AQA

A-level
Mathematics

MFP3- Further Pure 3

Mark scheme

6360

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	The interval of integration is infinite	**B1**	**1**	OE. Do NOT apply ISW
(b)	(I=) $\int \frac{x-1}{e^x}\,dx = \int (x-1)e^{-x}\,dx$ $u = x-1,\quad \frac{dv}{dx} = e^{-x}$ (I) $= -(x-1)e^{-x} - \int -e^{-x}\,(dx)$	**M1**		$\frac{du}{dx}=1,\ v=\pm e^{-x}$ used in correct integration by parts formula. PI by next line ie correct integration
	$= -(x-1)e^{-x} - e^{-x}\ (+c)$ $(= -x\,e^{-x})$	**A1**		$-(x-1)e^{-x} - e^{-x}$ OE
	$\int_1^{\infty} \frac{x-1}{e^x}\,dx = \lim_{a\to\infty} \int_1^{a} (x-1)e^{-x}\,dx$	**M1**		Evidence of limit ∞ having been replaced by a (OE) at any stage and $\lim_{a\to\infty}$ seen or taken at any stage with no remaining lim relating to 1.
	$= \lim_{a\to\infty}\{-a\,e^{-a}\} - (-e^{-1})$ $= 0 + e^{-1} = e^{-1}$	**A1**	**4**	Be convinced
	Total		**5**	
(b)	May see integral split, ie $= \int x\,e^{-x}\,dx - \int e^{-x}\,dx = -x\,e^{-x} - \int -e^{-x}\,dx - \int e^{-x}\,dx = -x\,e^{-x}$. For the first M1 mark, apply the main scheme, with $u = x$ following a correct split.			

Q2	Solution	Mark	Total	Comment
	DO NOT ALLOW ANY MISREADS IN THIS QUESTION			
	$k_1 = 0.3\times(2+1.5) = 1.05 \quad \left(=\frac{21}{20}\right)$	**B1**		Correct exact value for k_1 seen or used
	$k_2 = 0.3\left[2.3+\frac{1}{2}\log_2(8+1.05)\right]$ $= 0.3\left[2.3+\frac{1}{2}(3.1779...)\right]$	**M1**		$k_2 = 0.3\left[2+0.3+\frac{1}{2}\log_2(7+1+\text{c's } k_1)\right]$ seen or used
	$= 1.166(687\ldots)$	**A1**		1.166 , 1.166… , 1.167 PI by later correct evaluation(s)
	$(y(2.3)=) \quad 1+\frac{1}{2}(1.05+1.166....)$ $(= 2.108(3\ldots))$	**dM1**		$1+\frac{1}{2}(\text{c's } k_1 \text{ value} + \text{c's } k_2 \text{ value})$ seen or used. If not seen, ft evaluation must be correct to at least 4sf.
	$= 2.108$ (to 4sf)	**A1**	5	CAO Must be 2.108
	Total		**5**	
	Any change of base must be carried out correctly eg $\log_2 N = \frac{\ln N}{\ln 2}$ or $\log_2 N = \frac{\log_{10} N}{\log_{10} 2}$			

Q3	Solution	Mark	Total	Comment
	Aux eqn $m^2-6m+10=0$ $(m-3)^2-9+10=0$	**M1**		Completing the square or using quadratic formula OE on correct aux. eqn. PI by correct value of '*m*' seen/used.
	$(m=3\pm\mathrm{i})$ $(y_{CF}=)$ $\mathrm{e}^{3x}(A\sin x+B\cos x)$	**A1**		Correct CF.
	$(y_{\mathrm{PI}}=)$ ax^2+bx+c	**M1**		Correct general form for particular integral If other term(s) included, candidate needs to show the corresponding coefficient is 0.
	$(y'_{PI}=)$ $2ax+b$; $(y''_{PI}=)$ $2a$ $2a-6(2ax+b)+10(ax^2+bx+c)$ $=34x-20x^2$	**dM1**		Dep only on the 2nd M1 above. Substitution into LHS of DE. PI by at least two correct equations in next line.
	$10a=-20$; $-12a+10b=34$; $2a-6b+10c=0$	**A1**		Three equations at least **two** correct, seen or used
	$a=-2,\ b=1, c=1; (y_{\mathrm{PI}}=)\ -2x^2+x+1$	**A1**		$a=-2,\ b=1,\ c=1$ or $-2x^2+x+1$
	$(y_{\mathrm{GS}}=)$ $\mathrm{e}^{3x}(A\sin x+B\cos x)-2x^2+x+1$	**A1**	**7**	Correct expression for the general solution
	Total		**7**	

Q4	Solution	Mark	Total	Comment
	$\sqrt{9-kx^4} = 3\left[1-\frac{k}{18}x^4+.......\right]$	B1		Correct first two terms in the expansion of $\sqrt{9-kx^4}$ seen or used. OR $\left[\frac{3-\sqrt{9-kx^4}}{7x^6+8x^4}\right]=$ $=\frac{9-(9-kx^4)}{8x^4(3+\sqrt{9-kx^4})+7x^6(3+\sqrt{9-kx^4})}$ seen or used.
	$\left[\frac{3-\sqrt{9-kx^4}}{7x^6+8x^4}\right]=\frac{\frac{3k}{18}x^4+O(x^8)}{7x^6+8x^4}$ $=\frac{\frac{3k}{18}+O(x^4)}{7x^2+8}$	M1		Dividing numerator and denominator by x^4 to get constant term in each, leading to a finite limit. Must be at least a total of three 'terms' divided by x^4.
	$\lim_{x\to 0}\left[\frac{3-\sqrt{9-kx^4}}{7x^6+8x^4}\right]=\frac{3k}{144}$	A1		Correct expression in terms of k for the value of the limit. OE seen or used.
	$\frac{3k}{144}=\frac{1}{32}\Rightarrow k=\frac{3}{2}$	A1F	4	OE Dep on no incorrect power of x seen after division by x^4 above. Only ft on the sign error leading to B0 $\sqrt{9-kx^4} = 3\left[1+\frac{k}{18}x^4+...\right]$ which would lead to $k=-\frac{3}{2}$ OE
	Total		**4**	
Example	$\left[\frac{3-\sqrt{9-kx^4}}{7x^6+8x^4}\right]=\frac{\frac{3k}{18}x^4+O(x^8)}{7x^6+8x^4}$ (B1) = $\frac{\frac{3k}{18}+O(x^2)}{7x^2+8}$ (error seen in power of x) (M1) $\lim_{x\to 0}.......=\frac{3k}{144}$ (A1) $=\frac{1}{32}\Rightarrow k=\frac{3}{2}$ (A0)			

Q5	Solution	Mark	Total	Comment
(a)	$3r+2r\sin\theta=5 \Rightarrow 3r+2y=5$	M1		$r\sin\theta=y$ used at any stage.
	$9r^2=(5-2y)^2$ $\Rightarrow 9(x^2+y^2)=(5-2y)^2$	M1		$r=\sqrt{x^2+y^2}$ used to form a Cartesian equation.
	$9x^2+5(y+2)^2=(45)$	A1		$9x^2+5(y+2)^2=$ constant or $9x^2+5y^2+20y-25=0$ or $x=0$, $5y^2+20y-25=0$ OE
	(Tangents parallel to coordinate axes:) $x=\sqrt{5}$, $x=-\sqrt{5}$, $y=1$, $y=-5$	A2,1,0	5	**A2** all four correct; **A1** any two correct.
(b)	$\frac{x^2}{5}+\frac{(y+2)^2}{9}=1$; $a=\sqrt{5}$; $b=3$	M1		Finding semi-axes with at least one value correct; seen or used
	(Area =) $\pi(\sqrt{5})(3)$	A1		
	(Area =) $\frac{1}{2}\int_0^{2\pi}\left(\frac{5}{3+2\sin\theta}\right)^2(\mathrm{d}\theta)$	M1		Seen or used
	$\Rightarrow$ ($\int_0^{2\pi}\frac{1}{(3+2\sin\theta)^2}\,\mathrm{d}\theta$) $=\frac{6\sqrt{5}}{25}\pi$	A1	4	ACF but must be exact.
	Total		9	

(a) For the A1 accept eg $\frac{9}{5}x^2+(y+2)^2=$ constant and accept other equally equivalent forms

(a) Using differentiation can be applied to any suitable form of the Cartesian eqn of the ellipse.

(b) Altn

$\frac{1}{2}r^2=\frac{1}{(3+2\sin\theta)^2}$ to a Cartesian eqn using correct (polar $\rightarrow$ Cartesian) conversion formulae **(M1)**

$r=\frac{\sqrt{2}}{(3+2\sin\theta)}$ Cartesian eqn $9x^2+5\left(y+\frac{2\sqrt{2}}{5}\right)^2=\frac{18}{5}$ (**A1**)

$a=\sqrt{\frac{2}{5}}$; $b=\sqrt{\frac{18}{25}}$; **(M1** Finding semi-axes with at least one value correct; seen or used)

(Area =) $\pi\sqrt{\frac{2}{5}}\sqrt{\frac{18}{25}}$;

($\int_0^{2\pi}\frac{1}{(3+2\sin\theta)^2}\,\mathrm{d}\theta$) $=\frac{6\sqrt{5}}{25}\pi$ (**A1** as in main scheme)

Q6	Solution	Mark	Total	Comment
(a)	$\frac{du}{dx}=\frac{d^2y}{dx^2}+\frac{dy}{dx}\cot x-y\operatorname{cosec}^2 x$	M1		Product rule used to differentiate $y\cot x$ or quotient rule used to differentiate $y/\tan x$
		A1		$\frac{du}{dx}=\frac{d^2y}{dx^2}+\frac{dy}{dx}\cot x-y\operatorname{cosec}^2 x$ OE
	$\frac{d^2y}{dx^2}+(\cot x+\tan x)\frac{dy}{dx}=$ $=\frac{du}{dx}+y\operatorname{cosec}^2 x+\tan x(u-y\cot x)$ $=\frac{du}{dx}+y(\operatorname{cosec}^2 x-1)+u\tan x$ $=\frac{du}{dx}+y\cot^2 x+u\tan x$			
	2^{nd} order DE $\rightarrow\frac{du}{dx}+u\tan x=0$	A1	3	AG Be convinced
(b)	$\int\frac{1}{u}(du)=\int-\tan x\,(dx)$	M1		Separation of variables or IF$=\sec x$
	$\ln u=\ln\cos x\ (+\ln A)$	A1		$\ln u=\ln\cos x$ or $\frac{d}{dx}(u\sec x)=0$ OE
	$u=A\cos x\Rightarrow\frac{dy}{dx}+y\cot x=A\cos x$	M1		Equating c's non-zero u to $\frac{dy}{dx}+y\cot x$
	IF = $e^{\int\cot x\,(dx)}$	M1		PI
	$=e^{\ln\sin x}=\sin x$	A1		
	$\frac{d}{dx}(y\sin x)=A\sin x\cos x$	dM1		
	$y\sin x=\frac{A}{2}\sin^2 x\ (+B)$	A1		OE Correct integration of $\sin x\cos x$ eg $0.5\sin^2 x$ or $-0.5\cos^2 x$ or $-0.25\cos 2x$
	When $x=\frac{\pi}{6}, y=0, \frac{dy}{dx}=\sqrt{3}$, $u=\sqrt{3}$ $A=2$, $B=-0.25$	A1		Correct value for **one** of the constants of integration. Dep only on 1st **M1** as could be found from $u=A\cos x$
	$y=\sin x-\frac{1}{4\sin x}$	A1	9	Correct expression for y in terms of $\sin x$
	Total		**12**	

Q7	Solution	Mark	Total	Comment
(a)	Aux eqn $m^2+4m+4=0$ $(m+2)^2=0$	M1		Factorising or using quadratic formula OE on correct aux eqn. PI by correct value of 'm' seen/used.
	$(y_{CF}=)\ (Ax+B)\,\mathrm{e}^{-2x}$	A1		
	Try $(y_{PI}=)\ a\sin 2x+b\cos 2x$	M1		Correct form for y_{PI} used. If other term(s) included, candidate needs to show the corresponding coefficient is 0
	$(y'_{PI}=)\ 2a\cos 2x-2b\sin 2x$; $(y''_{PI}=)\ -4a\sin 2x-4b\cos 2x$			
	$-4a\sin 2x-4b\cos 2x$ $+4(2a\cos 2x-2b\sin 2x)+$ $4(a\sin 2x+b\cos 2x)=4\sin 2x+8\cos 2x$	dM1		Correct substitution into DE, dep on previous **M** only. PI by correct a and b seen or used.
	$-8b=4$; $8a=8\Rightarrow\ a=1,\ b=-\frac{1}{2}$	A1		Finding correct coefficients for particular integral. PI by next line
	$(y_{\mathrm{GS}}=)\ \ (Ax+B)\,\mathrm{e}^{-2x}+\sin 2x-\frac{1}{2}\cos 2x$	A1	6	Correct GS with two arbitrary constants.
(b)	When $x=0,\ y=\frac{1}{2},\ \frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\mathrm{f}(0)=B-0.5=0.5\Rightarrow B=1$	M1		$\mathrm{f}(0)=0.5$ OE used with c's GS as far as finding a value for one of the arbitrary constants.
	$\mathrm{f}'(0)=A-2B+2=0\Rightarrow A=0$	M1		$\mathrm{f}'(0)=0$ OE used with c's GS as far as finding a value for the remaining arbitrary constant.
	$(y=\mathrm{f}(x)=)\ \mathrm{e}^{-2x}+\sin 2x-\frac{1}{2}\cos 2x$	A1		$\mathrm{e}^{-2x}+\sin 2x-\frac{1}{2}\cos 2x$ OE PI by the next line
	$(\mathrm{f}\left(\frac{\pi}{6}\right)=)\ \mathrm{e}^{-\frac{\pi}{3}}+\frac{\sqrt{3}}{2}-\frac{1}{4}$	A1	4	ACF but must be exact with no trig fns
	Total		**10**	
(b)	Altn for the two **M1** marks : Relevant expansions of e^{-2x}, $\sin 2x$, $\cos 2x$ used and coefficients of x^0 or x equated; relevant equations are $B-0.5=0.5$, $A-2B+2=0$ and a value for the arbitrary constant must be found as in main scheme.			
(b)	$\mathrm{f}''(0)=6$ used as far as finding a value for an arbitrary constant is an OE for M1; $[-4A+4B+2=6]$			

Q8	Solution	Mark	Total	Comment
(a)	$r\sec^2\theta = 4$; $(r-1)^2 = \tan^2\theta$ $r\{1+(r-1)^2\} = 4$	M1		Eliminating θ to reach a cubic equation in r, or eliminating r to reach a cubic equation in $\tan\theta$ or $\cos^2\theta$ or $\sin^2\theta$.
	$r^3 - 2r^2 + 2r - 4 = 0$	A1		Correct cubic in r or $\tan\theta$ or $\cos^2\theta$ or $\sin^2\theta$ eg $\tan^3\theta + \tan^2\theta + \tan\theta - 3 = 0$ PI by factorised form
	$(r-2)(r^2+2) = 0$	A1		Correct factorisation of correct cubic. eg $(\tan\theta - 1)(\tan^2\theta + 2\tan\theta + 3) = 0$ or stating the roots exactly or to at least 2sf
	$r^2 + 2 \neq 0$ so (only value of) r is 2	E1		Showing that cubic equation in r or $\tan\theta$ or $\cos^2\theta$ or $\sin^2\theta$ has only one relevant root. eg $\tan^2\theta + 2\tan\theta + 3 = 0$ has no real root ($2^2 - 4(3) < 0$) so $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$ only (allow 3sf dec)
	When $r = 2$, **only** value of θ is $\frac{\pi}{4}$ so **single** point of intersection ($P\left(2, \frac{\pi}{4}\right)$) with ***OP* = 2**.	E1	5	Showing that there is only one point of intersection which is at a distance 2 from the pole. Values must be exact and previous 4 marks must have been scored
(b)	$A(1, 0)$ (or $r = 1$, $\theta = 0$)	B1		May be indicated on the diagram. PI by use of both $OA = 1$ and limit 0. Condone (0,1) if later used as r=1 or $\theta = 0$
	Area $\Delta OAP = \frac{1}{2}(OA)2\sin(\theta_P)$	M1		$\frac{1}{2}(OA)2\sin(\theta_P)$ OE
	Area $\Delta OAP = \frac{1}{\sqrt{2}}$	A1		OE….must be in an exact form.
	(Area bounded by *OP*, arc *AP* and *OA*) $= \frac{1}{2}\int_{(0)}^{(\frac{\pi}{4})}(1+\tan\theta)^2\ (\mathrm{d}\theta)$	M1		Use of $k\int r^2\ (\mathrm{d}\theta)$, $k > 0$
	$= \frac{1}{2}\int_0^{\frac{\pi}{4}}\left(1+\tan^2\theta + 2\tan\theta\right)(\mathrm{d}\theta)$	B1		Correct expansion, correct limits and k=0.5
	$= \frac{1}{2}\left[\tan\theta + 2\ln\sec\theta\right]_0^{\pi/4}$	M1		Correct integration of $\tan^2\theta$
	$= \frac{1}{2}\left(1 + 2\ln\sqrt{2}\right)$	A1		OE….must be in an exact form.
	(Required area) $= \frac{1}{2}(1+\ln 2) - \frac{1}{\sqrt{2}}$	A1	8	OE….must be in an exact form.
	Total		**13**	
(a)	eg $16\cos^6\theta - 8\cos^4\theta + 2\cos^2\theta - 1 = 0$ (**A1**); eg $(\cos^2\theta - 0.5)(16\cos^4\theta + 2) = 0$ (**A1**); eg $16\sin^6\theta - 40\sin^4\theta + 34\sin^2\theta - 9 = 0$ (**A1**); eg $(2\sin^2\theta - 1)(8\sin^4\theta - 16\sin^2\theta + 9) = 0$ (**A1**);			

Q9	Solution	Mark	Total	Comment
(a)	$\ln(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \frac{y^5}{5} \ldots$ $\ln(1-y) = -y - \frac{y^2}{2} - \frac{y^3}{3} - \frac{y^4}{4} - \frac{y^5}{5} \ldots$	**M1**		Expansions for each attempted with at least one expansion correct up to y^5 term or at least two of the three terms in the next line.
	$\ln(1+y) - \ln(1-y) = 2y + \frac{2}{3}y^3 + \frac{2}{5}y^5$	**A1**		
	(valid for) $-1 < y < 1$	**B1**	**3**	or $\lvert y \rvert < 1$.
(b)	$\ln\left(\frac{1-x+x^2}{1+x+x^2}\right) = \ln(1-x+x^2) - \ln(1+x+x^2)$	**M1**		$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$ used at any stage
		B1		$1-x^3 = (1-x)(1+x+x^2)$ seen or used
	$= \ln(1+x^3) - \ln(1-x^3) - [\ln(1+x) - \ln(1-x)]$	**A1**		Seen in grouped form or used with (a)
	$= 2x^3 + \frac{2}{3}x^9 + \frac{2}{5}x^{15} + ..$	**dM1**		c's **(a)** used for y as x and x^3
	$-2\left[..+\frac{x^3}{3}+..+\frac{x^9}{9}+..+\frac{x^{15}}{15}+..\right]$	**A2,1,0**		**A2…**At least 5 of these 6 terms seen/used **A1…**At least 4 of these 6 terms seen/used [Combined terms would lead to $\frac{4}{3}x^3 + \frac{4}{9}x^9 + \frac{4}{15}x^{15}$]
	Coefficient of x^{6r-3}: $\frac{2}{2r-1} - \frac{2}{6r-3} = \frac{4}{6r-3}$	**A1**	**7**	Dep on previous 6 marks scored. Either $\frac{4}{6r-3}$ or $\frac{4}{3(2r-1)}$
	Total		**10**	
	TOTAL		**75**	
(a) (b)	If eg x is used instead of y in answer(s), then maximum penalty is 1 mark Candidate who uses expns of ln(1+X) for both X=(x+x^2) and X=(-x+x^2) must go as far as X^{15}			