Please write clearly in block capitals.

Centre number						Candidate number				

Surname ________________

Forename(s) ________________

Candidate signature ________________

A-level
MATHEMATICS

Unit Further Pure 2

Friday 23 June 2017 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from **the** booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
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6	
7	
8	
TOTAL	

J U N 1 7 M F P 2 0 1

IB/G/Jun17/E7

MFP2

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Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Express $\dfrac{r+1}{(2r+1)(2r+3)}$ in partial fractions.

[2 marks]

(b) Use the method of differences to find $\displaystyle\sum_{r=1}^{n}\frac{(-1)^{r+1}(r+1)}{(2r+1)(2r+3)}$.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 1

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QUESTION PART REFERENCE	Answer space for question 1

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2 The cubic equation

$$z^3 + (6-3\mathrm{i})z^2 + pz + q = 0$$

where p and q are complex numbers, has roots α, β and γ.

(a) Given that $\beta + \gamma = -3 + 3\mathrm{i}$, find the value of α.

[2 marks]

(b) (i) Given that $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \mathrm{i}$, find the value of q.

[3 marks]

(ii) Hence find the value of p.

[2 marks]

(c) Find the value of $\alpha^2 + \beta^2 + \gamma^2$.

[2 marks]

QUESTION PART REFERENCE	Answer space for question 2

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3 The curve C has equation $y = 1 + \cosh x$.

(a) The curve C intersects the curve with the equation $y = 2\sinh x$ at the point P. Find the exact value of the x-coordinate of P.

[5 marks]

(b) The finite region bounded by the curve C, the coordinate axes and the line $x = \ln 2$ is rotated through 2π radians about the x-axis. Show that the volume of the solid generated is $\frac{\pi}{32}(m + n\ln 2)$, where m and n are integers.

[5 marks]

QUESTION PART REFERENCE	Answer space for question 3

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4 (a) Express $9(k+1)^2-(k+1)-2$ in the form $9k^2+bk+c$, where b and c are integers.

[1 mark]

(b) Prove by induction that, for all integers $n \geqslant 1$,

$$\sum_{r=1}^{n} r(2r-1)(3r-1) = \frac{1}{6}n(n+1)(9n^2-n-2)$$

[6 marks]

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5 The complex number ω is given by $\omega = -\sqrt{3} + 3\,\mathrm{i}$.

(a) (i) Find the argument of ω, giving your answer in terms of π.

[2 marks]

(ii) Find $|\omega - 2\mathrm{i}|$.

[2 marks]

(b) The complex number z satisfies both $|z - 2\mathrm{i}| \leqslant 2$ **and** $\frac{\pi}{2} \leqslant \arg z \leqslant \frac{2\pi}{3}$

(i) Sketch, on the Argand diagram opposite, the locus of z.

[5 marks]

(ii) Mark ω on the Argand diagram opposite.

[1 mark]

(iii) Find the greatest possible value of $\left| z - \frac{1}{2}\omega \right|$.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

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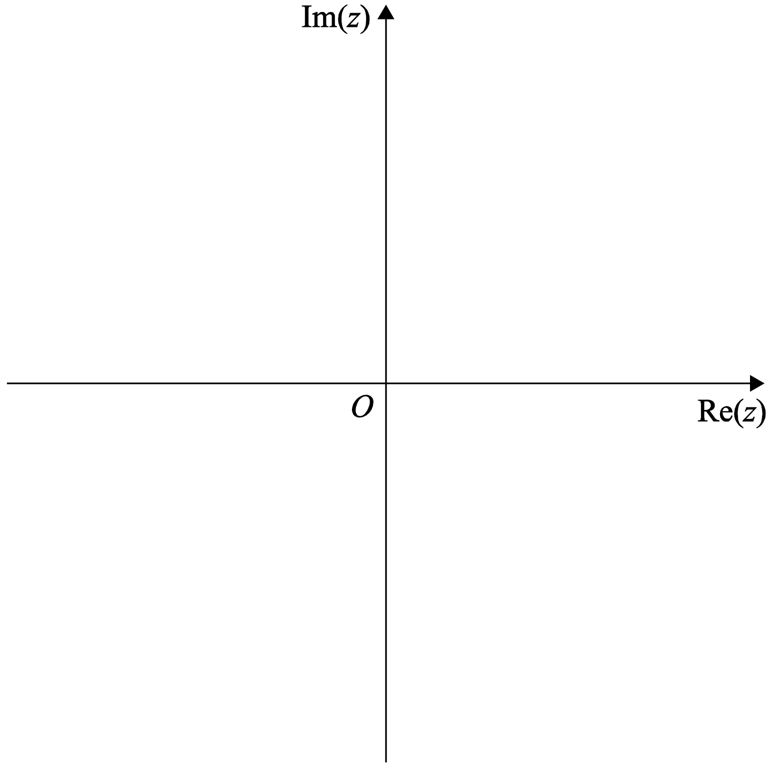

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6 Show that the exact value of $\int_0^{\sqrt{3}} x \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) \mathrm{d}x$ is $p\pi + q$, where p and q are rational numbers.

[6 marks]

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7 (a) Given that $f(\theta) = \ln\left[\frac{\sinh\theta}{1+\cosh\theta}\right]$, where $\theta > 0$, show that $f'(\theta) = \frac{1}{\sinh\theta}$.

[4 marks]

(b) The curve with the equation $y = \ln x$ from the point where $x = 1$ to the point where $x = 2\sqrt{2}$ has length s.

(i) Show that $s = \int_1^{2\sqrt{2}} \frac{\sqrt{x^2+1}}{x}\, dx$.

[3 marks]

(ii) Hence show that $s = a + b\sqrt{2} + \ln\left(1 + \frac{\sqrt{2}}{2}\right)$, where a and b are integers.

[7 marks]

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8 (a) Use de Moivre's theorem to show that

$$\frac{\sin 7\theta}{\sin\theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta$$

[6 marks]

(b) (i) Explain why $x = \sin^2\frac{\pi}{7}$ is a root of the equation

$$64x^3 - 112x^2 + 56x - 7 = 0$$

and write down the two other roots in trigonometric form.

[3 marks]

(ii) Hence show that the value of

$$\operatorname{cosec}^2\frac{\pi}{7} + \operatorname{cosec}^2\frac{2\pi}{7} + \operatorname{cosec}^2\frac{3\pi}{7}$$

is an integer.

[3 marks]

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END OF QUESTIONS

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