AQA

A-LEVEL
Mathematics

MM04 Mechanics 4
Mark scheme

6360

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1	Tan $60^0 = \dfrac{h/2}{r}$ $h = 2\sqrt{3}r$	M1 A1 A1	3	Use of tangent to form an equation or suitable moments taken Fully correct equation Accept decimal equivalent $h = 3.46r$
	Total		**3**	

Q	Solution	Mark	Total	Comment
2 (a)	Take moments about *A* $Pl = 50(3l)$	**M1**		Use of moments for complete system – one side correct
	$P = 150$ N	**A1**	2	CAO
(b)(i)	Balancing forces for the whole system Vertical component at A = 50 N (upwards) Horizontal component at A = 150 N (left)			
	Magnitude = $\sqrt{150^2 + 50^2} = 50\sqrt{10}$ N	**M1**		Balances system and uses Pythagoras theorem
	Hence $k = 50$	**A1**	2	Correct k value obtained – implied by correct magnitude
(ii)		**B1**	1	Correct direction clearly shown
(c)	Rods *CD*, *ED* and *BE* are in compression	**B1**	1	All correct – no extras
(d)	Resolve vertically at *C* $T_{CD} \sin\theta = 50$	**M1**		Resolves correctly at C at least once and uses $\sin\theta = \frac{1}{\sqrt{5}}$ or $\cos\theta = \frac{2}{\sqrt{5}}$ or $\theta = 26.6^0$
	$T_{CD} = 50\sqrt{5}$ N (AWRT 112N)	**A1**		
	Resolves horizontally at *C* $T_{BC} = T_{CD} \cos\theta$ $T_{BC} = 100$ N	**A1F**		Resolves correctly and uses their answer for T_{CD}
	Resolve vertically at E $T_{BE} \cos 45^0 = T_{AE}$ Resolving vertically at *A* $T_{AE} = 50$	**M1**		Resolves correctly and uses sufficient equations to find T_{BE}
	$T_{BE} = 50\sqrt{2}$ N (AWRT 71N)	**A1**	5	
	Total		**11**	

Q	Solution	Mark	Total	Comment
3 (a)	$\int y dx = \int_{-a}^{a} a^2 - x^2 dx$			
	$= \left[a^2x - \frac{x^3}{3}\right]_{-a}^{a}$	M1		Applies correct formula to find area and integrates – one term correct
	$= \frac{4a^3}{3}$	A1		Substitutes correct limits to obtain correct answer
	$\frac{1}{2}\int y^2 dx = \frac{1}{2}\int_{-a}^{a}(a^2 - x^2)^2 dx$			
	$= \frac{1}{2}\int_{-a}^{a}(a^4 - 2a^2x^2 + x^4)dx$	M1		Applies correct formula, expands and integrates – one term correct
	$= \frac{1}{2}\left[a^4x - \frac{2a^2x^3}{3} + \frac{x^5}{5}\right]_{-a}^{a}$	A1		Integrates correctly – all terms
	$= \frac{8a^5}{15}$	M1		Substitutes correct limits
	Hence $\bar{Y} = \frac{8a^5}{15} \Big/ \frac{4a^3}{3} = \frac{2a^2}{5}$	M1		Forms centre of mass equation
	Coordinates are $\left(0, \frac{2a^2}{5}\right)$	A1	7	Coordinates must be stated correctly
(b)	Takes moments about point A			
	$(2ab)(\frac{b}{2})(2\rho) = (\frac{4a^3}{3})(\frac{2a^2}{5})(\rho)$	M1 A1 A1 M1		Forms moment equation Correct use of area x distance - rectangle Correct use of area x distance – curved Correct use of density ratio applied to their equation
	$b = \frac{2\sqrt{15}}{15}a^2$	A1		Rearranged - any equivalent form
			5	
	Total		**12**	

Q	Solution	Mark	Total	Comment
4 (a)	Force $\mathbf{F_3}$ has zero moment about origin			
	Moment of couple = $\mathbf{r_1} \times \mathbf{F_1} + \mathbf{r_2} \times \mathbf{F_2}$	**M1**		Forms correct sum of moments
	$\mathbf{r_1} \times \mathbf{F_1} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$	**A1**		CAO
	$\mathbf{r_2} \times \mathbf{F_2} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 12 \end{pmatrix}$	**A1**		CAO
	Total moment of couple = $\begin{pmatrix} 5 \\ 6 \\ 14 \end{pmatrix}$	**A1F**	4	Correct total of their two non-zero moments
(b)	System in equilibrium so resultant force = $\mathbf{0}$			
	Hence $\mathbf{F_3} = -(\mathbf{F_1} + \mathbf{F_2})$	**M1**		Sets up equation to find $\mathbf{F_3}$
	$\mathbf{F_3} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$	**A1**		Obtains correct $\mathbf{F_3}$
	Let $\mathbf{F_3}$ act through point (x , y , z) Moment of $\mathbf{F_3}$ about origin = $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} y+4z \\ 2z-x \\ -4x-2y \end{pmatrix}$	**M1** **A1**		Finds moment of their $\mathbf{F_3}$ about origin All components correct
	Total moment of all forces must be zero to be in equilibrium hence $\begin{pmatrix} y+4z \\ 2z-x \\ -4x-2y \end{pmatrix} = \begin{pmatrix} -5 \\ -6 \\ -14 \end{pmatrix}$	**M1**		Sets up equation to ensure zero total moment
	By inspection $x = 0$ $y = 7$ $z = -3$	**M1** **A1**		Solves their equations Any correct point found
	A possible vector equation is $\mathbf{r} = \begin{pmatrix} 0 \\ 7 \\ -3 \end{pmatrix} + t\begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$	**M1** **A1F**	9	Correct structure used – their point and $\mathbf{F_3}$ used Fully correct – LHS and RHS – follow through their values Possible answers – for any constant a $\mathbf{r} = \begin{pmatrix} 2a \\ 7-4a \\ -3+a \end{pmatrix} + t\begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$ or

				$\mathbf{r} = \begin{pmatrix} 6+2c \\ -5-4c \\ c \end{pmatrix} + t\begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$ for any constant c
	Total		**13**	

Q	Solution	Mark	Total	Comment
5 (a)	$I_G = \frac{1}{3}m\left[(2a)^2+(1.5a)^2\right] = \frac{25}{12}ma^2$	B1		I_G correctly obtained
	Using parallel axis theorem $I_A = I_G + md^2 = \frac{25ma^2}{12} + m\left[(1.5a)^2+(2a)^2\right]$	M1		Use of I_G and correct d
	$= \frac{25ma^2}{3}$	A1F	3	Follow through incorrect I_G
(b)(i)	Gain in KE $= \frac{1}{2}(\frac{25ma^2}{3})\dot{\theta}^2 = \frac{25ma^2}{6}\dot{\theta}^2$	B1F		Correct KE – follow through (a)
	Loss in PE = $\frac{5}{2}mga\sin\theta$	B1		Correct PE
	Using gain in KE = loss in PE $\frac{25ma^2}{6}\dot{\theta}^2 = \frac{5}{2}mga\sin\theta$	M1		Forms energy equation
	Hence $\dot{\theta}^2 = \frac{3g\sin\theta}{5a}$	A1	4	CSO - AG
(ii)	Differentiating gives $2\dot{\theta}\ddot{\theta} = \frac{3g\cos\theta\,\dot{\theta}}{5a}$	M1		Differentiates both sides – must differentiate
	$\ddot{\theta} = \frac{3g\cos\theta}{10a}$	A1	2	Correct answer obtained
(iii)	Newton's law along AG gives			

	$Y - mg\sin\theta = \frac{5ma}{2}\dot{\theta}^2$	M1A1		**M1** – one side correct **A1** both correct
	$Y = mg\sin\theta + \frac{5ma}{2}\left(\frac{3g\sin\theta}{5a}\right) = \frac{5mg\sin\theta}{2}$	A1		Substitutes given expression from 9(b)(i) to obtain force along AG
	Newton's law perpendicular to AG $mg\cos\theta - X = \frac{5ma}{2}\ddot{\theta}$	M1A1		**M1** – one side correct **A1** both correct
	$X = mg\cos\theta - \frac{5ma}{2}\left(\frac{3g\cos\theta}{10a}\right) = \frac{mg\cos\theta}{4}$	A1F		Substitutes their expression from 9(b)(ii) to obtain force perpendicular to AG
	Resultant force = $\sqrt{\left(\frac{mg\cos\theta}{4}\right)^2 + \left(\frac{5mg\sin\theta}{2}\right)^2}$	M1		Use of Pythagoras for resultant force – dependent on two **M1**s above
	$= \frac{mg}{4}\sqrt{\cos^2\theta + 100\sin^2\theta}$ $= \frac{mg}{4}\sqrt{(1-\sin^2\theta) + 100\sin^2\theta}$ $= \frac{mg}{4}\sqrt{1 + 99\sin^2\theta}$	A1F	8	Correct simplification and use of trig identity to obtain result – follow through their (b)(ii)
(iv)	$\frac{5mg}{2}$	B1F	1	Follow through part (iii)
	Total		**18**	

Q	Solution	Mark	Total	Comment
6 (a)	$\rho = \dfrac{M}{\pi r^2}$	B1		ρ and M linked and used anywhere
	Mass of elemental 'hoop' = $2\pi\rho\partial x x$	M1		Considers elemental hoop - mass correct
	MI of each hoop = $2\pi\rho\partial x x^3$	A1		Use of Mr^2 with elemental hoop
	MI disc= $\int_0^r 2\pi\rho x^3 dx = \int_0^r 2\dfrac{M}{r^2}x^3 dx$	M1		Integrates – integrand must be of correct form
	$= \left[\dfrac{2Mx^4}{4r^2}\right]_0^r = \dfrac{Mr^2}{2}$	A1	5	CSO – AG
(b)(i)	$6mg - T_2 = 6mr\ddot{\theta}$	M1		Forms a correct acceleration equation
	$T_1 - 3mg = 3mr\ddot{\theta}$	A1		Both equations correct and $r\ddot{\theta}$ used
	Ratio gives $3T_2 = 4T_1$			
	$3(6\text{mg} - 6mr\ddot{\theta}) = 4(3\text{mg} + 3mr\ddot{\theta})$	M1		Use of tension ratio to reduce to a single equation
	$30mr\ddot{\theta} = 6mg$			
	$\ddot{\theta} = \dfrac{g}{5r}$	A1	4	CSO
(ii)	Using part (b)(i) $T_1 = \dfrac{18mg}{5}$	B1F		Obtains their expression for T_1
	$T_2 = \dfrac{24mg}{5}$	B1F		Obtains their expression for T_2
	For pulley $T_2 r - T_1 r = I\ddot{\theta}$	M1		Forms correct pulley equation
	$\dfrac{6mg}{5}r = I\dfrac{g}{5r}$			
	$I = 6mr^2$	M1		Substitutes $T_2, T_1, \ddot{\theta}$ and makes comparison with standard result to obtain mass
	MI for disc = $\dfrac{1}{2}Mr^2$ Mass of pulley = $12m$	A1	5	Correct mass obtained - CAO
(iii)				

	Using $C = I\ddot{\theta}$ Gives $-mgr = 6mr^2\ddot{\theta}$ $\ddot{\theta} = -\dfrac{g}{6r}$ Using laws of constant angular acceleration $0^2 - \omega^2 = 2\left(-\dfrac{g}{6r}\right)\theta$ Hence $\theta = \dfrac{3r\omega^2}{g}$	**M1** **A1** **M1** **A1**	4	Forms equation to find new acceleration Correct acceleration found Forms equation to find the angle required Correct expression obtained – CSO – must ensure negative sign is dealt with correctly
	Total		**18**	