A-level

MATHEMATICS

Unit Further Pure 2

Report on the Examination

6360

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General

The early questions gave students the opportunity to demonstrate their understanding of particular topics such as summation of series, complex numbers, hyperbolic functions and proof by induction for a sequence. When a result is given in the question to be proved, students must write their final result in exactly the form as the printed answer. The slightly more unfamiliar integration questions caused some problems to those who could not apply basic techniques from A-Level Mathematics.

Some of the writing and presentation this year was very poor and this makes students' work particularly difficult to read when being marked as a scanned image. In particular it is very difficult to mark graphs when several attempts have been made on the same set of axes since it is impossible to distinguish between lines drawn in pencil or ink.

Question 1

(a) Most students found the correct value of k but did not score full marks if they missed out brackets when writing the numerator as $2r+5-2r+1$ even if they then wrote this as 4. It was rather surprising to see quite a few writing their common denominator as $(2r+1)(2r+3)^2(2r+5)$, making unnecessary extra work of factorising and cancelling.

(b) The difference method was generally well understood. Far too many students lost the final mark because they failed to write their final expression in terms of N, choosing to write their final answer in terms of r or n, ignoring the upper limit of the summation given in the question.

Question 2

The structure here resulted in a high success rate with students doing much better than the last time a similar question was set.

(a)(i) The words "show that" were ignored by many students who never wrote "r =" in their solution and lost a mark. They needed to show some working such as $r = 4\sqrt{2} = \left(\sqrt{2}\right)^5$, even though the value $4\sqrt{2}$ was undoubtedly obtained from a calculator.

(ii) Probably because of the widespread use of calculators with advanced features, eighty percent of students obtained the correct value of θ. It is a pity that the previous good practice of sketching an appropriate diagram, indicating that the complex number was in the second quadrant, seemed to be less in evidence these days and many were happy to just write $\theta = -\frac{\pi}{3}$.

(b) Students were more successful this year in finding the correct five solutions to the given equation with all the angles stated in the required interval. More than half the students scored full marks on this part. Some credit was given to those with incorrect answers from parts **(a)** and **(b)** if they used de Moivre's theorem correctly.

Question 3

This was one of the easier proofs by induction because it involved a sequence and those who coped with the fractional expressions in both the numerator and denominator scored high marks in this question. About a third of the students scored full marks this year. Over the years as students have had access to model answers to past papers this has improved the quality of solutions in proof by induction. There was less fabricating of working this year as students moved towards a target expression, although examiners still had to be vigilant to spot incorrect working leading to a correct formula for u_{k+1}. A few students found the value of u_2 and yet seemed to feel they had proved the result true for $n=1$.

Question 4

(a) Unsurprisingly most students multiplied out the brackets correctly but many then left their simplified answer as $2+2\cosh 2x$. Some were given credit later when tackling the integral if they went a step further and wrote the denominator in the expected form $4\cosh^2 x$. As a result almost two thirds of the students earned full marks for this part.

(b) Those who wrote the integrand as $\frac{1}{4}\text{sech}^2 x$ usually went on to complete and over a quarter of students scored full marks. Students were expected to simplify e^1 when writing their final answer.

Question 5

(a) The vast majority drew a straight line with most of these students recognising that they needed to draw the perpendicular bisector of $2+0\text{i}$ and $0-4\text{i}$. Just over half the students scored full marks for their sketch.

(b) Most students worked with a general complex number $x+\text{i}y$ and solved the simultaneous equations $x+2y+3=0$ and $y=2x$ to find z_1 ; others used an optimisation method based on minimising the distance from O to the straight line $x+2y+3=0$, considering an expression such as $x^2+y^2=(2y+3)^2+y^2$ then either using calculus or completing the square; a few used methods based on trigonometry or similar triangles but only the best students persevered to obtain the correct answer. About one in five students scored full marks. A common mistake was finding the distance from O to the straight line rather than the complex number z_1. It is also fair to say that many students made no attempt to answer this part of the question.

Question 6

Many students had no real understanding of the function $\cosh^{-1} x$ and its associated domain and range and failed to see the important difference between the graphs of $y=\cosh^{-1} x$ and $x=\cosh y$.

(a) Some students simply drew the graph of $y=\cosh x$ whereas others sketched the graph of $y=\text{cosech}x$. The vast majority realised that they should consider the reflection of $y=\cosh x$ in the line $y=x$ but many then drew the graph of $x=\cosh y$ failing to realise that this was not a function. Only the portion of the graph for which y was non negative should have been drawn. It was also necessary to have an infinite gradient at the end point (1,0). As a result less than half of the sketches were worthy of full marks.

(b) Two main approaches were seen here but just over one tenth of the attempts scored full marks. The majority wrote $x = \cosh y$ and then found that $\frac{\mathrm{d}x}{\mathrm{d}y} = \sinh y$ or differentiated implicitly to obtain $1 = \sinh y \frac{\mathrm{d}y}{\mathrm{d}x}$. Students then used the identity $\cosh^2 y - \sinh^2 y = 1$ and in order to complete the proof needed to obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \pm \frac{1}{\sqrt{x^2 - 1}}$ with an explanation as to why the negative sign should be rejected. Because the vast majority of students failed to appreciate that the gradient of $y = \cosh^{-1} x$ was always positive very few were able to earn the final mark when using this approach.
The other approach was using the identity $\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$. The difficulty here was even when the derivative had been found correctly students struggled to simplify an expression such as $\frac{1 + x(x^2 - 1)^{-\frac{1}{2}}}{x + \sqrt{x^2 - 1}}$ into the form of the printed answer and many tried to convince examiners with erroneous steps in their working.

(c) Most students made an attempt at this part but many failed at the first hurdle when they were unable to differentiate $\cosh^{-1}(3x)$. Nevertheless they were able to earn a method mark for isolating and squaring their expression involving a square root term. The better students were able to find the correct values of x and y but very few made a correct statement regarding the domain of $\cosh^{-1}(3x)$ being $x \ldots \frac{1}{3}$ when explaining why the only stationary point occurred when $x = \frac{5}{12}$ and consequently fewer than ten percent of students earned full marks.

Question 7

(a) Sloppy work denied some students the mark assigned for reaching the correct printed expression for the surface area of revolution; this required, $S =$, a correct integrand, 32π , the correct limits and $\mathrm{d}t$ in the integral. It was pleasing to see that most students could differentiate the parametric equations correctly followed by an expression for $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2$. The majority of students realised that they had to use the double angle identity $\sin 2t = 2\sin t \cos t$ but arithmetic errors sometimes occurred when this was squared and there was quite a lot of crossing out as they worked towards the printed answer. Others went down a cul-de-sac by converting $\sin^2 2t$ into an expression involving $\cos^2 2t$ and then in terms of either $\sin^2 t$ or $\cos^2 t$; some eventually returned to their earlier work after half a page of working whereas most aborted when they obtained an expression looking nothing like the printed answer. Just under half the students earned full marks for this proof.

(b) More confident students immediately realised that the integral was of the form $p(1+\sin^2 t)^{\frac{3}{2}}$ and then found the value of p by anti-differentiation. Others used a number of sensible substitutions such as $u=1+\sin^2 t$ to arrive at a similar stage, namely $\frac{32\pi}{3}u^{\frac{3}{2}}$. Examiners are aware of the capabilities of modern calculators and no doubt these were used to check that their final answer was correct. An important stage in any solution was writing down the correctly integrated expression before substituting limits. Therefore it was not expected that students would simply use their advanced calculator and copy down from its screen something such as $\int_0^1 u\sqrt{1+u^2}\,\mathrm{d}u=\frac{2\sqrt{2}}{3}-\frac{1}{3}$. The instruction on the front of the question paper booklet says "Show all necessary working; otherwise marks for method may be lost."

Question 8

(a)(i) Very high scoring; the only errors were from those who forgot to divide by 2 or who inserted a minus sign.

(a)(ii) Similarly here the success rate was very high apart from those few who forgot the minus sign or the division by 2.

(b) Those who had part **(a)** correct inevitably scored full marks here for this well-learned technique.

(c)(i) This was meant to be challenging and it proved to be the case with most students only earning the first easy method mark. More able students wrote $2z^3+5z=-3$, squared both sides and substituted $z^2=\frac{1}{x}$ and obtained the correct polynomial equation in x in just a few lines. The most common approach seen by students was substituting $z=\frac{1}{\sqrt{x}}$; only a few had the insight to write $2+5x+3x\sqrt{x}=0$ in a form which would eliminate the square root term. The expected approach $(2+5x)^2=9x^3$ was seen far less than attempts at $\left(2+5x+3x\sqrt{x}\right)^2=0$ or $\left(2+5x+3x\sqrt{x}\right)\left(2+5x-3x\sqrt{x}\right)=0$ but the maximum mark for each of these approaches was only 3 marks unless they used $z=\pm\frac{1}{\sqrt{x}}$.

(c)(ii) The word "**hence**" in bold print was intended as a trigger that the sum of the squares of the roots of the equation derived in part **(c)(i)** was required. Although only about one in ten found the correct value of $\frac{1}{\alpha^4}+\frac{1}{\beta^4}+\frac{1}{\gamma^4}$, most of the students who attempted this part usually wrote down a correct identity such as $\alpha_1^2+\beta_1^2+\gamma_1^2=(\alpha_1+\beta_1+\gamma_1)^2-2(\alpha_1\beta_1+\beta_1\gamma_1+\gamma_1\alpha_1)$ but often made errors when substituting values from the correct polynomial equation.

Question 9

(a) A mark was available for all students who wrote down de Moivre's theorem for $n=5$. Those students who then used Pascal's triangle and immediately worked with the real part arrived at the printed result after just a few lines of working. Most realised the need to use $\sin^2\theta = 1-\cos^2\theta$ and $\sin^4\theta = \left(1-\cos^2\theta\right)^2$ but some students were rather sloppy in their working and failed to indicate when they were considering $\cos 5\theta + \mathrm{i}\sin 5\theta$, or $\cos 5\theta$ in the various lines of their proof. In general this was a formula that seemed well rehearsed and most students scored well on this part with just less than sixty percent scoring full marks.

(b)(i) Most students who understood what was required scored full marks, most of them using their calculators to solve the quadratic equation in $\cos^2\theta$ to obtain answers in simplified surd form. A few did not read the question properly and tried to find values of θ that satisfied $\cos 5\theta = 0$.

(ii) Students were expected to state two possible roots of the earlier quadratic such as $\cos^2\frac{\pi}{10}$ and $\cos^2\frac{3\pi}{10}$ and then deduce that $\cos^2\frac{3\pi}{10} = \frac{5-\sqrt{5}}{8}$ since $\cos^2\frac{3\pi}{10} < \cos^2\frac{\pi}{10}$. This explanation was rarely seen and so the only mark available was for using the double angle formula and writing $\cos\frac{3\pi}{5} = 2\cos^2\frac{3\pi}{10} - 1 = \frac{5-\sqrt{5}}{4} - 1 = \frac{1-\sqrt{5}}{4}$. This was a challenging end to the paper and was marked fairly strictly so only a handful of students were awarded full marks.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.

Converting Marks into UMS marks

Convert raw marks into Uniform Mark Scale (UMS) marks by using the link below.

UMS conversion calculator www.aqa.org.uk/umsconversion