AQA

Surname ____________________

Other Names ____________________

Centre Number ____________________

Candidate Number ____________________

Candidate Signature ____________________

A-level
MATHEMATICS

Paper 1

7357/1

Wednesday 5 June 2019 Morning

Time allowed: 2 hours

For this paper you must have:
- an AQA Formulae for A-level Mathematics booklet.
- a graphical or scientific calculator that meets the requirements of the specification.

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

[Turn over]

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INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do NOT use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 Given that $a > 0$, determine which of these expressions is NOT equivalent to the others.

Circle your answer. [1 mark]

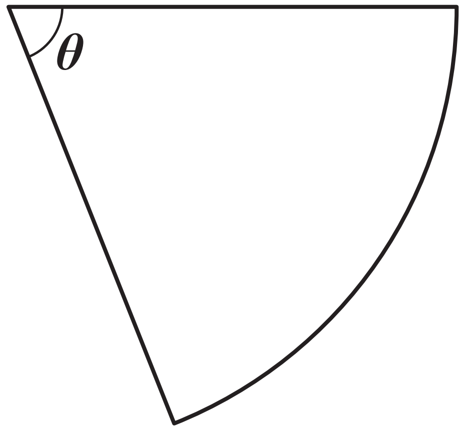
$-2\log_{10}\left(\frac{1}{a}\right)$ $\quad$ $2\log_{10}(a)$ $\quad$ $\log_{10}(a^2)$ $\quad$ $-4\log_{10}(\sqrt{a})$

2 Given $y = e^{kx}$, where k is a constant, find $\frac{dy}{dx}$

Circle your answer. [1 mark]

$\frac{dy}{dx} = e^{kx}$ $\quad$ $\frac{dy}{dx} = ke^{kx}$ $\quad$ $\frac{dy}{dx} = kxe^{kx-1}$ $\quad$ $\frac{dy}{dx} = \frac{e^{kx}}{k}$

3 The diagram below shows a sector of a circle.



The radius of the circle is 4 cm and $\theta = 0.8$ radians.

Find the area of the sector.

Circle your answer. [1 mark]

$1.28\ \text{cm}^2$ $3.2\ \text{cm}^2$ $6.4\ \text{cm}^2$ $12.8\ \text{cm}^2$

[Turn over]

4 **The point *A* has coordinates (-1, a) and the point *B* has coordinates (3, b)**

The line *AB* has equation $5x + 4y = 17$

Find the equation of the perpendicular bisector of the points *A* and *B*. [4 marks]

[Turn over]

5 **An arithmetic sequence has first term a and common difference d.**

The sum of the first 16 terms of the sequence is 260

5 (a) **Show that $4a + 30d = 65$ [2 marks]**

5 (b) **Given that the sum of the first 60 terms is 315, find the sum of the first 41 terms. [3 marks]**

[Turn over]

5 (c) S_n **is the sum of the first** n **terms of the sequence.**

Explain why the value you found in part (b) is the maximum value of S_n **[2 marks]**

[Turn over]

6 The function f is defined by

$$\mathrm{f}(x) = \frac{1}{2}(x^2 + 1),\ x \geq 0$$

6 (a) Find the range of f. [1 mark]

6 (b) (i) Find $\mathrm{f}^{-1}(x)$ [3 marks]

6 (b) (ii) State the range of $f^{-1}(x)$ [1 mark]

[Turn over]

6 (c) **State the transformation which maps the graph of $y = f(x)$ onto the graph of $y = f^{-1}(x)$**
[1 mark]

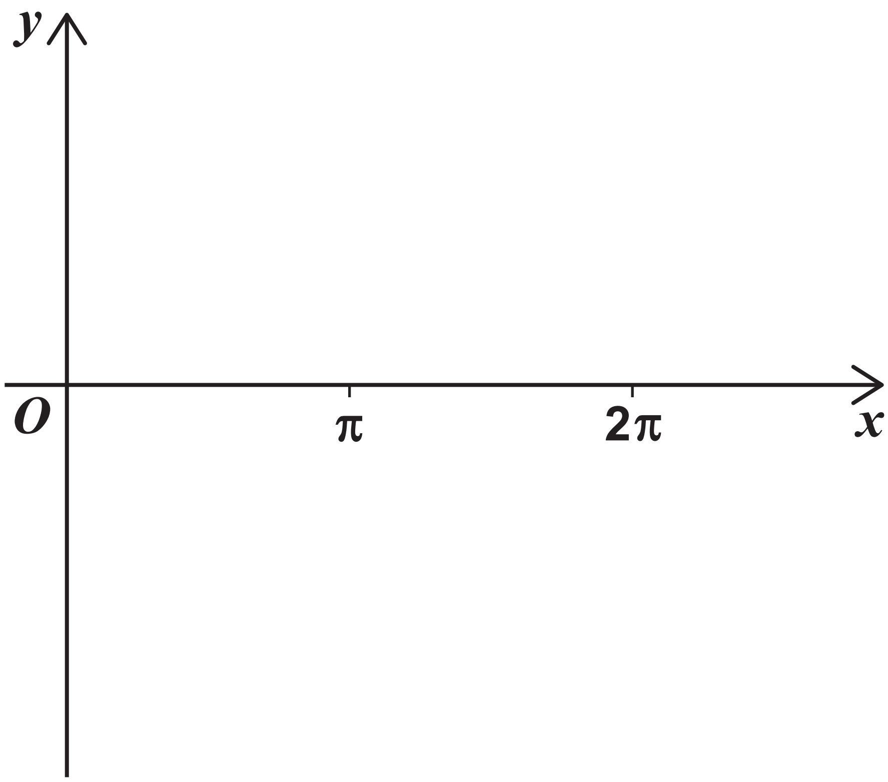
6 (d) **Find the coordinates of the point of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$**
[2 marks]

[Turn over]

7 (a) By sketching the graphs of $y = \frac{1}{x}$ and $y = \sec 2x$ on the axes below, show that the equation

$$\frac{1}{x} = \sec 2x$$

has exactly one solution for $x > 0$ **[3 marks]**



7 (b) By considering a suitable change of sign, show that the solution to the equation lies between 0.4 and 0.6 [2 marks]

[Turn over]

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7 (c) **Show that the equation can be rearranged to give**

$$x = \frac{1}{2}\cos^{-1} x$$

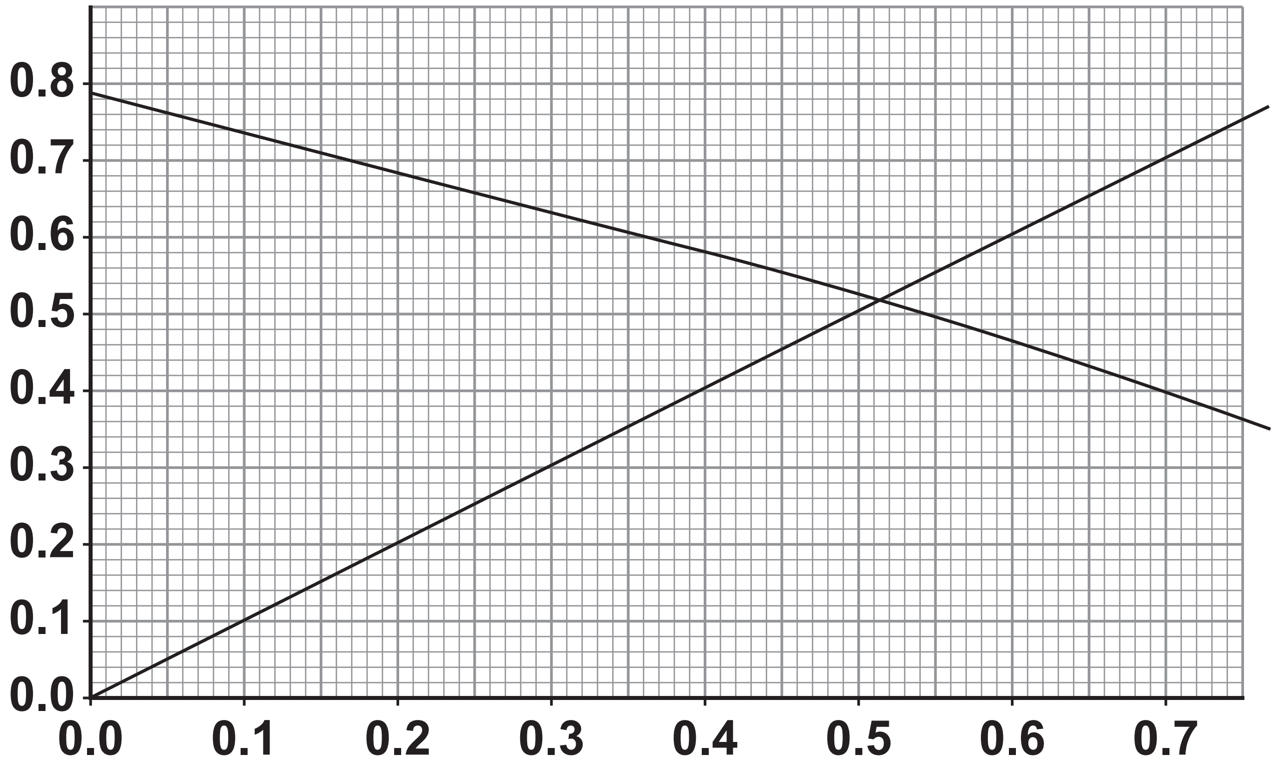
[2 marks]

7 (d) (i) Use the iterative formula

$$x_{n+1} = \frac{1}{2}\cos^{-1} x_n$$

with $x_1 = 0.4$, to find x_2, x_3 and x_4, giving your answers to four decimal places. [2 marks]

7 (d) (ii) On the graph below, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2, x_3 and x_4.
[2 marks]



[Turn over]

8 $P(n) = \sum_{k=0}^{n} k^3 - \sum_{k=0}^{n-1} k^3$ **where n is a positive integer.**

8 (a) **Find $P(3)$ and $P(10)$ [2 marks]**

8 (b) **Solve the equation $P(n) = 1.25 \times 10^8$ [2 marks]**

[Turn over]

9 **Prove that the sum of a rational number and an irrational number is always irrational. [5 marks]**

[Turn over]

10 **The volume of a spherical bubble is increasing at a constant rate.**

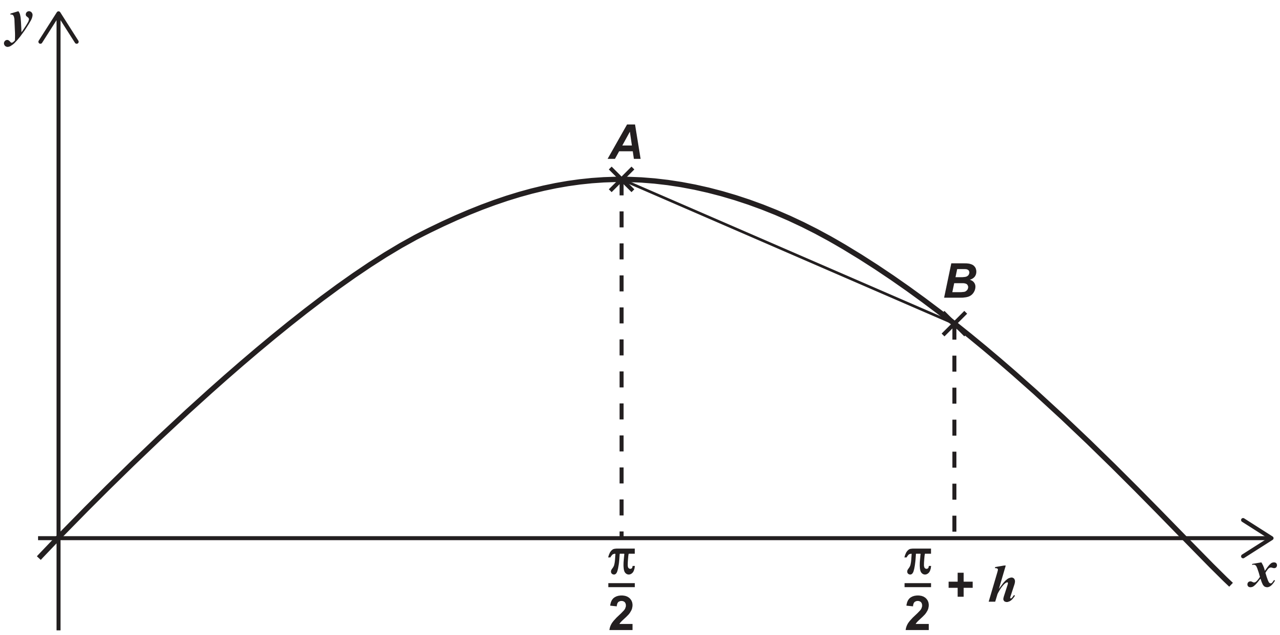
Show that the rate of increase of the radius, r, of the bubble is inversely proportional to r^2

Volume of a sphere $= \frac{4}{3}\pi r^3$

[4 marks]

11 Jodie is attempting to use differentiation from first principles to prove that the gradient of $y = \sin x$ is zero when $x = \frac{\pi}{2}$

Jodie's teacher tells her that she has made mistakes starting in Step 4 of her working. Her working is shown on page 29.



Step 1 Gradient of chord $AB = \dfrac{\sin\left(\frac{\pi}{2} + h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 2 $= \dfrac{\sin\left(\frac{\pi}{2}\right)\cos(h) + \cos\left(\frac{\pi}{2}\right)\sin(h) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 3 $= \sin\left(\frac{\pi}{2}\right)\left(\dfrac{\cos(h) - 1}{h}\right) + \cos\left(\frac{\pi}{2}\right)\dfrac{\sin(h)}{h}$

Step 4 For gradient of curve at A,

let $h = 0$ then

$\dfrac{\cos(h) - 1}{h} = 0$ and $\dfrac{\sin(h)}{h} = 0$

Step 5 Hence the gradient of the curve at A is given by

$\sin\left(\frac{\pi}{2}\right) \times 0 + \cos\left(\frac{\pi}{2}\right) \times 0 = 0$

[Turn over]

Complete Steps 4 and 5 of Jodie's working below, to correct her proof. [4 marks]

Step 4 For gradient of curve at *A*,

Step 5 Hence the gradient of the curve at *A* is given by

12 (a) Show that the equation

$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$

can be written in the form

$a\operatorname{cosec}^2 x + b\operatorname{cosec} x + c = 0$

[2 marks]

[Turn over]

12 (b) Hence, given x is obtuse and

$$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$$

find the exact value of $\tan x$

Fully justify your answer. [5 marks]

[Turn over]

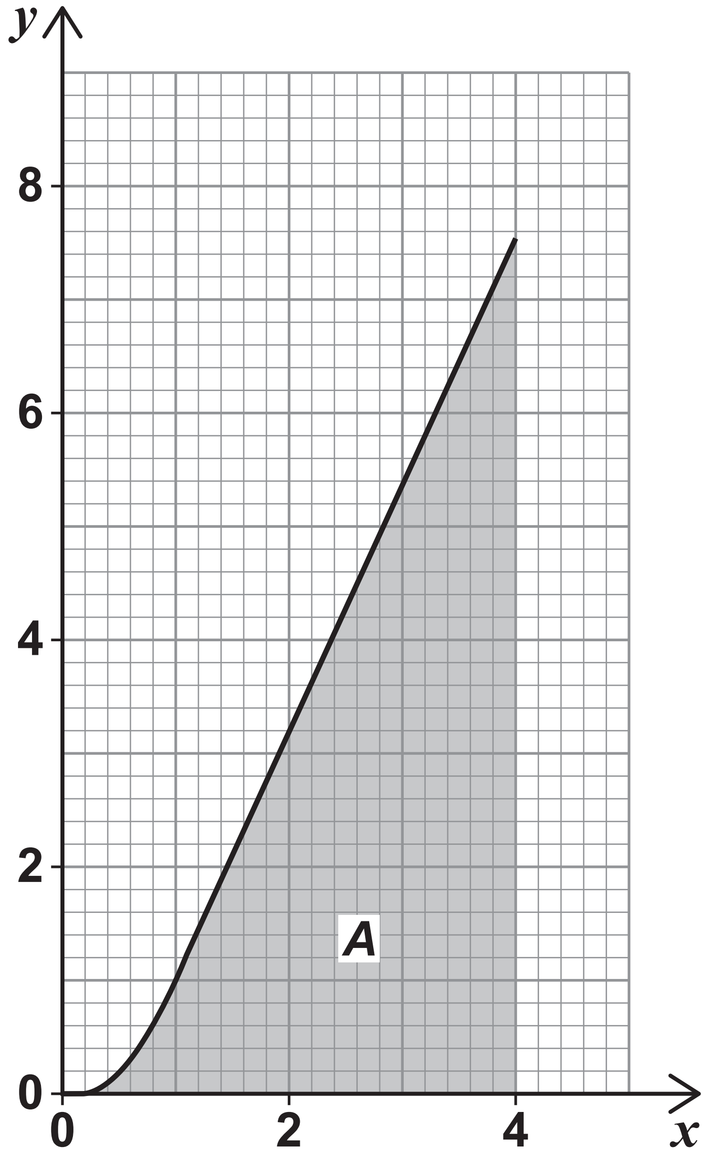
13 A curve, *C*, has equation

$$y = \frac{e^{3x-5}}{x^2}$$

Show that *C* has exactly one stationary point.

Fully justify your answer. [7 marks]

14 The graph of $y = \dfrac{2x^3}{x^2 + 1}$ is shown for $0 \le x \le 4$



Caroline is attempting to approximate the shaded area, *A*, under the curve using the trapezium rule by splitting the area into n trapezia.

14 (a) When $n = 4$

14 (a) (i) State the number of ordinates that Caroline uses. [1 mark]

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14 (a) (ii) **Calculate the area that Caroline should obtain using this method.**

Give your answer correct to two decimal places. [3 marks]

14 (b) **Show that the exact area of *A* is**

$16 - \ln 17$

Fully justify your answer. [5 marks]

14 (c) **Explain what would happen to Caroline's answer to part (a)(ii) as $n \to \infty$ [1 mark]**

[Turn over]

15 (a) **At time t hours AFTER A HIGH TIDE, the height, h metres, of the tide and the velocity, v knots, of the tidal flow can be modelled using the parametric equations**

$$v = 4 - \left(\frac{2t}{3} - 2\right)^2$$

$$h = 3 - 2\sqrt[3]{t - 3}$$

High tides and low tides occur alternately when the velocity of the tidal flow is zero.

A high tide occurs at 2 am.

15 (a) (i) **Use the model to find the height of this high tide. [1 mark]**

15 (a) (ii) Find the time of the first LOW tide after 2 am. [3 marks]

[Turn over]

15 (a)(iii) Find the height of this low tide. [1 mark]

15 (b) Use the model to find the height of the tide when it is flowing with maximum velocity. [3 marks]

15 (c) **Comment on the validity of the model.**
[2 marks]

[Turn over]

16 (a) $y = e^{-x}(\sin x + \cos x)$

Find $\frac{dy}{dx}$

Simplify your answer. [3 marks]

16 (b) **Hence, show that**

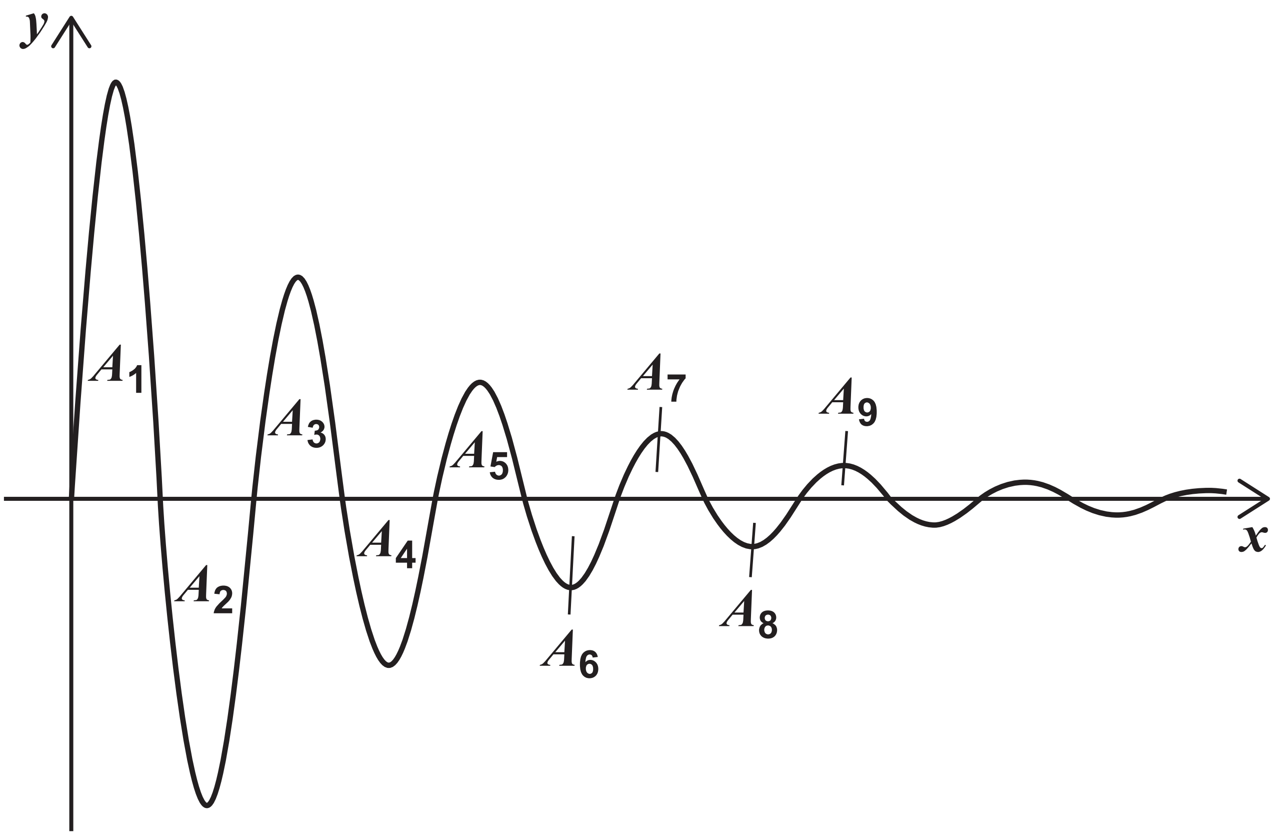
$$\int e^{-x} \sin x \, dx = a e^{-x}(\sin x + \cos x) + c$$

where a is a rational number. [2 marks]

[Turn over]

16 (c) **A sketch of the graph of $y = e^{-x} \sin x$ for $x \geq 0$ is shown below.**

The areas of the finite regions bounded by the curve and the x-axis are denoted by A_1, A_2, ..., A_n, ...



16 (c) (i) Find the exact value of the area A_1 [3 marks]

[Turn over]

16 (c) (ii) Show that

$$\frac{A_2}{A_1} = e^{-\pi}$$

[4 marks]

[Turn over]

16 (c)(iii) Given that

$$\frac{A_{n+1}}{A_n} = e^{-\pi}$$

show that the exact value of the total area enclosed between the curve and the x-axis is

$$\frac{1 + e^{\pi}}{2(e^{\pi} - 1)}$$

[4 marks]

END OF QUESTIONS

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For Examiner's Use	
Question	Mark
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