Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

A-level
MATHEMATICS
Unit Further Pure 2

Friday 22 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Given that $\mathrm{f}(r) = \dfrac{1}{(2r+3)(2r+5)}$, show that

$$\mathrm{f}(r-1) - \mathrm{f}(r) = \frac{k}{(2r+1)(2r+3)(2r+5)}$$

where k is an integer.

[2 marks]

(b) Use the method of differences to find $\displaystyle\sum_{r=1}^{N} \frac{1}{(2r+1)(2r+3)(2r+5)}$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 1

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QUESTION PART REFERENCE	Answer space for question 1

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2 (a) The complex number $-2\sqrt{2}+2\sqrt{6}\,\mathrm{i}$ can be expressed in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$.

(i) Show that $r = (\sqrt{2})^n$ where n is an integer.

[2 marks]

(ii) Find the exact value of θ.

[1 mark]

(b) Hence solve the equation $z^5 = -2\sqrt{2}+2\sqrt{6}\,\mathrm{i}$ giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE	Answer space for question 2

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3 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = -1, \qquad u_{n+1} = \frac{u_n - 5}{3u_n - 7}$$

Prove by induction that $u_n = \dfrac{2^{n+1} - 5}{2^{n+1} - 3}$, for all integers $n \geqslant 1$.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE	Answer space for question 3

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4 (a) Express $(1+\mathrm{e}^{2x})(1+\mathrm{e}^{-2x})$ in terms of $\cosh x$.

[3 marks]

(b) Hence, find the value of

$$\int_0^1 \frac{1}{(1+\mathrm{e}^{2x})(1+\mathrm{e}^{-2x})}\,\mathrm{d}x$$

giving your answer in terms of e.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 4

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QUESTION PART REFERENCE	Answer space for question 4

Do not write outside the box

5 (a) The locus of points L, representing the complex number z, satisfies the equation

$$|z-2| = |z+4\mathrm{i}|$$

Sketch L on the Argand diagram below.

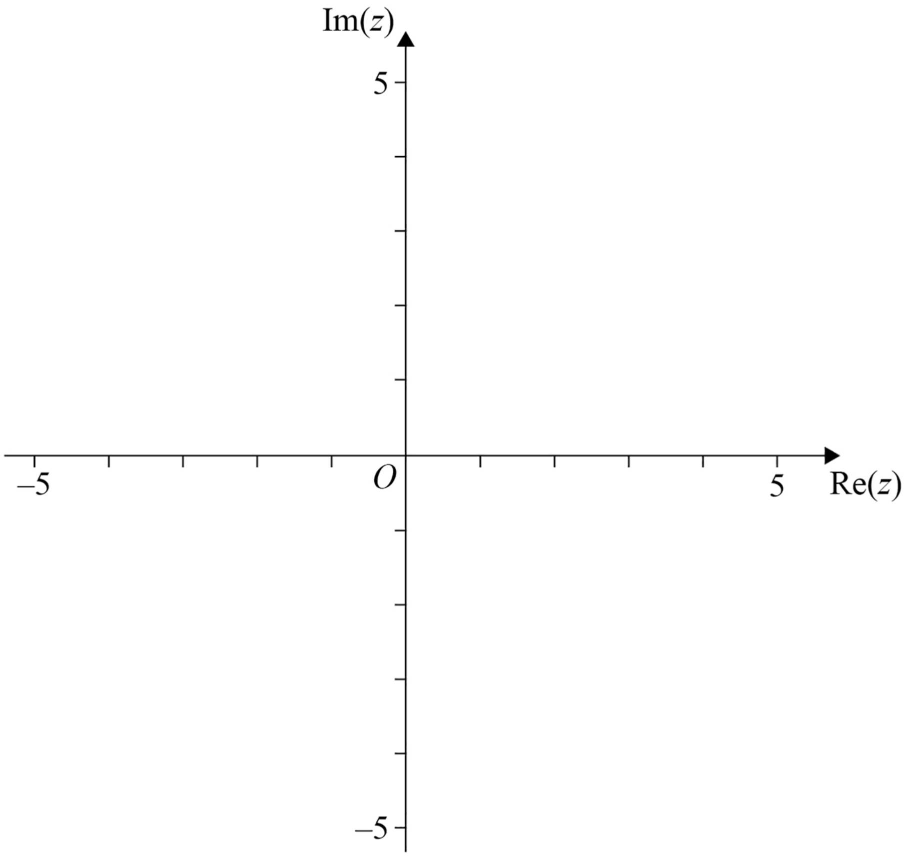
[3 marks]

(b) Given that $|z-2| = |z+4\mathrm{i}|$, find the complex number z_1 for which $|z|$ has its least value.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 5



QUESTION PART REFERENCE	Answer space for question 5

6 (a) Sketch the graph of $y = \cosh^{-1} x$ on the axes below.

[2 marks]

(b) Given that $y = \cosh^{-1} x$, show that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2 - 1}}$.

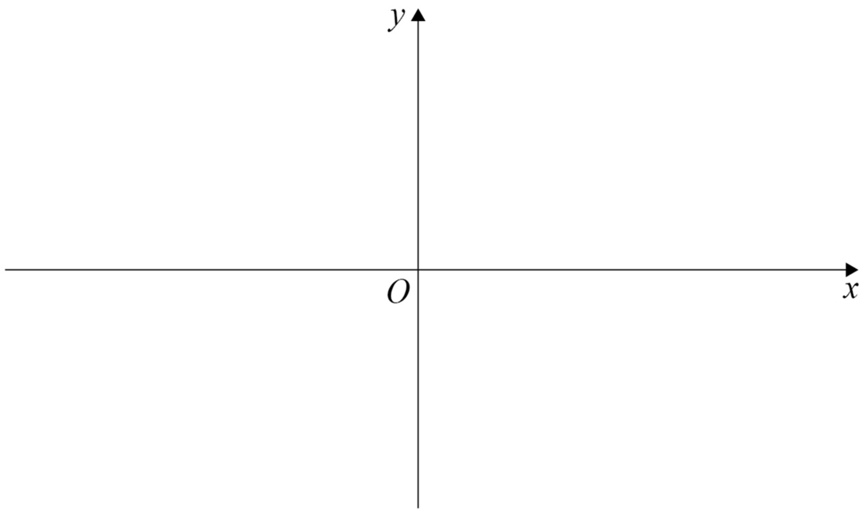
[3 marks]

(c) A curve has equation $y = \dfrac{5}{3} - 4x + \cosh^{-1}(3x)$. Show that the curve has a single stationary point, M, and express the y-coordinate of M as a natural logarithm.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 6



QUESTION PART REFERENCE	Answer space for question 6

QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE	Answer space for question 6

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7 A curve is defined parametrically by

$$x = 3 - \cos 2t, \qquad y = 4\sin t$$

The arc of the curve from $t = 0$ to $t = \frac{\pi}{2}$ is rotated through 2π radians about the x-axis to generate a surface with area S.

(a) Show that $S = k\pi\int_0^{\frac{\pi}{2}} \sin t \cos t\sqrt{(1 + \sin^2 t)}\,\mathrm{d}t$, where k is an integer.

[5 marks]

(b) Hence find the value of S, giving your answer in the form $\frac{\pi}{3}(m\sqrt{2} + n)$, where m and n are integers.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 7**

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8 The cubic equation $2z^3 + 5z + 3 = 0$ has roots α, β and γ.

(a) Write down the value of:

(i) $\alpha\beta + \beta\gamma + \gamma\alpha$;

[1 mark]

(ii) $\alpha\beta\gamma$.

[1 mark]

(b) Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

[2 marks]

(c) (i) Use the substitution $z^2 = \frac{1}{x}$ to show that

$$9x^3 - 25x^2 + mx + n = 0$$

where m and n are integers.

[4 marks]

(ii) **Hence** find the value of $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4}$.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 8

QUESTION PART REFERENCE	Answer space for question 8

9 (a) Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta + A\cos^3\theta + B\cos\theta$$

where A and B are integers.

[5 marks]

(b) (i) Given that $\cos 5\theta = 0$ and $\cos\theta \neq 0$, find the possible values of $\cos^2\theta$, giving your answers in simplified surd form.

[2 marks]

(ii) Hence find the exact value of $\cos^2\dfrac{3\pi}{10}$ and deduce that $\cos\dfrac{3\pi}{5} = \dfrac{1-\sqrt{5}}{4}$.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 9

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END OF QUESTIONS