AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Mechanics 3

Wednesday 6 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working, otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The time, t seconds, taken for a torsional oscillator to make a single oscillation is thought to depend on the moment of inertia of the disc, I $\mathrm{kg\,m^2}$, the torsion constant of the wire, K $\mathrm{kg\,m^2\,s^{-2}}$, and a dimensionless constant, C, such that $t = CI^{\alpha}K^{\beta}$.

By using dimensional analysis, find the values of α and β.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

2 A golf ball is projected from a point O on a horizontal ground with initial velocity $(6\mathbf{i} + 8\mathbf{j})\,\mathrm{m\,s^{-1}}$, where **i** and **j** are horizontal and upward vertical unit vectors respectively. The golf ball travels in a vertical plane through O. During its flight, the horizontal and upward vertical displacements of the ball from O are x metres and y metres respectively.

(a) Show that, during the flight, the equation of the trajectory of the golf ball is given by

$$y = \frac{4x}{3} - \frac{49x^2}{360}$$

[4 marks]

(b) Find the change in the horizontal displacement of the golf ball whilst it is at least 2 metres above the ground.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

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3 A particle, of mass $1.5\,\text{kg}$, is moving in a straight line on a smooth horizontal surface. The particle moves under the action of a single force which acts in the direction of motion.

At time t seconds, the force has magnitude $k\left(5t^{\frac{3}{2}} + 2t\right)$ newtons, where k is a constant and $t \geqslant 0$.

When $t = 1$, the velocity of the particle is $6\,\text{m s}^{-1}$.
When $t = 4$, the velocity of the particle is $10\,\text{m s}^{-1}$.

(a) Find the value of k.

[5 marks]

(b) Find the velocity of the particle when $t = 3$.

[2 marks]

(c) Find the magnitude of the impulse exerted by the force on the particle between the times $t = 1$ and $t = 3$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 3

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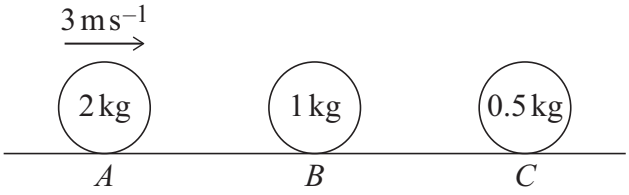
Answer space for question 3

4 Three uniform smooth spheres, A, B and C, have equal radii and masses $2\,\text{kg}$, $1\,\text{kg}$ and $0.5\,\text{kg}$ respectively. The spheres lie at rest in a straight line on a smooth horizontal surface.

The sphere A is set in motion with velocity $3\,\text{m s}^{-1}$ and collides directly with B.



The coefficient of restitution between A and B is e.

(a) Find, in terms of e, the velocity of A and the velocity of B immediately after the collision.

[6 marks]

(b) Given that the magnitude of the impulse exerted by A on B is $\frac{10}{3}\,\text{N s}$, find the value of e.

[3 marks]

(c) Subsequently, B collides directly with C.

The coefficient of restitution between B and C is $\frac{4}{5}$.

Find the velocity of B after this collision.

[3 marks]

(d) Determine, with a reason, whether or not A and B will collide again.

[2 marks]

QUESTION PART REFERENCE

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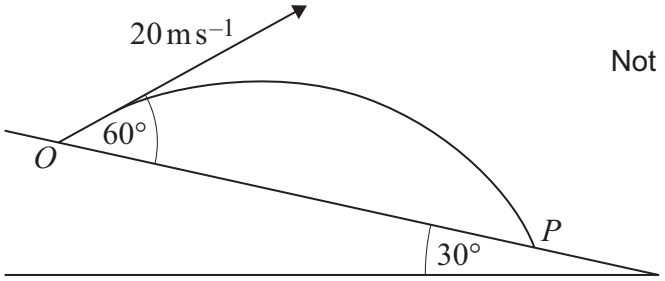
5 A projectile is fired with a speed of $20\,\mathrm{m\,s^{-1}}$ from a point O on a plane which is inclined at an angle $30°$ to the horizontal. The projectile is fired at an angle $60°$ to the inclined plane and moves in a vertical plane through a line of greatest slope of the inclined plane. The projectile lands at a point P, lower down the inclined plane, as shown in the diagram.



Not drawn to scale

(a) Find the greatest perpendicular distance of the projectile from the inclined plane.

[4 marks]

(b) The coefficient of restitution between the projectile and the plane is $\frac{1}{2}$.

Find, to three significant figures, the speed of the projectile as it rebounds from the inclined plane at P.

[9 marks]

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6 The unit vectors **i** and **j** are directed east and north respectively.

Two ships, A and B, are moving on straight courses with constant velocities of $24\mathbf{i}\,\text{km}\,\text{h}^{-1}$ and $18\mathbf{j}\,\text{km}\,\text{h}^{-1}$ respectively. At noon, A and B have position vectors $-6\mathbf{i}\,\text{km}$ and $-12\mathbf{j}\,\text{km}$ respectively relative to a lighthouse.

(a) The position vector of A relative to B at time t hours after noon is $\mathbf{r}\,\text{km}$. Find $\mathbf{r}$.

[2 marks]

(b) Find the time when A and B are closest together.

[5 marks]

(c) When A and B are closest together, a motor boat, M, leaves B and travels on a straight course with constant speed. Given that M intercepts A 10 minutes later, find the bearing, to the nearest degree, on which M has travelled.

[4 marks]

QUESTION PART REFERENCE

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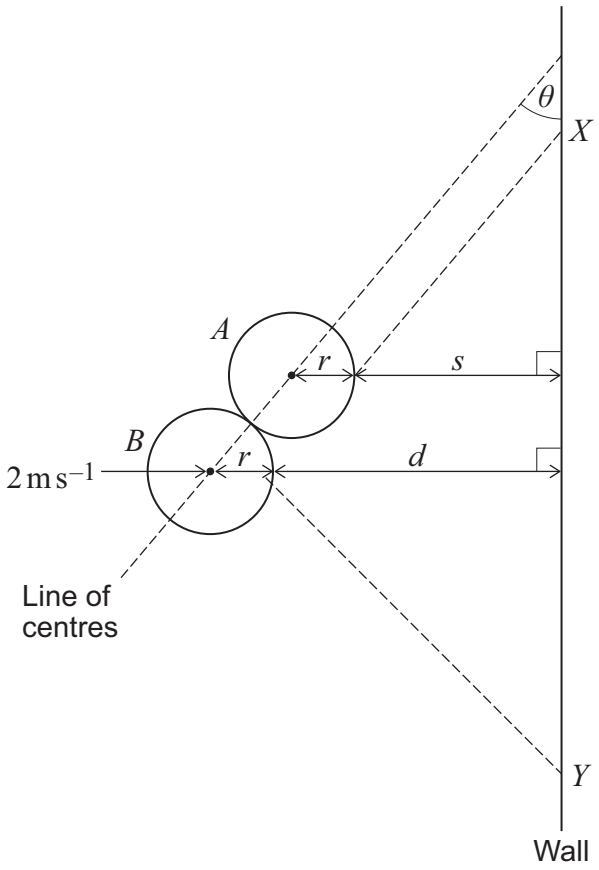
7 A small uniform sphere, A, of radius r, lies at rest on a smooth horizontal floor and at a distance s metres from a fixed smooth vertical wall. An identical sphere, B, is moving on the floor with speed $2\,\mathrm{m\,s^{-1}}$ in a direction perpendicular to the wall. The sphere B collides with the sphere A. At the instant of collision, B is at a distance d from the wall and the line of centres of A and B makes an angle θ with the wall, as shown in the diagram.



The coefficient of restitution between A and B is $\frac{2}{3}$.

(a) Show that, immediately after the collision, the component of the velocity of B parallel to the line of centres is $\frac{1}{3}\sin\theta\,\mathrm{m\,s^{-1}}$ and find the component of the velocity of B perpendicular to the line of centres in terms of θ.

[6 marks]

(b) Show that, immediately after the collision, the component of the velocity of B perpendicular to the wall is $\left(2-\frac{5}{3}\sin^2\theta\right)\mathrm{m\,s^{-1}}$ and find the component of the velocity of B parallel to the wall in terms of θ.

[5 marks]

(c) Subsequently, A and B collide with the wall at the points X and Y respectively. Given that $\tan\theta=\frac{3}{4}$, find, in terms of r, s and d, the distance XY.

[5 marks]

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END OF QUESTIONS

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