Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

A-level MATHEMATICS

Unit Statistics 4

Wednesday 28 June 2017 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working, otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 During a study of the effect of the drug *Choldrop* on the level of cholesterol in blood, a randomly selected group of 11 men, all with high levels of cholesterol, was treated with the drug for three months.

The level of low density lipoprotein, LDL, in each man's blood was measured immediately before the start of the treatment and again after the three-month course of treatment.

The results, in grams of LDL per litre of blood, are shown in the table.

Patient	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**
Before	2.98	2.70	2.61	2.93	2.56	3.05	2.94	3.41	2.22	3.07	2.88
After	2.63	2.43	2.87	2.54	2.22	2.76	2.88	2.98	2.37	2.84	2.63

Assuming that the differences in measurements of LDL are approximately normally distributed, investigate, at the 1% level of significance, whether *Choldrop* reduces the mean level of LDL in men with high levels of cholesterol.

[9 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE	**Answer space for question 1**

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2 The random variable X has a geometric distribution with parameter p, mean μ and variance σ^2.

(a) Given that $\mathrm{P}(X \leqslant 2) = 0.36$, calculate the value of p, and hence find values for μ and σ^2.

[5 marks]

(b) Calculate $\mathrm{P}(\mu - 0.5\sigma < X < \mu + 0.5\sigma)$.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

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3 In a comparison of two concrete mixes, M15 and M45, the compressive strengths, in newtons per square millimetre $(\mathrm{N/mm^2})$, of randomly sampled cubes of each mix were measured.

The results are summarised in the table.

		Sample size (n)	**Mean** $(\bar{x})$	$\sum(x-\bar{x})^2$
Concrete mix	**M15**	16	15.01	0.1352
	M45	10	45.13	0.1898

The compressive strength of concrete cubes may be assumed to be normally distributed. Mix M15 has mean μ_1 and variance σ_1^2, and mix M45 has mean μ_4 and variance σ_4^2.

(a) (i) Construct a 95% confidence interval for the ratio $\dfrac{\sigma_4}{\sigma_1}$.

[6 marks]

(ii) Comment on what may be concluded from your answer to part **(a)(i)**.

[2 marks]

(b) Investigate, at the 5% level of significance, the hypothesis that $\mu_4 - \mu_1 > 30$.

[9 marks]

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4 It is suggested that the response time, X minutes, to an emergency request for the attendance of an ambulance, at an incident within 3 miles of an ambulance station, can be modelled by the following probability density function.

$$\mathrm{f}(x) = \begin{cases} \dfrac{x^2}{18} & 0 \leqslant x < 3 \\ \dfrac{1}{4}(5 - x) & 3 \leqslant x \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

To investigate this suggestion, the value of X was recorded for a sample of 324 requests, with the following results.

x	0–1	1–2	2–3	3–4	4–5	>5
Frequency	4	49	129	112	30	0

Use a χ^2 goodness of fit test and the 5% level of significance to investigate whether X can be modelled by the probability density function given above.

[13 marks]

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5 (a) The continuous random variable X has the cumulative distribution function $\mathrm{F}(x)$ where

$$\mathrm{F}(x)=\begin{cases}0 & x<0\\ 1-\mathrm{e}^{-\lambda x} & 0\leqslant x<\infty\end{cases}$$

and $\mathrm{E}(X)=\dfrac{1}{\lambda}$.

(i) Deduce the probability density function, $\mathrm{f}(x)$, of X for $0\leqslant x<\infty$.

[1 mark]

(ii) Use integration to find an expression for $\mathrm{E}\left(X^2\right)$.

[3 marks]

(iii) Hence show that $\mathrm{Var}(X)=\dfrac{1}{\lambda^2}$.

[1 mark]

(b) The number of emails received by a helpdesk may be modelled by a Poisson distribution with an average of 3 emails per hour.

(i) Determine the probability that the helpdesk receives fewer than 15 emails during a four-hour period.

[2 marks]

(ii) Calculate the probability that the time between successive emails is:

(A) exactly 20 minutes;

(B) less than 15 minutes;

(C) between 15 and 25 minutes.

[6 marks]

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6 The random variable X has a normal distribution with mean μ and variance σ^2.

The variable $\bar{X}_1$ denotes the mean of a random sample of **10** observations on X.
The variable $\bar{X}_2$ denotes the mean of an independent random sample of **30** observations on X.

(a) Two estimators proposed for μ are

$$Y_1 = \frac{1}{2}\left(\bar{X}_1 + \bar{X}_2\right) \quad \text{and} \quad Y_2 = \frac{1}{4}\left(\bar{X}_1 + 3\bar{X}_2\right)$$

(i) Show that both Y_1 and Y_2 are unbiased estimators for μ.

[2 marks]

(ii) Derive simplified expressions, in terms of σ^2, for each of $\mathrm{Var}\left(Y_1\right)$ and $\mathrm{Var}\left(Y_2\right)$.

[3 marks]

(iii) Calculate the efficiency of Y_1 relative to Y_2.

[2 marks]

(b) The variable S_1^2 denotes the unbiased estimator for σ^2 from the same sample of **10** observations on X. The variable S_2^2 denotes the unbiased estimator for σ^2 from the same sample of **30** observations on X.

Two estimators proposed for σ^2 are

$$T_1 = \frac{1}{2}\left(S_1^2 + S_2^2\right) \quad \text{and} \quad T_2 = \frac{1}{38}\left(9S_1^2 + 29S_2^2\right)$$

(i) Show that both T_1 and T_2 are unbiased estimators for σ^2.

[2 marks]

(ii) Derive simplified expressions, in terms of σ^4, for each of $\mathrm{Var}\left(T_1\right)$ and $\mathrm{Var}\left(T_2\right)$.

You may assume, for a random sample of n observations from the distribution $\mathrm{N}\left(\mu, \sigma^2\right)$, that $\mathrm{Var}\left(S^2\right) = \dfrac{2\sigma^4}{n-1}$.

[3 marks]

(iii) Hence state, with justification, which of T_1 and T_2 is the better unbiased estimator for σ^2.

[2 marks]

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