AS

Mathematics

MFP2- Further Pure 2
Mark scheme

6360
June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
dM	mark is dependent on one or more **M** marks and is for method
A	mark is dependent on **M** or **dM** marks and is for accuracy
B	mark is independent of **M** or **dM** marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\mathrm{f}(r-1)=\frac{1}{(2r+1)(2r+3)}$			
	$\mathrm{f}(r-1)-\mathrm{f}(r)=\frac{2r+5-(2r+1)}{(2r+1)(2r+3)(2r+5)}$	**M1**		$\mathrm{f}(r-1)$ **correct and** attempt at common denominator
	$=\frac{4}{(2r+1)(2r+3)(2r+5)}$	**A1 cso**	**2**	correct use of brackets and k=4 correct
(b)	$\mathrm{f}(0)-\mathrm{f}(1)+\mathrm{f}(1)-\mathrm{f}(2)+\ldots$ **or** $\frac{1}{3\times5}-\frac{1}{5\times7}+\frac{1}{5\times7}-\frac{1}{7\times9}+\ldots$	**M1**		clear attempt to use **method of differences** (generous)
	$[A]\ \left\{\frac{1}{3\times5}\ldots-\frac{1}{(2N+3)(2N+5)}\right\}$	**dM1**		may be seen on several lines with terms cancelled; may have r, n for N for **dM1**
	$\frac{1}{60}-\frac{1}{4(2N+3)(2N+5)}$ **OE**	**A1**	**3**	must have N
	Total		**5**	
(a)	For **A1cso**, denominator must have factors in order given and numerator must be 4 or perhaps k with k=4 stated.			
(b)	Withhold **dM1** if errors seen in terms cancelled **Example** $\frac{1}{3\times5}-\cancel{\frac{1}{5\times7}}+\cancel{\frac{1}{5\times7}}-\cancel{\frac{1}{6\times8}}+\ldots-\frac{1}{(2N+3)(2N+5)}$ scores **M1 dM0** and hence **A0** Final **A1** is earned for $\frac{1}{4}\left\{\frac{1}{15}-\frac{1}{(2N+3)(2N+5)}\right\}$, $\frac{N(N+4)}{15(2N+3)(2N+5)}$ **OE**			

Q2	Solution	Mark	Total	Comment
(a)(i)	$\left[r^2=\right]\ \left(-2\sqrt{2}\right)^2+\left(2\sqrt{6}\right)^2$ **or** 32 OE	**M1**		may have $\left(2\sqrt{2}\right)^2+\left(2\sqrt{6}\right)^2$ **or** $r=\sqrt{32}$ **or** $r=4\sqrt{2}$
	$r=\left(\sqrt{2}\right)^5$	**A1**	**2**	$n=5$; must have "$r=$"
(ii)	$\theta=\frac{2\pi}{3}$	**B1**	**1**	
(b)	$r=\sqrt{2}$	**B1F**		r = fifth root of "their" $\left(\sqrt{2}\right)^n$ **OE**
	Use of de Moivre "their arg"/5	**M1**		or correct
	adding $\frac{2\pi}{5}\,or\,\frac{6\pi}{15}$ to obtain at least 2 other values of θ	**dM1**		
	$\theta=\frac{2\pi}{15},\ \frac{8\pi}{15},\ \frac{14\pi}{15},$ $-\frac{10\pi}{15}\left(or\,\frac{20\pi}{15}\right),-\frac{4\pi}{15}\left(or\,\frac{26\pi}{15}\right)etc$	**A1**		5 correct values of θ mod 2π
	Roots are $\sqrt{2}\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{15}},\sqrt{2}\,\mathrm{e}^{\mathrm{i}\frac{8\pi}{15}},\sqrt{2}\,\mathrm{e}^{\mathrm{i}\frac{14\pi}{15}},$ $\sqrt{2}\,\mathrm{e}^{\mathrm{i}\left(\frac{-10\pi}{15}\right)},\sqrt{2}\,\mathrm{e}^{\mathrm{i}\left(\frac{-4\pi}{15}\right)}$	**A1**	**5**	must be in exponential form with $\sqrt{2}$ and these 5 arguments for final mark $\sqrt{2}\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{15}}$ may be written as $\sqrt{2}\,\mathrm{e}^{\frac{2\pi\mathrm{i}}{15}}$ etc $\sqrt{2}\,\mathrm{e}^{\mathrm{i}\left(\frac{-10\pi}{15}\right)}$ may be written as $\sqrt{2}\,\mathrm{e}^{-\mathrm{i}\frac{2\pi}{3}}$ etc
	Total		**8**	
(a)	Condone $r=\sqrt{2}^{\,5}$ without the brackets round the square root for **A1** but $r=\sqrt{2^5}$ scores **M1A0**			
(b)	For first **A1** may have $\theta=\frac{2\pi}{15}+\frac{2k\pi}{5}$ **OE** but must also have $k=-2,-1,0,1,2$ or $k=0,1,2,3,4$ **OE** Withhold final **A1** if answer left as $\sqrt{2}\,\mathrm{e}^{\mathrm{i}\left(\frac{2\pi}{15}+\frac{2k\pi}{5}\right)}$ $k=-2,-1,0,1,2$ **OE**			

Q 3	Solution	Mark	Total	Comment
	When $n=1$, $u_1=\left(\frac{2^2-5}{2^2-3}\right)=\frac{4-5}{4-3}=-1$			be convinced they are evaluating u_1 from formula
	Therefore formula is true when *n* =1	**B1**		must also have statement
	Assume result is true for *n*=*k* (*)			
	$u_{k+1}=\dfrac{\left(\dfrac{2^{k+1}-5}{2^{k+1}-3}\right)-5}{3\left(\dfrac{2^{k+1}-5}{2^{k+1}-3}\right)-7}$	**M1**		condone one consistent error (may use different letter than *k*)
	$=\dfrac{2^{k+1}-5-5(2^{k+1}-3)}{3(2^{k+1}-5)-7(2^{k+1}-3)}$	**dM1**		attempt to multiply numerator and denominator by $2^{k+1}-3$ to obtain unsimplified single fraction
	$=\dfrac{-4\times2^{k+1}+10}{-4\times2^{k+1}+6}$	**A1**		single fraction with terms collected
	$=\dfrac{2\times2^{k+2}-10}{2\times2^{k+2}-6}\quad=\dfrac{2^{k+2}-5}{2^{k+2}-3}$	**A1**		must show one further step from previous line dealing with either 2 or minus signs
	Therefore true for *n*=*k*+1 (**) and since true for *n* =1 , formula is true for *n* =1,2,3, … by induction (***)	**E1**	**6**	must score previous five marks and have (*), (**) and (***)
	Total		**6**	

For **B1**, must mention here or later that the result is "true when *n* =1"; it is not enough to simply say "true for all integers $n \ldots 1$ " at the end to earn this **B1** mark – this on its own earns **B0**.

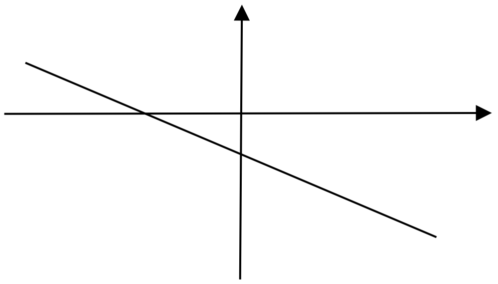
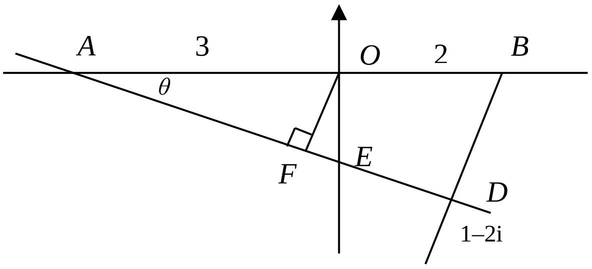
Condone statements such as "it works for *n* =1" or "therefore RHS=LHS" for **B1** mark but withhold **E1** mark; however simply putting a "tick" earns **B0** and hence **E0**.

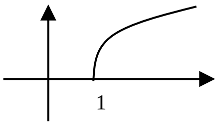
Alternative to (***) is "therefore true for all positive integers *n*" or "so true for all integers $n \ldots 1$" etc
Accept set notation such as "true $\forall n \in$ □ $^{+}$ " etc
However "so true for all $n \ldots 1$" is incorrect and immediately scores **E0**.

Allow **A1** marks for incorrect/missing/poor use of brackets if recovered later, but withhold **E1** mark..

May define P(*k*) as the "proposition that the formula is true when $n = k$ " and earn full marks. However, if P(*k*) is not defined then allow **B1** for showing P(1) is true but withhold **E1** mark.

Q 4	Solution	Mark	Total	Comment
(a)	$\left[\left(1+e^{2x}\right)\left(1+e^{-2x}\right)=\right]\;\; 1+e^{2x}+e^{-2x}+1$	**B1**		**or** $\left(1+e^{2x}\right)=e^{x}(e^{-x}+e^{x})$ **or** $\left(1+e^{-2x}\right)=e^{-x}(e^{x}+e^{-x})$
	Use of $e^{2x}+e^{-2x}=2\cosh 2x$ **OE**	**M1**		**or** use of $\cosh x=\frac{1}{2}\left(e^{x}+e^{-x}\right)$ **OE** **or** $\cosh^2 x=\frac{1}{4}\left(e^{2x}+2+e^{-2x}\right)$
	$\left[\left(1+e^{2x}\right)\left(1+e^{-2x}\right)=\right]\;\; 4\cosh^2 x$	**A1**	3	accept $\left(2\cosh x\right)^2$ **(*)**
(b)	$\int k\,\text{sech}^2 x\left(dx\right)$	**M1**		integrand of form $k\,\text{sech}^2 x$
	$\frac{1}{4}\tanh x$	**A1**		
	$\tanh 1=\frac{e^{(1)}-e^{-1}}{e^{(1)}+e^{-1}}$	**dM1**		tanh(1) written correctly in terms of e condone e^1 for **dM1**
	Integral = $=\frac{e-e^{-1}}{4(e+e^{-1})}$ **or** $\frac{e^2-1}{4(e^2+1)}$ **OE**	**A1**	4	must be in terms of e and not left with e^1 or $\frac{1}{2}$ in numerator and denominator
	Total		7	
(a)	Any formula in $\cosh x$ or $\cosh 2x$ must be correct for **M1** (*) May earn **M1** in part **(b)** and even **A1** if $4\cosh^2 x$ is explicitly seen.			
(b)	Accept $\frac{1}{e}$ for e^{-1} for final **A1**			

Q 5	Solution	Mark	Total	Comment
(a)		**M1**		line with intention of being perpendicular bisector of 2+0i and 0 – 4i
		**A1**		passing through 1–2i "by eye"
	correct line in three quadrants indicated by crossing Re axis closer to –3 than –2.5 and crossing Im axis between –1 and –2	**A1**	3	withhold final **A1** if freehand line is totally unacceptable– otherwise condone "sketch" if it is clearly intended to be a straight line provided it satisfies criteria on LHS
(b)	$(x-2)^2+y^2=x^2+(y+4)^2$ $x+2y+3=0$ **or** $y=-\frac{1}{2}x-\frac{3}{2}$	**B1**		**or** $y+2=-\frac{1}{2}(x-1)$ **OE**
	$y=2x$	**B1**		perpendicular line through origin **PI** by **M1**
	"their" $x+2(2x)+3=0$ **OE**	**M1**		correctly eliminating x or y from "their" equations
	$x=-\frac{3}{5}\,;\,y=-\frac{6}{5}$			
	$[z_1=]$ $-0.6-1.2\text{i}$ **OE**	**A1**	4	must write as complex number
	Total		**7**	
(b)	**Alternative:**  Gradient of AD = – 0.5 ; $\tan\theta=0.5$; F corresponds to z_1 ; may use $\lvert z_1\rvert$ for OF Sim triangles or trig : $\left(\frac{OF}{3}=\frac{\sqrt{5}}{5}\Rightarrow\right)OF=\frac{3}{\sqrt{5}}$ OE **B1** ; $\sin\theta=\frac{1}{\sqrt{5}},\ \cos\theta=\frac{2}{\sqrt{5}}$ or $\tan\theta=\frac{1}{2}$ **B1** $x_1=-OF\sin\theta$ and $y_1=-OF\cos\theta$ **or** use of $\lvert x_1\rvert$ and $\lvert y_1\rvert$ **or** $z_1=-OF\sin\theta\ -\left(OF\cos\theta\right)\text{i}$ **M1** $\left[z_1=-\frac{3\sqrt{5}}{5}\times\frac{1}{\sqrt{5}}--\frac{3\sqrt{5}}{5}\times\frac{2}{\sqrt{5}}\text{i}\right]$; $[z_1=]$ $-\frac{3}{5}-\frac{6}{5}\text{i}$ **OE (**must write as complex number) **A1**			

Q 6	Solution	Mark	Total	Comment
(a)	Graph roughly correct in 1st quadrant	**M1**		condone extra branch in 4th quadrant for **M1** 
	$y=\cosh^{-1}x$ (only drawn for $y \ldots 0$); “vertical” at (1,0); 1 marked on x-axis	**A1**	**2**	
(b)	$x=\cosh y \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\sinh y$ **or** $\frac{\mathrm{d}y}{\mathrm{d}x}\sinh y=1$	**M1**		
	Use of $\cosh^2 y-\sinh^2 y=1$	**dM1**		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\pm\frac{1}{\sqrt{x^2-1}}$ but graph has positive gradient so $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{x^2-1}}$	**A1**	**3**	must see ± and – sign rejected because of positive gradient for final mark **AG**
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-4+\frac{3}{\sqrt{(3x)^2-1}}$	**B1**		
	$\frac{3}{\sqrt{9x^2-1}}=4\Rightarrow\frac{9}{9x^2-1}=16$	**M1**		isolating and squaring **FT** “their ”$\frac{\mathrm{d}y}{\mathrm{d}x}$
	$x^2=\frac{25}{144}$	**A1**		or $x=\pm\frac{5}{12}$
	$\cosh^{-1}3x$ not defined for $3x<1$ [Hence only SP when] $x=\frac{5}{12}$	**E1**		correct **and** reason given for rejection of invalid solution such as $x \ldots \frac{1}{3}$
	$y=\frac{5}{3}-\frac{5}{3}+\cosh^{-1}\left(\frac{5}{4}\right)$			$y=\ln\left(\frac{5}{4}+\sqrt{\frac{25}{16}-1}\right)$
	$y=\ln 2$	**B1**	**5**	may simply use calculator if x is correct but award **B0** if clearly **FIW**
	Total		**10**	

(a) Gradient at (1,0) must be clearly >1 for **A1 ;** award **A0** if graph has stationary point

(b) **Alternative:** $y=\ln\left(x+\sqrt{x^2-1}\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1+\frac{1}{2}\times 2x\Big/\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$ **M1**

multiplying top and bottom by $\sqrt{x^2-1}$ or by $(x-\sqrt{x^2-1})$ **dM1** showing that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{x^2-1}}$ **A1**

(c) Not enough to simply say $x\neq-\frac{5}{12}$; this scores **E0,**

Referring to graph in part **(a)** or wrong reason for rejection such as $x \ldots 0$ etc also scores **E0.**
Candidates may score **B1 M1 A1 E0 B1** in this part.
For final **B1** may find $y=\ln 2$ and $y=-\ln 2$ and then reject negative value.

Q 7	Solution	Mark	Total	Comment
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}=2\sin 2t$ **;** $\frac{\mathrm{d}y}{\mathrm{d}t}=4\cos t$	**B1**		PI by next line; ***make sure there are no minus signs here or inside brackets on next line***
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2=(2\sin 2t)^2+(4\cos t)^2$	**M1**		their $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2$
	Use of $\sin 2t=2\sin t\cos t$	**dM1**		
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2=16\cos^2 t\sin^2 t+16\cos^2 t$	**A1**		condone sign error in earlier differentiation for first **A1 only**
	$16\cos^2 t(1+\sin^2 t)$ **or** $4\cos t\sqrt{(1+\sin^2 t)}$			
	$S=32\pi\int_0^{\frac{\pi}{2}}\sin t\cos t\sqrt{\left(1+\sin^2 t\right)}\,\mathrm{d}t$	**A1**	**5**	**AG** be convinced – must have $S=\ldots,32\pi$ $\mathrm{d}t$ and limits and $1+\sin^2 t$ with terms in that order but condone $\sqrt{1+\sin^2 t}$
(b)	$p\left(1+\sin^2 t\right)^{\frac{3}{2}}$	**M1**		where p is a constant
	$\left[\frac{k\pi}{3}\left(1+\sin^2 t\right)^{\frac{3}{2}}\right]$	**A1F**		**FT** their k (for 32) from part **(a)**
	$\frac{k\pi\left(2^{\frac{3}{2}}-1\right)}{3}$	**dM1**		correct sub of limits into expression of form $p\left(1+\sin^2 t\right)^{\frac{3}{2}}$
	$(S=)\ \frac{\pi\left(64\sqrt{2}-32\right)}{3}$	**A1**	**4**	condone $\frac{32\pi\left(2\sqrt{2}-1\right)}{3}$
	Total		**9**	
(a)	A candidate starting with $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2=(-2\sin 2t)^2+(4\cos t)^2$, for example, and concluding with correct final expression can score at most **B0 M1 dM1 A1 A0** May have " $S=$ " on earlier line with equals signs on each line before final line for **A1** Condone " $S_x=$ " since this is given in the Formulae booklet.			
(b)	May use substitution such as $u=1+\sin^2 t$ integrating to $pu^{\frac{3}{2}}$ for **M1** and $\frac{k\pi}{3}u^{\frac{3}{2}}$ for **A1F** or use of double angles with **M1** for $p(3-\cos 2t)^{\frac{3}{2}}$ **A1F** for $\frac{k\pi\sqrt{2}}{3}(3-\cos 2t)^{\frac{3}{2}}$ etc			

Q 8	Solution	Mark	Total	Comment
(a)(i)	$[\alpha\beta+\beta\gamma+\gamma\alpha=]\quad \frac{5}{2}$	B1	1	
(ii)	$[\alpha\beta\gamma=]\quad -\frac{3}{2}$	B1	1	
(b)	$\sum\frac{1}{\alpha}=\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$	M1		**or** use of $z=1/y$ *etc* to obtain $3y^3+5y^2+2=0$.
	$=-\frac{5}{3}$	A1cso	2	must follow from correct part **(a)** values unless starts again with $z=1/y$ *etc*.
(c)(i)	$2z\left(\frac{1}{x}\right)+5z+3=0\Rightarrow z\left(\frac{2}{x}+5\right)+3=0$	M1		substituting $z^2=\frac{1}{x}$ into equation and attempt to collect terms in z
	$z^2\left(\frac{2}{x}+5\right)^2=9$ and attempt to eliminate z	dM1		squaring both sides & **only in terms of** $\boldsymbol{x}$
	Alternative : $z(2z^2+5)=-3$ & squaring first $z^2(2z^2+5)^2=9$ **M1** then substituting $z^2=\frac{1}{x}$ **dM1**			
	$\frac{1}{x}\left(\frac{2}{x}+5\right)^2=9$	A1		correct with z eliminated
	$(2+5x)^2=9x^3$ $9x^3-25x^2-20x-4=0$	A1	4	$m=-20;\quad n=-4$
(ii)	$\sum\frac{1}{\alpha^4}=\sum\ \alpha_1^{\ 2}$	B1		recognition that sum of squares of roots of equation in x required
	$=(\alpha_1+\beta_1+\gamma_1)^2-2(\alpha_1\beta_1+\beta_1\gamma_1+\gamma_1\alpha_1)$	M1		correct identity but must have earned **B1**
	$=\left(\frac{25}{9}\right)^2-\frac{2\times\text{"}their\text{"}m}{9}$	dM1		correct substitution **FT** their m
	$=\frac{985}{81}$	A1cso	4	
	Total		12	
(c)(i)	**Alternative:** condone use of $z=\frac{1}{\sqrt{x}}$ giving $\frac{2}{x\sqrt{x}}+\frac{5}{\sqrt{x}}+3=0$ or $2+5x+3x\sqrt{x}=0$ **M1** then $\frac{1}{x}\left(\frac{2}{x}+5\right)^2=9$ or $(2+5x)^2=9x^3$ **dM1 A1** but **only award final A1** if $z=\pm\frac{1}{\sqrt{x}}$ is used or $\left(2+5x+3x\sqrt{x}\right)\left(2+5x-3x\sqrt{x}\right)=0$ and expansion attempt to eliminate $\sqrt{x}$ for **dM1** then $(2+5x)^2-9x^3=0$ **OE** for **A1** but **only award final A1** if $z=\pm\frac{1}{\sqrt{x}}$ is used			
(c)(ii)	Candidates may write; Sum of roots $=\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{25}{9}$ for **B1** and the identity as $\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)^2=\left(\frac{1}{\alpha^2}\right)^2+\left(\frac{1}{\beta^2}\right)^2+\left(\frac{1}{\gamma^2}\right)^2+2\left(\frac{1}{\alpha^2\beta^2}+\frac{1}{\beta^2\gamma^2}+\frac{1}{\gamma^2\alpha^2}\right)$ **OE** for **M1** even if **B0** earned			

Q 9	Solution	Mark	Total	Comment
(a)	$\cos 5\theta + \mathrm{i}\sin 5\theta = (\cos\theta + \mathrm{i}\sin\theta)^5$	**B1**		**or** $\cos 5\theta = \mathrm{Re}\left[(\cos\theta + \mathrm{i}\sin\theta)^5\right]$
	$(c+\mathrm{i}s)^5 = c^5 + 5c^4(\mathrm{i}s) + 10c^3(\mathrm{i}s)^2$ $+10c^2(\mathrm{i}s)^3 + 5c(\mathrm{i}s)^4 + (\mathrm{i}s)^5$	**M1**		accept real part only (condone one error in one real term) (*)
	$(\cos 5\theta =)$ $c^5 + 10c^3(\mathrm{i})^2(1-c^2) + 5c(\mathrm{i})^4(1-c^2)^2$	**dM1**		taking real part and use of $s^2 = 1-c^2$ twice
	$= c^5 - 10c^3(1-c^2) + 5c(1-2c^2+c^4)$	**A1**		correct – with powers of i simplified – and final bracket squared correctly
	$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	**A1**	5	must have $\cos 5\theta = \ldots$; $A = -20, \quad B = 5$
(b)(i)	$\cos 5\theta = 0$ **(but** $\cos\theta \neq 0$ **)**			
	$(\cos^2\theta =)\dfrac{20 \pm \sqrt{80}}{32}$	**M1**		using their *A* and *B* from part **(a)** **PI** by correct simplified surd values cannot earn this **M1** in part **(b)(ii)**
	$= \dfrac{5 \pm \sqrt{5}}{8}$	**A1**	2	$\dfrac{5+\sqrt{5}}{8}$, $\dfrac{5-\sqrt{5}}{8}$
(ii)	**Two** roots of **quadratic** are $\cos^2\dfrac{\pi}{10}$ and $\cos^2\dfrac{3\pi}{10}$ **OE**	**B1**		one root must be $\cos^2\dfrac{3\pi}{10}$ and other a correct equivalent alternative to $\cos^2\dfrac{3\pi}{10}$
	$\cos^2\dfrac{3\pi}{10} = \dfrac{5-\sqrt{5}}{8}$ **since** $\cos^2\dfrac{3\pi}{10} < \cos^2\dfrac{\pi}{10}$	**E1**		must justify choice and be correct
	$\left[\cos\dfrac{3\pi}{5} = 2\cos^2\dfrac{3\pi}{10} - 1 = \right]$ $\dfrac{2(5-\sqrt{5})}{8} - 1$	**M1**		**FT** their surd value of $\cos^2\dfrac{3\pi}{10}$
	$\left[= \dfrac{5-\sqrt{5}}{4} - 1 \Rightarrow\right]$ $\cos\dfrac{3\pi}{5} = \dfrac{1-\sqrt{5}}{4}$	**A1cso**	4	**AG** must score other 3 marks to earn final **A1cso**
	Total		11	
(a)	$\cos 5\theta = (\cos\theta + \mathrm{i}\sin\theta)^5$ scores **B0** ; it is possible to earn **B0 M1 dM1 A1 A1** if there is no evidence of de Moivre’s theorem being used correctly. (*) Ignore errors in imaginary terms in a full expansion approach for **M1** but withhold final **A1** mark Other errors even though recovered should be penalised by withholding final **A1** mark May write $\cos 5\theta =$ with each subsequent line having “=” sign used correctly for final **A1**			
(b)(ii)	Roots cannot come from original quartic/quintic for **B1** and **E1** since “Hence” but can score **M1**			