Answer space for question 3**

4 Ben is a fencing contractor who is often required to repair a garden fence by replacing a broken post between fence panels, as illustrated.



The tasks involved are as follows.

U : detach the two fence panels from the broken post
V : remove the broken post
W : insert a new post
X : attach the two fence panels to the new post

The mean and the standard deviation of the time, in minutes, for each of these tasks are shown in the table.

Task	Mean	Standard deviation
U	15	5
V	40	15
W	75	20
X	20	10

The random variables U, V, W and X are pairwise independent, except for V and W for which $\rho_{VW} = 0.25$.

(a) Determine values for the mean and the **variance** of:

(i) $R = U + X$;

(ii) $F = V + W$;

(iii) $T = R + F$;

(iv) $D = W - V$.

[8 marks]

(b) Assuming that each of R, F, T and D is approximately normally distributed, determine the probability that:

(i) the total time taken by Ben to repair a garden fence is less than 3 hours;

(ii) the time taken by Ben to insert a new post is at least 1 hour more than the time taken by him to remove the broken post.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

QUESTION PART REFERENCE	**Answer space for question 4**

QUESTION PART REFERENCE	**Answer space for question 4**

5 (a) The random variable X, which has distribution $\mathrm{N}(\mu_X, \sigma^2)$, is independent of the random variable Y, which has distribution $\mathrm{N}(\mu_Y, \sigma^2)$.

In order to test $\mathrm{H}_0: \mu_X = 1.5\mu_Y$, samples of size n are taken on each of X and Y and the random variable $\overline{D}$ is defined as

$$\overline{D} = \overline{X} - 1.5\overline{Y}$$

State the distribution of $\overline{D}$ assuming that H_0 is true.

[4 marks]

(b) A machine that fills bags with rice delivers weights that are normally distributed with a standard deviation of 4.5 grams.

The machine fills two sizes of bags: large and extra-large.

The mean weight of rice in a random sample of 50 large bags is 1509 grams.

The mean weight of rice in an independent random sample of 50 extra-large bags is 2261 grams.

Test, at the 5% level of significance, the claim that, on average, the rice in an extra-large bag is $1\frac{1}{2}$ times as heavy as that in a large bag.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE	**Answer space for question 5**

6 (a) The discrete random variable X has probability distribution given by

$$\mathrm{P}(X = x) = \begin{cases} \dfrac{\mathrm{e}^{-\lambda}\lambda^{x}}{x!} & x = 0, 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases}$$

Show that $\mathrm{E}(X) = \lambda$ and that $\mathrm{Var}(X) = \lambda$.

[7 marks]

(b) In light-weight chain, faults occur randomly and independently, and at a constant average rate of 0.075 per metre.

(i) Calculate the probability that there are no faults in a 10-metre length of this chain.

[2 marks]

(ii) Use a distributional approximation to estimate the probability that, in a 500-metre reel of light-weight chain, there are:

(A) fewer than 30 faults;

(B) at least 35 faults but at most 45 faults.

[7 marks]

(c) As part of an investigation into the quality of a new design of medium-weight chain, a sample of **fifty** 10-metre lengths was selected.

Subsequent analysis revealed a total of 49 faults.

Assuming that faults occur randomly and independently, and at a constant average rate, construct an approximate 98% confidence interval for the average number of faults **per metre**.

[6 marks]

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE	**Answer space for question 6**

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED