AQA

A-LEVEL
FURTHER MATHEMATICS

MFP2 Further Pure 2
Report on the Examination

6360
June 17

Version: 1.0

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General

The early questions gave students the opportunity to demonstrate their understanding of particular topics such as summation of series, complex numbers, roots of cubic equations, hyperbolic functions and proof by induction for a series. The integration questions without structure caused some problems to those who could not apply basic techniques from A-level Mathematics. The presentation of solutions from a significant number of students was poor and did not meet the standard expected for this examination.

Question 1

Part **(a)** was an easy starter. A few students, after writing $\frac{A}{2r+1}+\frac{B}{2r+3}$,were unsettled by finding two positive values, namely $A=\frac{1}{4}, B=\frac{1}{4}$, and wrote the expression as $\frac{\frac{1}{4}}{2r+1}-\frac{\frac{1}{4}}{2r+3}$ in order to prepare for using the difference method.

In part **(b)**, the difference method caused problems for those students who could not handle $(-1)^{r+1}$. The better students realised that there were two positive terms followed by two negative terms in the summation and were able to see which terms cancelled.
Although the expected form of the final answer was $\frac{1}{12}+\frac{(-1)^{n+1}}{8n+12}$, those students who gave two separate expressions when n was odd or even scored full marks.

Question 2

The structure in part **(a)** resulted in a high success rate with almost every student finding the correct value of α , but a few weaker students thought the sum of the roots was $6-3\mathrm{i}$.

Most errors in part **(b)(i)** came from writing $q=\alpha\beta\gamma$ and some students took several lines to express $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}$ as $\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$. More than half of the students could find the correct value of q but students were expected to simplify expressions such as $\frac{6-3\mathrm{i}}{\mathrm{i}}$.

The most direct approach in part **(b)(ii)** was to substitute $z=-3$ and $q=-3-6\mathrm{i}$ into the cubic equation in order to find the value of p , but often errors in finding the correct value of q meant that only about a third of the students obtained the correct value of p . An alternative approach was to find $\beta\gamma=\frac{\alpha\beta\gamma}{\alpha}$ followed by $p=\alpha\beta+\beta\gamma+\gamma\alpha=\alpha(\beta+\gamma)+\beta\gamma$, but once again many using this approach had errors in their earlier calculations and so failed to find the correct value of p .

Although only a minority found the correct value of $\alpha^2+\beta^2+\gamma^2$ in part **(c)**, many students wrote down a correct identity such as $(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$ and, encouragingly, this identity continues to be well known.

Question 3

Part **(a)** involved solving the equation $1+\cosh x = 2\sinh x$. The expected approach was using a quadratic equation in e^x and those using this approach were generally successful in finding the single solution $x = \ln 3$. Some students decided to square both sides of the equation seemingly unaware that this approach introduced bogus solutions that had to be tested in the original equation before being discarded. Quite a few incorrectly thought that $\cosh x = \frac{5}{3} \Rightarrow x = \cosh^{-1}\frac{5}{3}$ and lost the final mark.

In part **(b)**, a large number of students failed to read the question properly and wrote down the formula for the curved surface area. These students were credited with the marks for integrating $\cosh^2 x$ since this was part of the correct solution to find the volume of revolution. Sloppy work denied some students the mark assigned for writing a correct expression for the volume of revolution; this required a correct integrand, π, the correct limits and $\mathrm{d}x$ in the integral. Others lost the final mark because of careless arithmetic when trying to express the final answer in the required form.

Question 4

In part **(a)**, many students failed to simplify the quadratic correctly which was unexpected at this level. This easy mark was intended to guide students in the final stages of the proof by induction and, in general, this was the case.

Students would be well advised to use previous mark schemes as a template as to how to set out a proof by induction. This year's question seen in part **(b)** was a straightforward example of induction in that it involved the sum of a series and yet the number scoring full marks was disappointing. Having shown that both sides of the formula yielded the value 2, it was expected that students would make a statement such as "hence the formula is true when $n = 1$" rather than simply ticking the result, writing QED or stating something like "induction starts" or "base case holds" etc. The crucial step was adding the correct $(k+1)\text{th}$ term to **both** sides of the formula that was assumed to be true when $n = k$. Less able students struggled with the next few lines of algebra but most were alert to the right hand side needing to have the quadratic factor found in part **(a)**.

Question 5

In part **(a)(i)** it was a generous two marks for finding the argument of a complex number. Nevertheless, quite a significant number gave their answer as $-\frac{\pi}{3}$ or $\frac{\pi}{3}$ when a sketch should have illustrated that this could not possibly be correct.

In part **(a)(ii)**, it was encouraging to see almost all the students finding the correct value of the modulus with the majority showing their correct working. Those who wrote $\left(\sqrt{3}\right)^2 + \mathrm{i}^2 = 3 - 1 = 2$ followed by modulus = 2, possibly extracted from their calculator, scored no marks since this was a result from incorrect working.

In part **(b)(i)**, most students drew a circle touching the real axis at O with centre at $2\mathrm{i}$, though some neglected to mark 2 on the imaginary axis. The half line from O was expected to be inclined at roughly $\frac{\pi}{3}$ to the negative real axis and many attempts were not clear enough to gain full marks. Some students thought that the locus was an arc of the circle in the second quadrant rather than the region bounded by the half line, the circle and the imaginary axis.

In part **(b)(ii)**, it was good to see many students using the earlier information to locate ω at the point of intersection of the half line and the circle.

Part **(b)(iii)** defeated most students. Some identified the critical point as being when $z = 4\mathrm{i}$ but did not write down a correct expression for the greatest possible value. The easiest method was by evaluating $\left|4\mathrm{i}+\frac{\sqrt{3}}{2}-\frac{3}{2}\mathrm{i}\right|$ but some chose far more complicated methods which were rarely successful in producing the correct answer.

Question 6

Most students realised the need to use integration by parts but many failed to earn the first method mark by writing the integral of x as x^2. Despite the comments in last year's report regarding the use of the formulae booklet, many students wrote the derivative of $\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$ as a multiple of $\frac{1}{1-\frac{x^2}{3}}$ and these scored no marks either. Having used integration by parts correctly, students needed to write $\frac{x^2}{x^2+3}=1-\frac{3}{x^2+3}$, or equivalent, in order to progress further with the integration and usually students who completed this step were able to find the correct value of the integral. It was good to see fewer students this year trying to work back from their "advanced calculator display", although possibly these better students sensibly used a calculator to check their final answer. Those who made the substitution $x=\sqrt{3}\,u$ initially were able to avoid the complications of the surds and were often successful in scoring full marks for the question.

Question 7

Two approaches were seen in part **(a)**; the majority used the quotient rule on the term inside the square brackets and multiplied by $\frac{1+\cosh\theta}{\sinh\theta}$ to complete the chain rule; others considered the difference of two logarithms and differentiated the two terms separately before combining with a common denominator. Both methods involved the simplification of $\cosh\theta+\cosh^2\theta-\sinh^2\theta$ which was usually done correctly so as to cancel with $1+\cosh\theta$ in the denominator, although quite a few used $\cosh^2\theta+\sinh^2\theta=1$ and fudged their working to obtain the printed answer. Just under half of the students scored full marks on this proof.

In part **(b)**, there was a slightly higher success rate than in part **(a)** in proving this result for the arc length. However, far too many showed insufficient steps in their working or failed to write the final result in exactly the form printed in the question.

The integral in part **(c)** caused major difficulties for a large number of students and the modal mark was zero. Since the numerator contained $\sqrt{x^2+1}$ the expected substitution was $x = \sinh\theta$ where the integrand then split into $\sinh\theta + \dfrac{1}{\sinh\theta}$ allowing students to use the result from part **(a)** to find the value of the integral in the form specified. Other sensible substitutions were $x = \tan\theta$ and $u = \sqrt{x^2+1}$ and each of these methods was seen successfully used by students.

Question 8

It was good to see more than just the most able students scoring several marks on this last question.

A mark was available in part **(a)** for all students who wrote down de Moivre's theorem for $n = 7$. Those students who then used Pascal's triangle and immediately worked with the imaginary part arrived at the printed result after just a few lines of working. It is worth reminding students to show their working in order to gain method marks, evidence was seen that some students did not expand $(1-\sin^2\theta)^3$ and $(1-\sin^2\theta)^2$, but tried to 'fudge' the result by simply writing down the printed answer without showing this important step. Some students were rather sloppy in their working and failed to indicate when they were considering $\cos 7\theta + i\sin 7\theta$, $\sin 7\theta$ or $\dfrac{\sin 7\theta}{\sin\theta}$ in the various lines of their proof.

In part **(b)**, it was not sufficient to write "$\theta = \dfrac{\pi}{7}$, $\sin 7\theta = 0$ so satisfies equation". The better students showed that $x = \sin^2\theta$ transformed the equation in θ into the cubic equation $64x^3 - 112x^2 + 56x - 7 = 0$ and hence that $x = \sin^2\dfrac{\pi}{7}$ was a root of the cubic equation.

Many students wrote down two other roots but often one of these, $\sin^2\dfrac{8\pi}{7}$ for example, was the same as $\sin^2\dfrac{\pi}{7}$ and so did not earn the mark.

There was no reward in part **(c)** for students who simply typed the expression into their calculators and who then stated that the value was 8. It was necessary to identify roots of the cubic equation α, β, γ with $\sin^2\dfrac{\pi}{7}, \sin^2\dfrac{2\pi}{7}, \sin^2\dfrac{3\pi}{7}$ and then to find $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ using $\sum\alpha\beta = \dfrac{56}{64}$ and $\alpha\beta\gamma = \dfrac{7}{64}$. Another effective approach was to obtain a new equation using the substitution $z = 1/x$ and then the sum of the roots of $7z^3 - 56z^2 + 112z - 64 = 0$ gave the required answer $\dfrac{56}{7} = 8$.

Use of statistics

Statistics used in this report may be taken from incomplete processing data. However, this data still gives a true account on how students have performed for each question.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.

Converting Marks into UMS marks

Convert raw marks into Uniform Mark Scale (UMS) marks by using the link below.
UMS conversion calculator