AQA

A-LEVEL
Mathematics

MM03 - Mechanics 3

Mark Scheme

6360

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Question	Solution	Marks	Total	Comments
1	$[t] = (\mathrm{ML}^2)^\alpha (\mathrm{ML}^2\mathrm{T}^{-2})^\beta$	M1		M1: Correct unsimplified dimensions of I and K
	$= \mathrm{M}^{\alpha+\beta}\mathrm{L}^{2\alpha+2\beta}\mathrm{T}^{-2\beta}$	A1		A1: Simplifying expression correctly PI
	$\beta = -\frac{1}{2}$	A1		A1:Correct value
	$\alpha = \frac{1}{2}$	A1		A1:Correct value
			4	
	Total		**4**	

Question	Solution	Marks	Total	Comments
2 (a)	$x = 6t$	M1		M1: Correct horizontal equation
	$y = 8t - \frac{1}{2}gt^2$	M1		M1: Correct vertical equation
	$y = 8 \times \frac{x}{6} - \frac{1}{2}(9.8)\left(\frac{x}{6}\right)^2$	dM1		dM1: Eliminating t from y
	$y = \frac{4x}{3} - \frac{49x^2}{360}$	A1	4	A1: CSO, AG
(b)	$2 = \frac{4x}{3} - \frac{49x^2}{360}$	M1		M1: Substituting 2 for y in vertical equation
	$49x^2 - 480x + 720 = 0$			
	$x = \frac{480 \pm \sqrt{(-480)^2 - 4(49)(720)}}{2(49)}$	dM1		dM1: Solving **their** quadratic equation by formula, or completing the square or calculator. Must see the method if the values of x are wron.
	$x = 7.9469...,\ 1.8490...$	A1		A1: For both solutions, PI from the answer
	The distance $= 6.10$ m	A1F	4	A1: Accept 6.1 m, FT on their solution of their equation
	Total		**8**	

Question	Solution	Marks	Total	Comments
3 (a)	$\int_1^4 k\left(5t^{\frac{3}{2}}+2t\right)\mathrm{d}t = 1.5(10)-1.5(6)$	M1 A1		M1: Using $\int F\mathrm{d}t = mv - mu$ or $\int F\mathrm{d}t = m\int \mathrm{d}v$ A1: All correct, condone missing units
	$k\left[2t^{\frac{5}{2}}+t^2\right]_1^4 = 1.5(10)-1.5(6)$	dM1		dM1: Correct integration, condone missing limits
	$k\left[2(4)^{\frac{5}{2}}+(4)^2-2(1)^{\frac{5}{2}}-(1)^2\right]=1.5(10)-1.5(6)$	A1		A1: Correct use of correct limits
	$k=\frac{6}{77}$ OE	A1	5	A1: CAO, Accept 0.0779 or better
(b)	$\frac{6}{77}\left[2(3)^{\frac{5}{2}}+(3)^2-2(1)^{\frac{5}{2}}-(1)^2\right]=1.5(v)-1.5(6)$	M1		M1: Correct impulse-momentum equation using their k
	$v = 7.93$ ms^{-1}	A1	2	A1: CAO, AWRT 7.93
(c)	1.5 (7.93126829) – 1.5 (6) =	M1		M1: Using $I = mv - mu$ with their velocity from (b)
	2.8969.... Ns	A1F	2	A1F: AWRT 2.9 FT their velocity from part (b)
	Total		**9**	

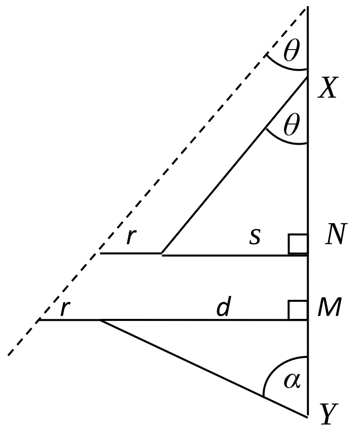
Question	Solution	Marks	Total	Comments
4 (a)	$2(3) = 2v_A + 1v_B$	M1 A1		M1: Three momentum terms, A1: All correct
	$3e = v_B - v_A$	M1 A1		M1: Restitution, allow sign error A1: All correct
	$v_B = 2(1+e)$ OE	A1		A1: CAO
	$v_A = 2 - e$ OE	A1	6	A1: CAO
(b)	$1(2(1+e)) = \frac{10}{3}$	M1 A1		M1: Using their v_A or v_B in an impulse-momentum equation A1: Correct equation
	$e = \frac{2}{3}$	A1	3	A1: CAO
(c)	$1\times 2\left(1+\frac{2}{3}\right) = 1\times v_B' + 0.5v_C$	M1		M1: Correct momentum equation with their v_B using their e but with $e < 1$
	$\frac{4}{5}\times 2\left(1+\frac{2}{3}\right) = v_C - v_B'$	M1		M1: Correct restitution equation with their v_B using their e but with $e < 1$
	$v_B' = \frac{4}{3}$	A1	3	A1: CAO
(d)	$v_A = 2 - \frac{2}{3} = \frac{4}{3}$	B1F		B1F: For $v_A = \frac{4}{3}$, OE, or their v_A using their e but with $e < 1$
	$v_A = v_B' \Rightarrow$ *A* and *B* will not collide again as they are travelling at the same velocity.	E1F	2	E1F: For reason and statement, FT their values
	Total		**14**	

Question	Solution	Marks	Total	Comments
5 (a)	$\dot{y} = 20\sin 60^\circ - g\cos 30^\circ t$	M1		M1: Correct perpendicular velocity equation
	$t = \dfrac{20\sin 60^\circ}{g\cos 30^\circ}$ or $\dfrac{20}{g}$ or awrt 2.041	A1		A1: Correct expression for t
	$y = 20\sin 60^\circ\left(\dfrac{20\sin 60^\circ}{g\cos 30^\circ}\right) - \dfrac{1}{2}g\cos 30^\circ\left(\dfrac{20\sin 60^\circ}{g\cos 30^\circ}\right)^2$	dM1		dM1: Correct perpendicular equation with **their** time
	$y = 17.7$ m	A1	4	A1: CAO, AWRT 17.7
(b)	Time from O to P = $2\times\dfrac{20\sin 60^\circ}{g\cos 30^\circ}$ or $\dfrac{40}{g}$ or awrt 4.08	B1		B1: Correct time
	$\dot{y} = 20\sin 60^\circ - g\cos 30^\circ\left(2\times\dfrac{20\sin 60^\circ}{g\cos 30^\circ}\right)$	M1		M1: Correct perpendicular velocity with **their** time
	$\dot{y} = -17.32$ or $-10\sqrt{3}$	A1		A1: Correct perpendicular velocity
	$\dot{x} = 20\cos 60^\circ + g\sin 30^\circ\left(2\times\dfrac{20\sin 60^\circ}{g\cos 30^\circ}\right)$	M1		M1: Correct parallel velocity with **their** time
	$\dot{x} = 30$	A1		A1: Correct parallel velocity
	Rebound: $\dot{y}' = \dfrac{1}{2}(17.32)$ or $\dfrac{1}{2}(10\sqrt{3})$ OE	dM1		dM1: Restitution FT their speed
	$\dot{x}' = 30$ or unchanged	B1		B1F: Parallel speed unchanged, PI from rebound speed, **FT their speed**
	Speed of rebound = $\sqrt{30^2 + \left(\dfrac{1}{2}(17.32)\right)^2}$	dM1		dM1: Dependent on all M1s and the dM1, **FT their speeds**
	$= 31.2$ ms^{-1}	A1	9	A1: CAO, AWRT 31.2
	Total		**13**	

Question	Solution	Marks	Total	Comments
5 (a)	**Alternative**: $\dot{y}^2 = (20\sin 60^\circ)^2 - 2g\cos 30^\circ\, y$ $0 = (20\sin 60^\circ)^2 - 2g\cos 30^\circ\, y_{\max}$ $y_{\max} = 17.7$ m	M1 A1 dM1 A1		M1: Equation with correct terms, allow sign error A1: All correct dM1: Substituting zero for $\dot{y}$ A1: CAO, AWRT 17.7

Question	Solution	Marks	Total	Comments
6 (a)	$\mathbf{r} = (-6\mathbf{i} + 12\mathbf{j}) + (24\mathbf{i} - 18\mathbf{j})t$ $\mathbf{r} = 6[(-1+4t)\mathbf{i} + (2-3t)\mathbf{j}]$	M1 A1	2	M1: Using $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$ A1: All correct
(b)	$s^2 = 6^2(-1+4t)^2 + 6^2(2-3t)^2$ $\frac{\mathrm{d}s^2}{\mathrm{d}t} = 6^2 \times 2(-1+4t) \times 4 + 6^2 \times 2(2-3t)(-3)$ $6^2 \times 2(-1+4t) \times 4 + 6^2 \times 2(2-3t)(-3) = 0$ $t = \frac{2}{5} \Rightarrow$ Closest time is 12 : 24 p.m.	M1 dM1 A1 dM1 A1	5	M1: Correct expression for s or s^2, 6^2 not needed dM1: Correct differentiation of s or s^2, 6^2 not needed A1: Setting the derivative to zero dM1: Solving the correct equation, PI by the answer A1: $t = \frac{2}{5}$ or 0.4 or Closest time is 12 : 24 p.m.
(c)	At 12:24 p.m. $\mathbf{r}_M = \left(-12 + 18 \times \frac{2}{5}\right)\mathbf{j} = -\frac{24}{5}\mathbf{j}$ At 12:34 p.m. $\mathbf{r}_M = \left(-6 + 24 \times \frac{34}{60}\right)\mathbf{i} = \frac{38}{5}\mathbf{i}$ $\tan^{-1} \frac{\frac{38}{5}}{\frac{24}{5}}$ $= 58^\circ$ The bearing is 058°	B1F B1F M1 A1	4	B1F: Position vector of *M* at 12:24 p.m. B1F: Position vector of *M* at 12:34 p.m. M1: Any correct trig ratio with their values A1: For 58°, CAO
	Total		**11**	

Question	**Solution**	Marks	Total	Comments
7 (a)	Along the line of centres: CLM : $2\sin\theta = v_A + v_B$	M1 A1		M1: Momentum equation with correct terms, but allow sign errors A1: Correct equation
	Restitution : $\left(\frac{2}{3}\right)2\sin\theta = v_A - v_B$	M1 A1		M1: Restitution equation with correct terms, but allow sign errors A1: Correct equation
	$v_B = \frac{1}{3}\sin\theta$	A1		A1: Correct parallel componentAG
	Perpendicular to line of centres : $v'_B = 2\cos\theta$	B1	6	B1: Correct perpendicular component
(b)	Component perpendicular to the wall = $\frac{1}{3}\sin\theta\sin\theta + 2\cos\theta\cos\theta =$	M1		M1:Resolving both components correctly
	$\frac{1}{3}\sin^2\theta + 2\left(1-\sin^2\theta\right) =$	dM1		dM1:Eliminating $\cos\theta$
	$2 - \frac{5}{3}\sin^2\theta$	A1		A1:Correct result, AG
	Component parallel to the wall = $2\cos\theta\sin\theta - \frac{1}{3}\sin\theta\cos\theta$	M1 A1	5	M1:Resolving both components, correct terms, allow sign errors A1:Correct result

(c)	 $NX = \dfrac{s}{\tan\theta} = \dfrac{s}{\frac{3}{4}} = \dfrac{4s}{3}$	B1		B1: NX in terms of s
	Component perpen. to the wall = $2 - \dfrac{5}{3}\left(\dfrac{3}{5}\right)^2$ or $\dfrac{7}{5}$ component parallel to the wall = $2\left(\dfrac{4}{5}\right)\left(\dfrac{3}{5}\right) - \dfrac{1}{3}\left(\dfrac{3}{5}\right)\left(\dfrac{4}{5}\right)$ or $\dfrac{4}{5}$	B1		B1 for both components
	$MY = \dfrac{d}{\tan\alpha} = \dfrac{d}{\frac{7}{5} \div \frac{4}{5}} = \dfrac{4d}{7}$	M1	5	M1: MY in terms of d
	$MN = 2r\cos\theta = 2r\left(\dfrac{4}{5}\right) = \dfrac{8r}{5}$	B1		B1: MN in terms of r
	$XY = \dfrac{4s}{3} + \dfrac{4d}{7} + \dfrac{8r}{5}$	A1		A1: XY, CAO
	Total		**16**	