# AS

# FURTHER MATHEMATICS

Paper 1

Report on the Examination

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## General

This question paper provided a suitable challenge for the most able, whilst also providing opportunities for weaker students. There was no evidence of students running out of time. However, those who employed elaborate and time-consuming methods may have had less time to check their work after completing the paper. Almost all students were able to make good progress with most of the questions, and some progress with the more demanding questions. It was evident that most students had a good understanding of at least the basics of each topic. However, particularly in questions 13 and 14, a number of students were let down by poor presentation, in the form of reduced legibility and unhelpful diagrams,.

## Question 1

This question was answered very well, with almost all students selecting the correct answer of 10.

## Question 2

The vast majority of students selected the correct answer of **AC**. The most common incorrect choice was **AB**.

## Question 3

The answer of cos $x$ was correctly chosen by most students, with the wrong choices evenly distributed amongst the other three options.

## Question 4

Many students struggled with this graph sketch, failing to notice that a rearrangement of the equation produces $r \cos \theta = a$ and hence $x = a$, which is a vertical line through ($a$, 0). Many students plotted some points — which rarely produced the correct answer — whilst others sketched a seemingly random curve, a cardioid being the most popular choice.

## Question 5

Most students realised that the given matrix was a rotation, although some struggled to describe it adequately. It was evident that the general matrix provided in the formulae book had been used by almost all students, but a common error was to solve only one of the trigonometric equations. A rotation of 60° from $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ was not uncommon. However, the majority of students answered this question well.

## Question 6

Many students failed to identify the error in the algebraic steps given in part (a). Of those who realised that $\pm$ was not acceptable, only half could explain why. A significant number of students suggested that completing the square was an invalid approach. Many others questioned the swapping of the variables $x$ and $y$, with some explaining that variables could not be swapped. Some students decided that the variables had been swapped at the wrong point in the solution.

When answering part (b), many students failed to notice the connection with part (a). Of those who did spot the link, some decided that $x$ was equal to $\sinh^{-1} 3$ instead of $\sinh 3$. However, a number of students were able to solve the equation by algebraic means, and the correct answer sometimes followed several lines of skilful algebra.

**Question 7**

The vast majority of students realised that $\mathbf{M}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$, and were able to multiply out to produce two equivalent equations. However, a significant number of these students were unable to use their equation(s) to find an invariant point. Many chose to solve them as simultaneous equations, and either made an error or confused themselves when every term cancelled out. A minority of students knew that (0, 0) would be invariant and gave this as the only possible point.

**Question 8**

This question was well answered with almost every student identifying the complex conjugate as a root of the equation. The methods for finding the third root were many and varied, as were the methods for finding the value of $m$. However, the majority of attempts were successful. Some students opted to find the three factors, expand the brackets and then compare coefficients. Although this is a perfectly valid method, it was quite time consuming, and prone to error.

A few students found $m$ whilst calculating the third root. Many of these then wasted time in rewriting their method in part (b). Some of the less successful attempts confused factors with roots.

**Question 9**

Most students drew a correct sketch of the curve, although some drew only the top half.

In part (b), the majority of students used centimetres and produced a bowl with sensible dimensions. This question was generally well answered, although a significant minority of students initially included $\pi$, but then lost it during their calculations.

In part (c), many students gave a sensible assumption, usually referring to the thickness of the material used. Some incorrect responses questioned the capabilities of 3-D printers.

**Question 10**

The vast majority of students clearly knew how to structure a proof by induction, but only a third of the students could produce a fully correct proof. Almost all attempts included a confirmation of the initial case of $n = 1$, and many of these then assumed the identity was true for $n = k$. Good students were able to successfully manipulate this equation into a form which satisfied the $n = k + 1$ case. Some of these, however, resorted to expanding their quartic expression and then demonstrating that it was equivalent to the required quartic. Although an acceptable method, this was a time consuming approach, and prone to error. However, those who succeeded in this usually went on to produce a satisfactory proof by induction, although a minority demonstrated a lack of understanding of the requirements of a proof by induction.

The majority of students were able to make some progress in part (b), correctly substituting 2$n$ into the appropriate formulae, but some of these then struggled with the algebra. Again, a minority of students opted to expand their quartic, instead of factorising, but then errors often crept in.

**Question 11**

Only a minority of students realised that this question was asking for the mean value. A common approach was to translate the graph $k$ units downwards and then solve $\int_1^4 \mathrm{f}(x) = 0$. It was not uncommon for students to assume that the intersection with $y = k$ occurred at $x = 2.5$. However, it was possible to use the unknown intersection point and solve $\int_p^4 y \, \mathrm{d}x = -\int_1^p y \, \mathrm{d}x$. Unfortunately, many students who attempted this method struggled with the subsequent algebra.

**Question 12**

The majority of students were able to calculate the determinant and show that it was equal to zero. However, a significant minority failed to make a conclusion after a correct calculation.

In part (b), most students were able to set up a correct inequality, although some wasted time searching for a root, having missed the connection with part (a). Those students who found the correct critical values, generally went on to identify the correct regions.

**Question 13**

In part (a), most students could write a function with at least one of the required asymptotes. Those students who found a function which satisfied all of the necessary criteria generally went for the straightforward rectangular hyperbola.

Most students were able to draw a suitable graph in part (b), although some sketches failed to show a correct approach to the asymptotes.

In part (c), many students found a correct intersection with $y = 5$, but a significant number failed to successfully progress further.

**Question 14**

Part (a) was well answered, although most circles were drawn freehand. However, an accurately drawn circle made both (b)(i) and (b)(ii) easier to solve.

Most students realised that the required line was a tangent to the circle, but many of these failed to realise that a simple sine ratio could be used to find the angle $\alpha$.

Part (b)(ii) was only answered well by only the most able students. Although most of the successful attempts used basic GCSE trigonometry, some elaborate methods were also employed, including the solution of quadratic simultaneous equations. A common error was to assume that the triangle with vertices 0, 3 and $w$ was right-angled. Another common mistake was to assume that $\theta$ and $\alpha$ were equal.

**Question 15**

Part (a) was well answered. A common error, however, was the incorrect use of brackets or missing brackets, for example $r + 3 – r + 2 = 1$.

Part (b) was also well answered. Many of those who failed to successfully prove the required identity wrote the list of differences beyond $r = n$.

**Question 16**

Only half of the students were able to make satisfactory progress with this question. Most of the successful attempts substituted $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (or equivalent) for **A**, and then solved the resulting simultaneous equations. Only the most able students were able to successfully isolate **A** or $\mathbf{A}^{-1}$ by rearranging the given matrix equation.

**Question 17**

Most successful attempts at question 17 came from using the definitions of sinh and cosh, and then solving the resulting exponential equation. Some weaker students struggled to simplify their exponential equation, resulting in a page of algebra which was rarely successful. Those students who swapped $\sinh\theta + \cosh\theta$ for $e^{\theta}$ produced very short and efficient solutions.

**Question 18**

Only the most able students were able to make progress with this question. Of these, there were some very good solutions which clearly explained why the expression had to be non-negative. Most students indicated that $-m$ was equal to the sum of the roots, and that $n$ was equal to the sum of the product pairs, but few were able to use these, along with the roots equation $\sum\alpha^2 = (\sum\alpha)^2 - 2\sum\alpha\beta$, to make progress. Many attempts were simply abandoned halfway.

**Question 19**

Almost all students correctly found the direction vectors, and most of these went on to write a vector equation for each of the two lines. Only the most able students were able to progress further, usually reducing the numbers involved to a more manageable size. The most common method was to find the two points which are closest and then find the distance between them. An alternative, but less popular, method found the scalar product of a unitised perpendicular vector with a vector between the two lines. Common errors included the assumption that the two lines were perpendicular to each other. The most common error in good attempts involved incorrect subtraction of negative values.

In part (b), the most common correct refinement of the model suggested that the lines should be treated as curves. Some were unable to gain credit as they did not express their answer as a refinement of the model.

## Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.