AS
FURTHER MATHEMATICS

Paper 2 – Discrete

7366/2D

Thursday 17 May 2018   Afternoon

Time allowed: 1 hour 30 minutes

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INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**

- **Answer ALL questions.**

- **You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).**

- **Do not write on blank pages.**

- **Show all necessary working; otherwise marks for method may be lost.**

- **Do all rough work in this book. Cross through any work you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**

- **The maximum mark for this paper is 40.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**

- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 **The table shows some of the outcomes of performing a modular arithmetic operation.**

	2	3
2		1
3	1	

Which pair are operations that could each be represented by the table?

Tick (✓) one box.

☐ **Addition mod 6 and multiplication mod 5**

☐ **Addition mod 6 and multiplication mod 6**

☐ **Addition mod 4 and multiplication mod 5**

☐ **Addition mod 4 and multiplication mod 6**

[1 mark]

2 The binary operation $\otimes$ is given by

$a \otimes b = 3\,a(5 + b) \pmod 8$

where $a, b \in \mathbb{Z}$

Given that $2 \otimes x = 6$, which of the integers below is a possible value of x?

Circle your answer.   [1 mark]

    0   1   2   3

[Turn over]

3   **Alex and Sam are playing a zero-sum game.**

The game is represented by the pay-off matrix for Alex.

Sam

	Strategy	S_1	S_2	S_3
Alex	A_1	2	2	3
	A_2	0	3	5
	A_3	−1	2	−2

3 (a)   **Explain why the value of the game is 2**
[3 marks]

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3 (b)   **Identify the play-safe strategy for each player.
[1 mark]**

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4   **The diagram shows a network of pipes.**

Each pipe is labelled with its upper capacity in $cm^3 \, s^{-1}$
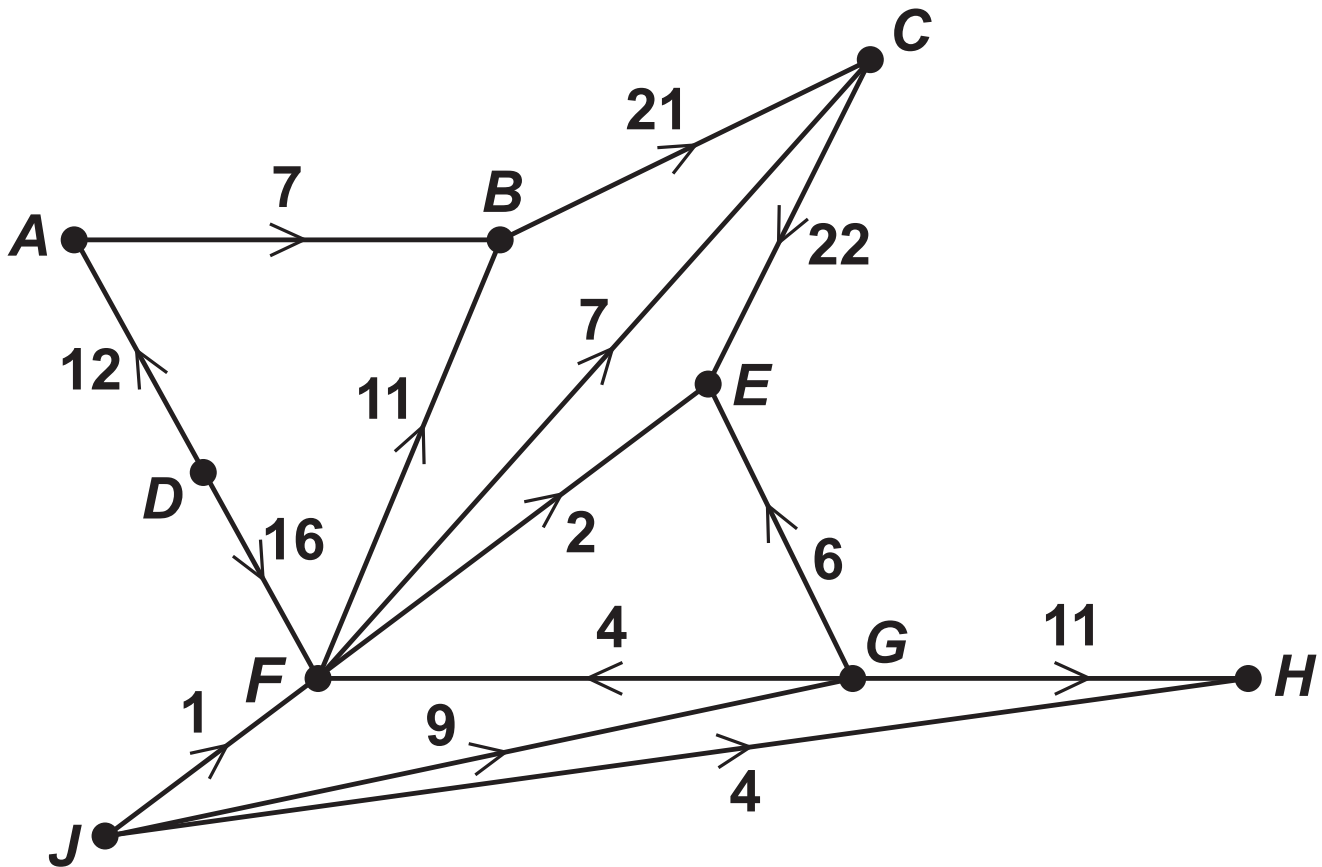
4 (a) (i) Find the value of the cut given by
{A, B, C, D, F, J} {E, G, H}.   [1 mark]

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4 (a) (ii) State what can be deduced about the maximum flow through the network.   [1 mark]

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4 (b) (i) List the nodes which are sources of the network.   [1 mark]

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4 (b) (ii) Add a supersource *S* to the network.   [1 mark]

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4 (c) (i) **List the nodes which are sinks of the network.**
[1 mark]

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4 (c) (ii) **Add a supersink _T_ to the network.** **[1 mark]**

[Turn over]

5 **A group of friends want to prepare a meal. They start preparing the meal at 6:30 pm**

Activities to prepare the meal are shown in Figure 1 below.

Figure 1

| Label	Activity	Duration (mins)	Immediate predecessors
A	Weigh rice	1	–
B	Cook rice	18	A
C	Drain rice	1	B
D	Chop vegetables	10	–
E	Fry vegetables	12	
F	Combine fried vegetables and drained rice	1	
G	Prepare sauce ingredients	4	–
H	Boil sauce	12	
I	Serve meal on plates	2	

5 (a) (i) Use Figure 2 shown below to complete Figure 1 opposite. [1 mark]

5 (a) (ii) Complete Figure 2 showing the earliest start time and latest finishing time for each activity. [2 marks]

Figure 2

Activity

Earliest
start time

Latest
finish time
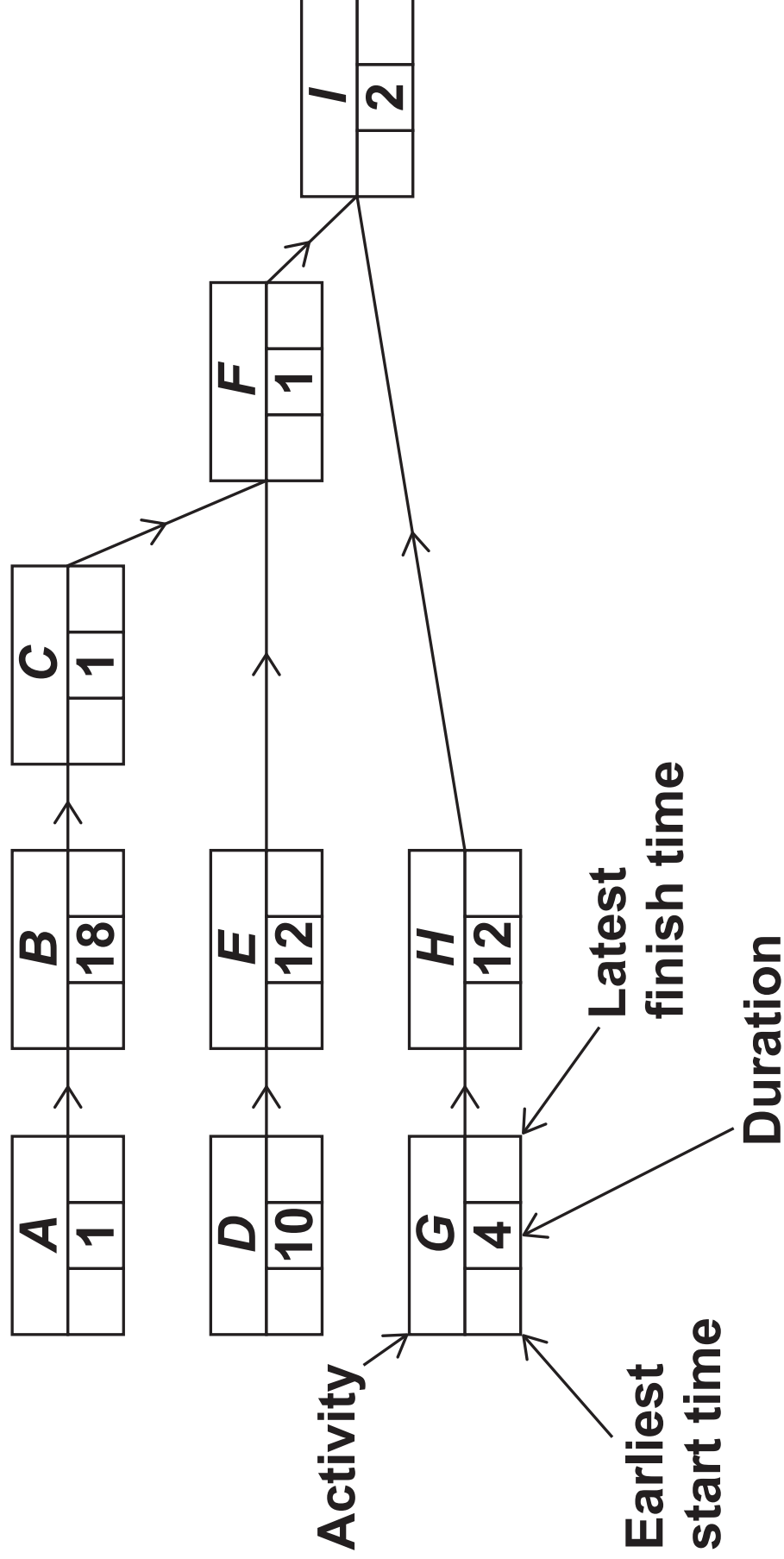
Duration

5 (b) (i) State the activity which must be started first so that the meal is served in the shortest possible time.

Fully justify your answer. [2 marks]

5 (b) (ii) Determine the earliest possible time at which the preparation of the meal can be completed.   [1 mark]

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5 (c) The group of friends want to cook spring rolls so that they are served at the same time as the rest of the meal. This requires the additional activities shown in Figure 3.

Figure 3

Label	Activity	Duration	Immediate predecessors
J	Switch on and heat oven		–
K	Put spring rolls in oven and cook		
L	Transfer spring rolls to serving dish		

It takes 15 seconds to switch on the oven.

The oven must be allowed to heat up for 10 minutes before the spring rolls are put in the oven.

It takes 15 seconds to put the spring rolls in the oven.

The spring rolls must cook in the hot oven for 8 minutes.

It takes 30 seconds to transfer the spring rolls to a serving dish.

5 (c) (i) Complete Figure 3 above. [1 mark]

5 (c) (ii) Determine the latest time at which the oven can be switched on in order for the spring rolls to be served at the same time as the rest of the meal.
[2 marks]

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6   An animal sanctuary has a rainwater collection site.

The manager of the sanctuary is installing a pipe system to connect the rainwater collection site to five other sites in the sanctuary.

Each site does not need to be connected directly to the rainwater collection site.

There are nine possible routes between the sites that are suitable for water pipes.

The distances, in metres, of the nine possible routes are given in the table below.

From/To	Henhouse (H)	Goatshed (G)	Kennels (K)	Cattery (C)
Rainwater collection site (R)	840	810	520	370
Cattery (C)	–	680	610	
Duckpond (D)	480	310		
Goatshed (G)	150			

Water pipe costs 60 pence per metre.

Find the minimum cost of connecting all the sites to the rainwater collection site.

Fully justify your answer.   [5 marks]

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7   **A linear programming problem has the constraints**

$$1 \leq x \leq 6$$
$$1 \leq y \leq 6$$
$$y \geq x$$
$$x + y \leq 11$$

7 (a) (i) Complete Figure 4 to identify the feasible region for the problem.   [2 marks]

Figure 4
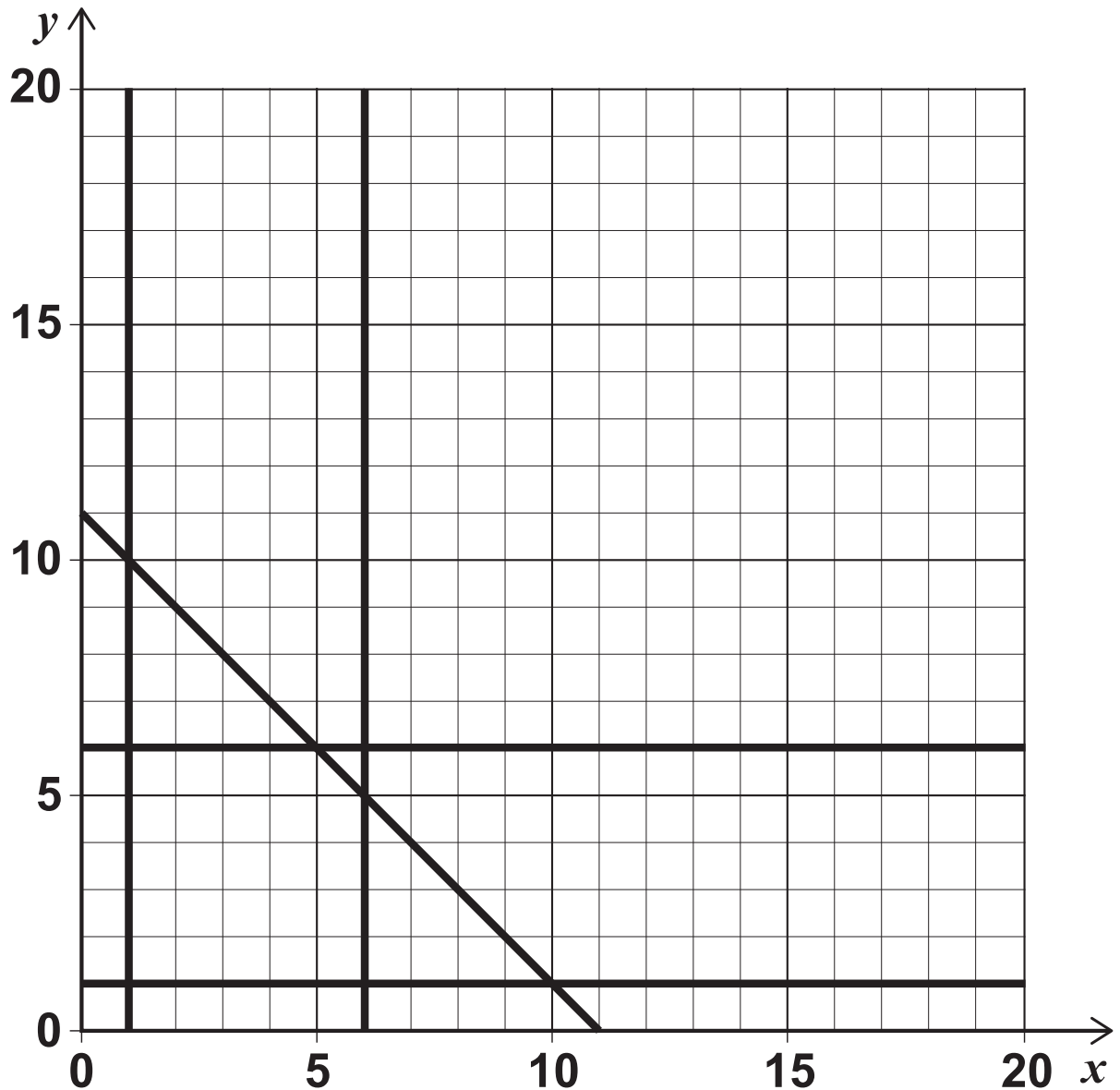
**7 (a) (ii) Determine the maximum value of $5x + 4y$
       subject to the constraints.   [2 marks]**

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7 (b) The simple-connected graph G has seven vertices.

The vertices of G have degree 1, 2, 3, v, w, x and y

7 (b) (i) Explain why $x \geq 1$ and $y \geq 1$ [1 mark]

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7 (b) (ii) Explain why $x \leq 6$ and $y \leq 6$ [1 mark]

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7 (b)(iii) Explain why $x + y \leq 11$ [2 marks]

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7 (b)(iv) **State an additional constraint that applies to the values of x and y in this context.   [1 mark]**

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7 (c) The graph G also has eight edges. The inequalities used in part (a)(i) apply to the graph G.

7 (c) (i) Given that $v + w = 4$, find all the feasible values of x and y. **[3 marks]**

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7 (c) (ii) It is also given that the graph G is semi-Eulerian.

On Figure 5, draw G.   [2 marks]

Figure 5
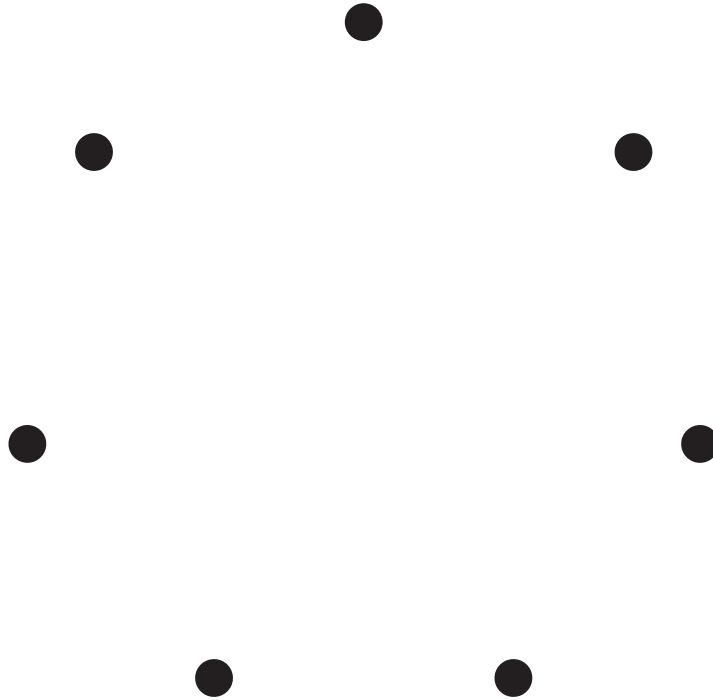
END OF QUESTIONS

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