A-LEVEL
Mathematics

MFP3 Further Pure 3
Mark scheme

6360

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	DO NOT ALLOW ANY MISREADS IN THIS QUESTION			
(a)	$y(1.2) = 4 + 0.2 \times \dfrac{1+2\sqrt{4}}{1+1}$	**M1**		Correct substitution into correct formula PI
	$= 4.5$	**A1**	**2**	4.5 OE
(b)	$(y(1.4)=)\quad y(1) + 2(0.2)\{\mathrm{f}[1.2, y(1.2)]\}$	**M1**		Seen or used
	$y(1.4) = 4 + 2(0.2) \times \dfrac{1.2+2\sqrt{4.5}}{1.2+1}$ $(= 4 + 0.4 \times 2.4739\ldots)$	**A1**		OE PI by 4.990 or 4.989…
	= 4.990 (to 3dp)	**A1**	**3**	CAO Must be 4.990
	Total		**5**	
(b)	Eg Treat $4 + 2(0.2) \times \dfrac{1+2\sqrt{4.5}}{1.2+1}$ as a slip scoring M1 A0 A0 (*x* has correctly been explicitly replaced by 1.2 in denominator but not so in the numerator.) If candidate had not shown this form and instead just given y(1.4)=4.953207…. score would be 0/3			
(b)	An OE for the 1st A1 is $4 + 2(0.2) \times \dfrac{6+15\sqrt{2}}{11}$.			

Q2	Solution	Mark	Total	Comment
(a)(i)	$f''(0)=5-5f'(0)-6f(0)=0$	B1		$f''(0)=0$ seen or used
	$f'''(x)=6x-5f''(x)-6f'(x)$ $f'''(0)=0-5f''(0)-6f'(0) \Rightarrow f'''(0)=-6$	B1		AG Be convinced
	$f^{(4)}(x)=6-5f'''(x)-6f''(x)$ $f^{(4)}(0)=6-5f'''(0)-6f''(0)=36$	B1	3	36
(a)(ii)	$f(x) \approx x(1)+\frac{x^2}{2}f''(0)+\frac{x^3}{3!}f'''(0)+\frac{x^4}{4!}f^{(4)}(0)$..	M1		Maclaurin's Thm used with $f(0)=0$, $f'(0)=1$ and c's non-zero values for at least two of $f''(0)$, $f'''(0)$ or -6, $f^{(4)}(0)$ leading to three non-zero terms
	$= x-x^3+\frac{3}{2}x^4$	A1F	2	OE Ft only on c's non-zero value for $f^{(4)}(0)$. Ignore any extra higher powers of x terms
(b)	Aux eqn $m^2+5m+6=0$ $(m+3)(m+2)=0$	M1		Correct factorising or correct substitution into quadratic formula OE on correct aux eqn. PI by correct values of 'm' seen/used.
	$(y_{CF}=)\ Ae^{-3x}+Be^{-2x}$	A1		
	Try $(y_{PI}=)\ ax^2+bx+c$	M1		Correct form for y_{PI}. If other term(s) included, cand needs to show the corresponding coefficient is 0
	$(y'_{PI}=)\ 2ax+b$; $(y''_{PI}=)\ 2a$			
	$2a+5(2ax+b)+6(ax^2+bx+c)=3x^2+5$	dM1		Substitution into DE, dep on previous **M** only. PI by at least two correct equations in next line provided previous M scored.
	$6a=3$; $10a+6b=0$; $2a+5b+6c=5$	A1		OE At least **two** correct, seen or used
	$a=\frac{1}{2}$; $b=-\frac{5}{6}$; $c=\frac{49}{36}$	A1		Seen or used; at least two correct
	$(y_{GS}=)\ Ae^{-3x}+Be^{-2x}+\frac{1}{2}x^2-\frac{5}{6}x+\frac{49}{36}$	A1	7	ACF but must be exact
	Total		**12**	
(a)(i)	Condone equivalent notation for the derivatives			

Q3	Solution	Mark	Total	Comment
(a)(i)	$\ln(1-\sin x) = -x - \frac{1}{2}x^2 - \frac{1}{6}x^3 - \frac{1}{12}x^4$	B1	1	
(a)(ii)	$\ln(1+\sin x) + \ln(1-\sin x) = \ln\cos^2 x$	M1		Must be using 'hence' otherwise 0/3 $\ln(1+\sin x)+\ln(1-\sin x)=\ln\cos^2 x$ seen or used.
	$0x - \frac{2}{2}x^2 + 0x^3 - \frac{2}{12}x^4 \ldots = 2\ln\cos x$	dM1		$\ln\cos^2 x = 2\ln\cos x$ seen or used equated to the sum of the c's expansions of $\ln(1+\sin x)$ and $\ln(1-\sin x)$. Like terms need not be combined for this dM1.
	$\ln\cos x = -\frac{1}{2}x^2 - \frac{1}{12}x^4$	A1	3	AG Be convinced. NMS scores 0/3
(a)(iii)	$\ln(\sec x + \tan x) = \ln\left(\frac{1+\sin x}{\cos x}\right)$	M1		$\ln\left(\frac{1+\sin x}{\cos x}\right)$
	$= \ln(1+\sin x) - \ln\cos x =$ $x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \frac{1}{2}x^2 + \frac{1}{12}x^4$	dM1		$\ln(1+\sin x) - \ln\cos x$ with the two given series used correctly. Condone one miscopy.
	$\ln(\sec x + \tan x) = x + \frac{1}{6}x^3$	A1	3	$x + \frac{1}{6}x^3$. NMS scores 3/3
Alt (a)(iii)	$[\ln(\sec x+\tan x)]' = \sec x$ $[\ln(\sec x+\tan x)]'' = \sec x\tan x$ $[\ln(\sec x+\tan x)]''' = \sec^3 x + \sec x\tan^2 x$	(M1)		3rd derivative of $\ln(\sec x + \tan x)$ attempted using product or quotient rule with at least 2nd derivative correct
	$\ln(\sec x + \tan x)$ $= 0 + (\sec 0)x + 0x^2 + k\frac{x^3}{3!}$, where $k = \text{c's } [\ln(\sec x+\tan x)]'''(0)$, $k \neq 0$	(dM1)		
	$\ln(\sec x + \tan x) = x + \frac{1}{6}x^3$	(A1)	(3)	$x + \frac{1}{6}x^3$
(b)	$\left[\frac{\ln(\sec x+\tan x)}{2x+5x^3}\right] = \frac{x + \frac{1}{6}x^3 \ldots}{2x+5x^3}$	M1		Using a series expansion for $\ln(\sec x + \tan x)$ in the given function
	$\lim_{x\to 0}\left[\frac{\ln(\sec x+\tan x)}{2x+5x^3}\right]$ $= \lim_{x\to 0} \frac{1+O(x^2)}{2+O(x^2)}$	dM1		Must see at least one '$O(x^2)$' or kx^2 term after division by x to get a constant term in both the numerator and the denominator.
	$= \frac{1}{2}$	A1F	3	ft only on c's (a)(iii) $= x + p\,x^n$, integer $n > 1$. No other errors seen
	Total		10	

Q4	Solution	Mark	Total	Comment
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}t}{\mathrm{d}x}\frac{\mathrm{d}y}{\mathrm{d}t}$	M1		OE Relevant chain rule eg $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}$	A1		OE eg $\frac{\mathrm{d}y}{\mathrm{d}t}=\mathrm{e}^{t}\frac{\mathrm{d}y}{\mathrm{d}x}$ or $x\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}$ etc
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}t}{\mathrm{d}x}\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}\right)$	M1		OE eg $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}=\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}x}\left(x\frac{\mathrm{d}y}{\mathrm{d}x}\right)$
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}t}{\mathrm{d}x}\left[-\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}+\mathrm{e}^{-t}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right]$	dM1		Product rule (dep on both previous **M**s) OE $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}=\frac{\mathrm{d}x}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\mathrm{e}^{-2t}\left[-\frac{\mathrm{d}y}{\mathrm{d}t}+\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right]$			$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}=\frac{\mathrm{d}y}{\mathrm{d}t}+x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$
	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-\frac{\mathrm{d}y}{\mathrm{d}t}$	A1	5	CSO Condone answer left as RHS=LHS
(b)	DE becomes $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-4\frac{\mathrm{d}y}{\mathrm{d}t}+5y=10\,\mathrm{e}^{t}$	B1F		Ft on c's integer n value ie $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+(n-3)\frac{\mathrm{d}y}{\mathrm{d}t}+5y=10\,\mathrm{e}^{t}$ PI by later work
	Auxl eqn $m^2-4m+5=0$; $(m-2)^2+1=0$	M1		$\left(m+\frac{n-3}{2}\right)^2+k=0$ or using quadratic formula on $m^2+(n-3)m+5=0$. PI by correct (ft on n) values of 'm' seen/used or PI by correct CF.
	$m=2\pm\mathrm{i}$ CF: $(y_C=)$ $\mathrm{e}^{2t}(A\cos t+B\sin t)$	B1F		If not correct CF, ft on $m=p\pm\mathrm{i}q$, $p\neq 0$, $q\neq 0$ and two arbitrary constants in CF.
	P.Int. $(y_P=)$ $5\mathrm{e}^{t}$	B1		
	$y=x^2[A\cos(\ln x)+B\sin(\ln x)]+5x$	A1	5	y = f(x) with correct f(x) simplified so no exponentials
	Total		**10**	
(a)	After the M1A1, cand writes $\left(\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)$ and stops, no further marks are scored due to the stated dependency even though the product rule has been used. If however the cand had subsequently replaced the $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)$ by $\frac{\mathrm{d}t}{\mathrm{d}x}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)$ or better then M1 dM1 would be scored at that stage.			

Q5	Solution	Mark	Total	Comment
	(I =) $\frac{2}{3}\ln x - \frac{1}{3}\ln(1-\cos 3x)$ $(+c)$	**B1**		Correct integration of $\frac{2}{3x}$
		B1		Correct integration of $\frac{\sin 3x}{1-\cos 3x}$. Can be a correct OE in terms of u if u defined.
	$= \lim_{a \to 0} \int_a^{\frac{\pi}{6}} \left(\frac{2}{3x} - \frac{\sin 3x}{1-\cos 3x}\right) dx$	**M1**		Evidence of limit 0 having been replaced by a (OE) at any stage and $\lim_{a \to 0}$ seen or taken at any stage with no remaining lim relating to $\frac{\pi}{6}$.
	$= \lim_{a \to 0} \left[\frac{1}{3}\ln \frac{x^2}{1-\cos 3x}\right]_a^{\pi/6}$	**M1**		Combining the relevant ln terms into a single ln term, ft after integration on one incorrect coeff of ln terms
	(I=) $\frac{2}{3}\ln\frac{\pi}{6} - \frac{1}{3}\ln 1$ $- \lim_{a \to 0}\left[\frac{1}{3}\ln\frac{a^2}{1-\cos 3a}\right]$	**M1**		$\mathrm{F}\left(\frac{\pi}{6}\right) - \mathrm{F}(a)$ taken at any stage
	Consider $\lim_{x \to 0}\left[\frac{x^2}{1-\cos 3x}\right]$ or with $\left[\frac{1-\cos 3x}{x^2}\right]$ $\lim_{a \to 0}\left[\frac{a^2}{1-\cos 3a}\right]$ or with $\left[\frac{1-\cos 3a}{a^2}\right]$	**B1**		$\cos 3x \approx 1 - \frac{(3x)^2}{2!} + ..$ seen/used; or $\cos 3a \approx 1 - \frac{(3a)^2}{2!} + ..$ seen/used; or L'Hopitals rule using either x or 'a' to $\lim_{x \to 0}\left[\frac{x^2}{1-\cos 3x}\right] = \lim_{x \to 0}\left[\frac{2}{9\cos 3x}\right]$ or OE for the reciprocal
	$= \frac{2}{9}$ (or $\frac{9}{2}$ if using $\lim_{x \to 0}\left[\frac{1-\cos 3x}{x^2}\right]$)	**B1**		Correct value for the relevant limit. NB if combining with the other ln term, the value will appear as an expression involving π. Check the 2/9 equivalent
	(I=) $\frac{1}{3}\ln\frac{\pi^2}{36} - \frac{1}{3}\ln\frac{2}{9} = \frac{1}{3}\ln\frac{\pi^2}{8}$	**A1**	8	**CSO** $\frac{1}{3}\ln\frac{\pi^2}{8}$ OE single term in exact form.
	Total		**8**	
	$\frac{2}{3}\ln 3x - \frac{1}{3}\ln(1-\cos 3x)$ **B1B1** leads to eg $\frac{1}{3}\ln\frac{9\pi^2}{36} - \frac{1}{3}\lim_{a \to 0}\left[\ln\frac{9a^2}{1-\cos 3a}\right]$ etc 1st M1, using a for lower limit…if splitting the integral and using subst for the trig part of the integral eg u=1-cos3x, and changing the limits, the lower limit must be linked to 1-cos3a with $a \to 0$, otherwise M0. Note: For final cso mark, using series method, cos series must have at least 3 terms so that an $O(a^2)$ or ka^2 (OE) term must be present after division of top and bottom by a^2			

Q6	Solution	Mark	Total	Comment
(a)	I.F. $e^{\int 2\,(\mathrm{d}x)}$	M1		Can be implied by a correct IF
	$= e^{2x\ (+c)}$	A1		A correct IF
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\,e^{2x}\right) = 2(x-1) + 4\,e^{2x}$	M1		OE Both sides must be considered correctly
	$y = \left(x^2 - 2x\right)e^{-2x} + 2 + c\,e^{-2x}$	A1	4	OE but must be in the form $y = \mathrm{f}(x)$ eg $y = (x-1)^2 e^{-2x} + 2 + ke^{-2x}$
(b)	As $x \to \infty$, $x^n\,e^{-2x} \to 0$	E1		Must consider explicitly either in general terms or both $x\,e^{-2x}$ and $x^2\,e^{-2x}$ separately considered
	$(\Rightarrow y \to 2)$; ie horizontal asymptote y=2	B1	2	$y = 2$ OE
(c)	$2 + 4e^2 = 3e^2 + 2 + c\,e^2 \Rightarrow c = 1$	M1		Sub $x = -1$, $y = 2 + 4e^2$ in c's ans **(a)** as far as finding a value for the arbitrary constant
	Eqn of C: $y = (x-1)^2\,e^{-2x} + 2$	A1		ACF for the equation of C. Condone y replaced by k for this mark.
	At st pt, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 2y = 2(x-1)e^{-2x} + 4$	M1		Either equating $\frac{\mathrm{d}y}{\mathrm{d}x}$ to 0 in the given DE or differentiating c's eqn of curve using product/quotient rule & equating derivative to 0
	$(x-1)^2\,e^{-2x} + 2 = (x-1)e^{-2x} + 2$ Only stationary points at $x = 1$ & $x = 2$	A1		A correct equation in x
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} = 2e^{-2x} - 4(x-1)e^{-2x}$ When $x = 1$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2e^{-2} > 0$ so min. When $x = 2$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2e^{-4} < 0$ so max.	E1		Dep on previous two M marks. Attempt to find the sign of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at each of the two stationary points or any other valid method to distinguish between maximum and minimum including a graphical approach
	y=k intersects C in three distinct points $\Rightarrow \quad 2 < k < 2 + e^{-4}$	B2,1,0	7	**B1** if one or both < replaced by $\le$.
	Total		**13**	
(b)	As $x \to \infty$, $y \to 2$ so HA $y = 2$ scores **E0 B1**; As $x \to \infty$, $(x^2 + c)\,e^{-2x} \to 0$, $x\,e^{-2x} \to 0$ scores **E1**			
(c)	Final two B marks, although unlikely, can be scored without any of the previous 5 marks scored			

Q7	Solution	Mark	Total	Comment
(a)(i)	$x^2+y^2=x^2+16-8x=(4-x)^2$	B1	1	$x^2+y^2=(4-x)^2$ must be in this form
(a)(ii)	$r^2=(4-x)^2$	M1		$x^2+y^2=r^2$ or $x=r\cos\theta$ & $y=r\sin\theta$ used
	$r^2=(4-r\cos\theta)^2$	dM1		x replaced by $r\cos\theta$ to form a polar eqn.
	$r=4-r\cos\theta=4-r\left(2\cos^2\dfrac{\theta}{2}-1\right)$	dM1		$\cos\theta=2\cos^2\dfrac{\theta}{2}-1$ used with $r=k-r\cos\theta$, where k is an integer
	$\Rightarrow r2\cos^2\dfrac{\theta}{2}=4\Rightarrow r=2\sec^2\dfrac{\theta}{2}$	A1	4	AG Be convinced
(b)(i)	$\tan\theta=\sqrt{3}\ \Rightarrow\theta=\dfrac{\pi}{3},\ \theta=-\dfrac{2\pi}{3}$	M1		At least one correct exact value for θ or $3r^2-32r+64=0$ or correct vals. for r.
	(P and Q are) $\left(\dfrac{8}{3},\dfrac{\pi}{3}\right)$ and $\left(8,-\dfrac{2\pi}{3}\right)$	A1 A1	3	A1 for $\left(\dfrac{8}{3},\dfrac{\pi}{3}\right)$; A1 for $\left(8,-\dfrac{2\pi}{3}\right)$; SC if A0A0 award 1mark for all 4 values correct
(b)(ii)	Length $PQ=8+\dfrac{8}{3}\quad(=10\dfrac{2}{3})$ or radius of circle = $(8+\dfrac{8}{3})\div 2$	B1F		Ft c's r_P+r_Q provided $\lvert\theta_P-\theta_Q\rvert=\pi$ or correct numerical expression for radius stated or used
	Area of circle $A_2=\left(5\dfrac{1}{3}\right)^2\pi$	B1		A correct numerical expression for the area of the circle
	(Area A_1 =) $\dfrac{1}{2}\int_{(-\frac{2\pi}{3})}^{(\frac{\pi}{3})}\left(2\sec^2\dfrac{\theta}{2}\right)^2(\mathrm{d}\theta)$	M1		Use of $k\int r^2(\mathrm{d}\theta)$, $k>0$. If using
		A1F		Cartesian eqn, correct expression for A_1 involving integrals ft on c's P and Q provided pts in the correct quadrants. Limits consistent with a correct value of k, if not $\pi/3$ & $-2\pi/3$, ft on c's θ_P and θ_Q
	$=2\int_{(-\frac{2\pi}{3})}^{(\frac{\pi}{3})}\sec^4\dfrac{\theta}{2}(\mathrm{d}\theta)$			$\sec^4\dfrac{\theta}{2}=\left(1+\tan^2\dfrac{\theta}{2}\right)\sec^2\dfrac{\theta}{2}$ used (*)
	$=2\int_{(-\frac{2\pi}{3})}^{(\frac{\pi}{3})}\left(1+\tan^2\dfrac{\theta}{2}\right)\sec^2\dfrac{\theta}{2}(\mathrm{d}\theta)$	dM1		could be in terms of eg u where $u=\tan\dfrac{\theta}{2}$. (*)If using Int by parts, after 1st application need to use $\tan^2(\ )\sec^2(\ )=\sec^4(\ )-\sec^2(\ )$
	$=2\left[2\tan\dfrac{\theta}{2}+\dfrac{2}{3}\tan^3\dfrac{\theta}{2}\right]_{(-2\pi/3)}^{(\pi/3)}$	dM1		$\lambda\left(\tan\dfrac{\theta}{2}+\dfrac{1}{3}\tan^3\dfrac{\theta}{2}\right)$ OE
		A1		$4\left(\tan\dfrac{\theta}{2}+\dfrac{1}{3}\tan^3\dfrac{\theta}{2}\right)$ OE
	$=\left(\dfrac{4}{\sqrt{3}}+\dfrac{4}{3}\times\dfrac{1}{3\sqrt{3}}\right)-4\left(-2\sqrt{3}\right)=\dfrac{256}{9\sqrt{3}}$	A1		$\dfrac{256}{9\sqrt{3}}$ OE Correct exact value for A_1
	$\dfrac{A_2}{A_1}=\dfrac{256\pi}{9}\div\dfrac{256}{9\sqrt{3}}=\pi\sqrt{3}$	A1	9	CSO AG
	Total		**17**	
	TOTAL		**75**	