AQA

Surname ______________________________

Other Names __________________________

Centre Number ________________________

Candidate Number _____________________

Candidate Signature ___________________

A-level MATHEMATICS

Paper 1

7357/1

Wednesday 5 June 2019 Morning

Time allowed: 2 hours

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

J U N 1 9 7 3 5 7 1 0 1

For this paper you must have:
- **an AQA Formulae for A-level Mathematics booklet.**
- **a graphical or scientific calculator that meets the requirements of the specification.**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do NOT use the space provided for a different question.**
- **Show all necessary working; otherwise marks for method may be lost.**

- **Do all rough work in this book. Cross through any work that you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 Given that $a > 0$, determine which of these expressions is NOT equivalent to the others.

Circle your answer. [1 mark]

$-2\log_{10}\left(\frac{1}{a}\right)$ $2\log_{10}(a)$

$\log_{10}(a^2)$ $-4\log_{10}(\sqrt{a})$

2 Given $y = e^{kx}$, where k is a constant, find $\frac{dy}{dx}$

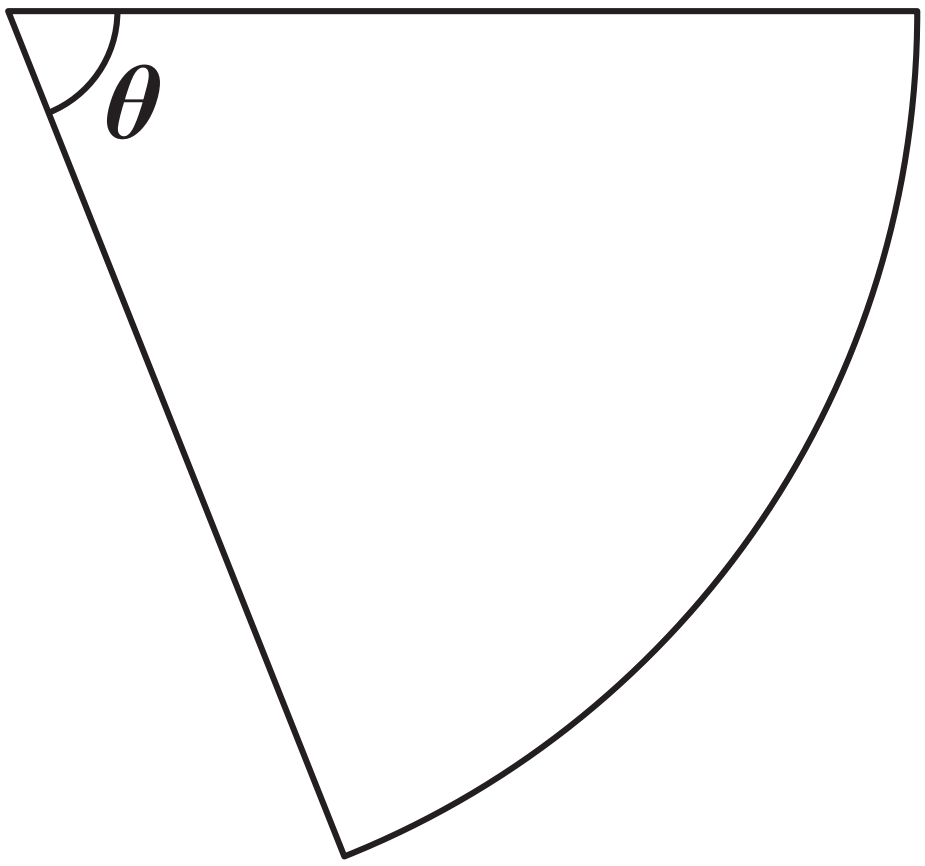
Circle your answer. [1 mark]

$\frac{dy}{dx} = e^{kx}$ $\frac{dy}{dx} = ke^{kx}$

$\frac{dy}{dx} = kxe^{kx-1}$ $\frac{dy}{dx} = \frac{e^{kx}}{k}$

[Turn over]

3 The diagram below shows a sector of a circle.



The radius of the circle is 4 cm and $\theta = 0.8$ radians.

Find the area of the sector.

Circle your answer. [1 mark]

1.28 cm^2 3.2 cm^2

6.4 cm^2 12.8 cm^2

4 The point *A* has coordinates $(-1, a)$ and the point *B* has coordinates $(3, b)$

The line *AB* has equation $5x + 4y = 17$

Find the equation of the perpendicular bisector of the points *A* and *B*. [4 marks]

[Turn over]

[Turn over]

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5 **An arithmetic sequence has first term a and common difference d.**

The sum of the first 16 terms of the sequence is 260

5 (a) **Show that $4a + 30d = 65$**
[2 marks]

5 (b) Given that the sum of the first 60 terms is 315, find the sum of the first 41 terms. [3 marks]

[Turn over]

5 (c) S_n **is the sum of the first** n **terms of the sequence.**

Explain why the value you found in part (b) is the maximum value of S_n **[2 marks]**

[Turn over]

6 The function f is defined by

$$\mathrm{f}(x) = \frac{1}{2}(x^2 + 1),\ x \geq 0$$

6 (a) Find the range of f. [1 mark]

6 (b) (i) Find $\mathrm{f}^{-1}(x)$ [3 marks]

[Turn over]

6 (b) (ii) State the range of $f^{-1}(x)$ [1 mark]

6 (c) State the transformation which maps the graph of $y = f(x)$ onto the graph of $y = f^{-1}(x)$ [1 mark]

[Turn over]

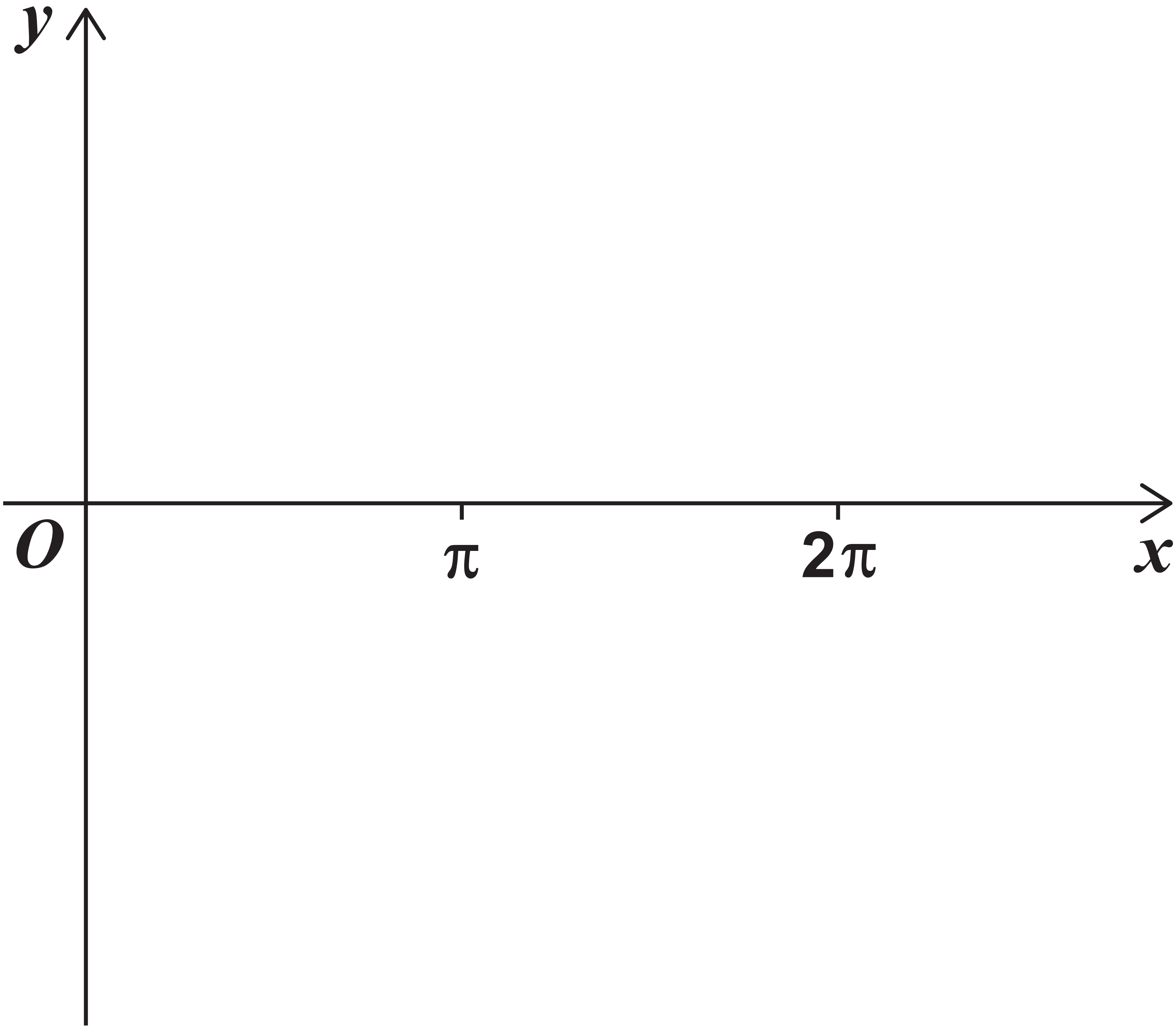
6 (d) **Find the coordinates of the point of intersection of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$** **[2 marks]**

[Turn over]

7 (a) **By sketching the graphs of $y = \frac{1}{x}$ and $y = \sec 2x$ on the axes below, show that the equation**

$$\frac{1}{x} = \sec 2x$$

has exactly one solution for $x > 0$
[3 marks]



7 (b) By considering a suitable change of sign, show that the solution to the equation lies between 0.4 and 0.6 [2 marks]

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7 (c) **Show that the equation can be rearranged to give**

$$x = \frac{1}{2}\cos^{-1} x$$

[2 marks]

[Turn over]

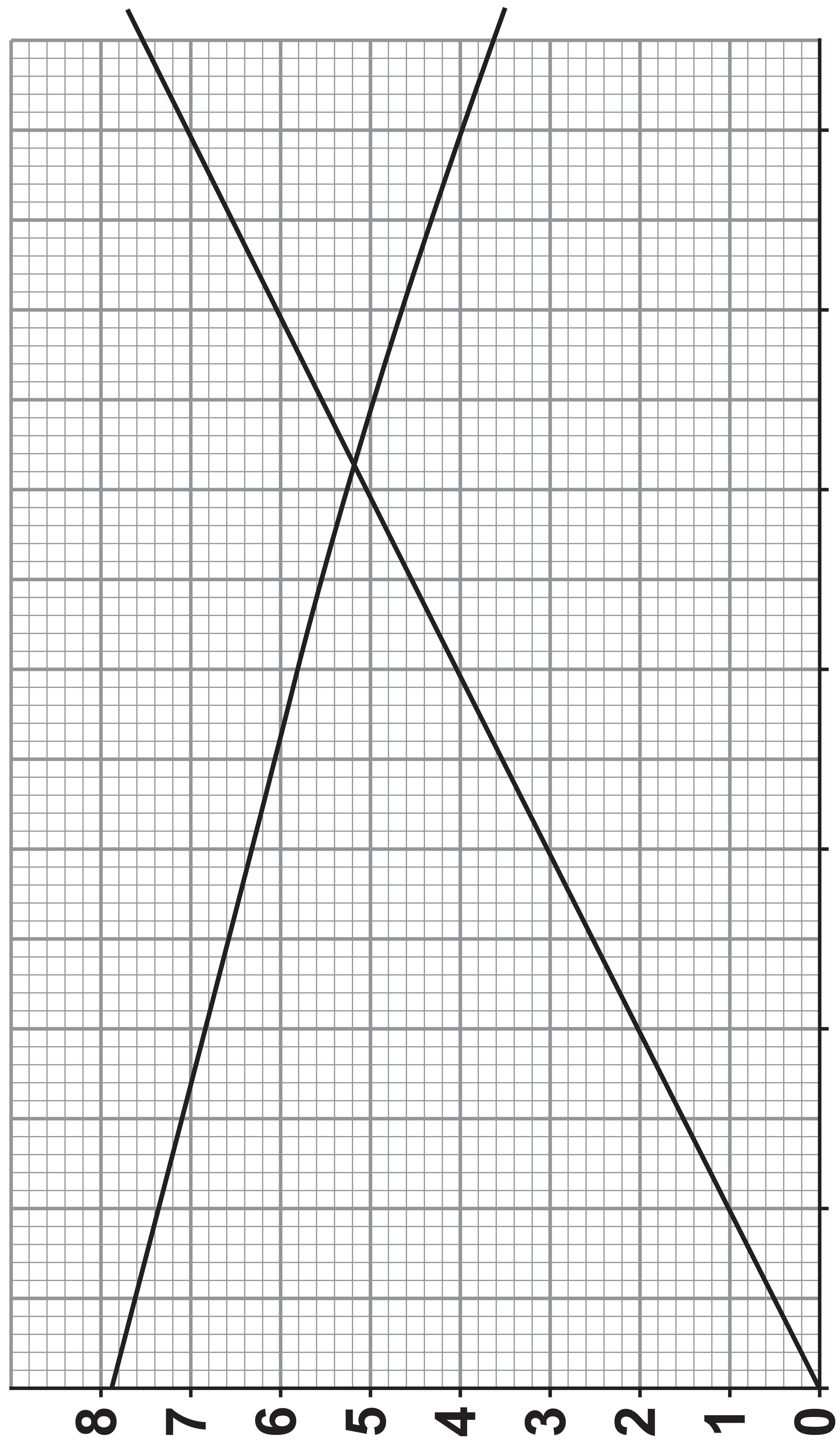
7 (d) (i) Use the iterative formula

$$x_{n+1} = \frac{1}{2}\cos^{-1} x_n$$

with $x_1 = 0.4$, to find x_2, x_3 and x_4, giving your answers to four decimal places. [2 marks]

[Turn over]

7 (d) (ii) On the graph on page 29, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2, x_3 and x_4.
[2 marks]


0.0
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0.8

8 $$P(n) = \sum_{k=0}^{n} k^3 - \sum_{k=0}^{n-1} k^3$$ where n is a positive integer.

8 (a) Find $P(3)$ and $P(10)$ [2 marks]

8 (b) Solve the equation

$P(n) = 1.25 \times 10^8$ [2 marks]

[Turn over]

9 **Prove that the sum of a rational number and an irrational number is always irrational. [5 marks]**

[Turn over]

10 **The volume of a spherical bubble is increasing at a constant rate.**

Show that the rate of increase of the radius, r, of the bubble is inversely proportional to r^2

Volume of a sphere $= \frac{4}{3}\pi r^3$

[4 marks]

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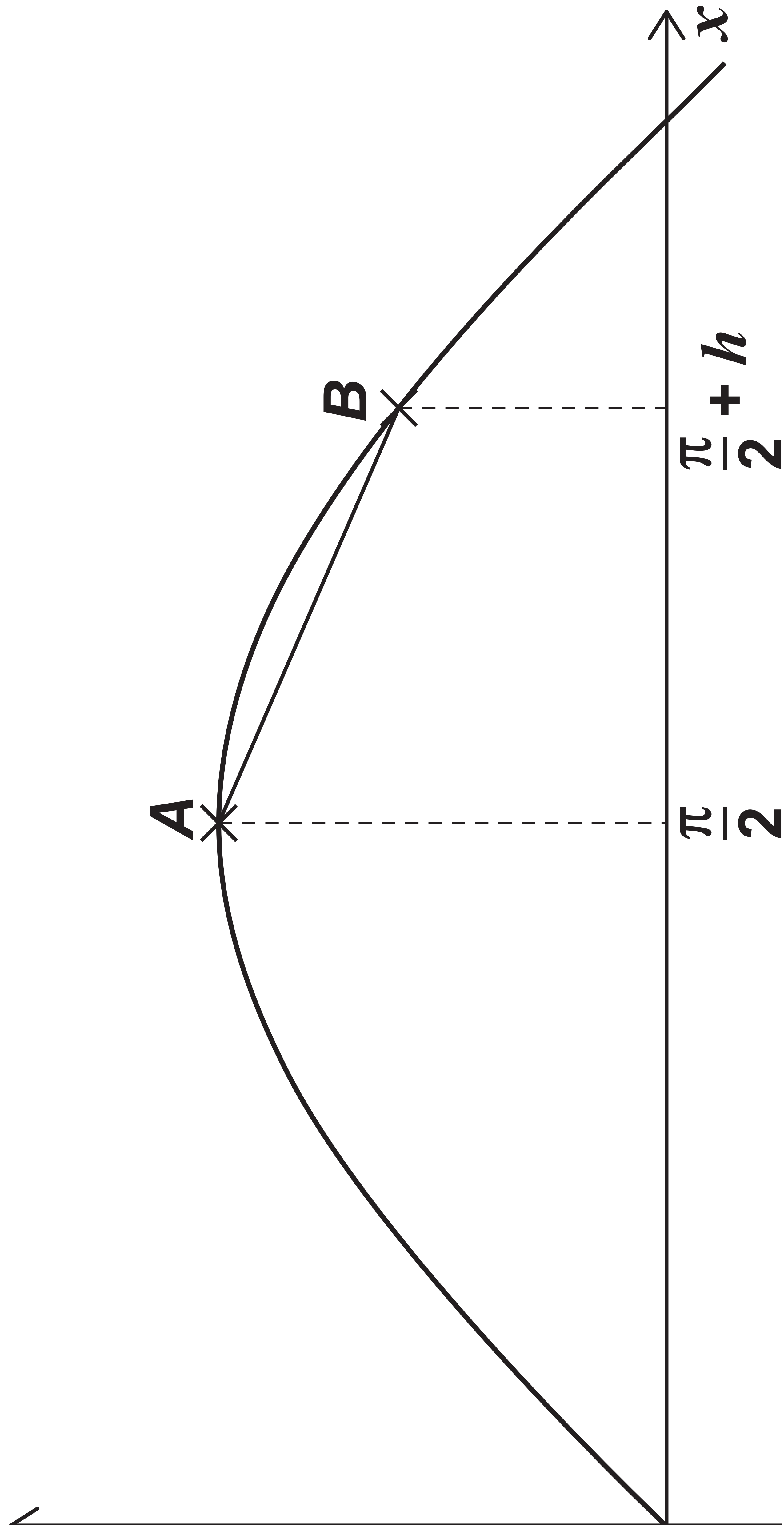
[Turn over for the next question]

11 **Jodie is attempting to use differentiation from first principles to prove that the gradient of $y = \sin x$ is zero when $x = \frac{\pi}{2}$**

Jodie's teacher tells her that she has made mistakes starting in Step 4 of her working. Her working is shown on pages 40 and 41.


y
x
A
B
π/2
π/2 + h

Step 1 **Gradient of chord** $AB = \dfrac{\sin\left(\frac{\pi}{2} + h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 2 $= \dfrac{\sin\left(\frac{\pi}{2}\right)\cos(h) + \cos\left(\frac{\pi}{2}\right)\sin(h) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 3 $= \sin\left(\frac{\pi}{2}\right)\left(\dfrac{\cos(h) - 1}{h}\right) + \cos\left(\frac{\pi}{2}\right)\dfrac{\sin(h)}{h}$

Step 4 For gradient of curve at *A*,

let $h = 0$ then

$$\frac{\cos(h) - 1}{h} = 0 \text{ and } \frac{\sin(h)}{h} = 0$$

Step 5 Hence the gradient of the curve at *A* is given by

$$\sin\left(\frac{\pi}{2}\right) \times 0 + \cos\left(\frac{\pi}{2}\right) \times 0 = 0$$

[Turn over]

Complete Steps 4 and 5 of Jodie's working below, to correct her proof. [4 marks]

Step 4 For gradient of curve at *A*,

Step 5 **Hence the gradient of the curve at *A* is given by**

[Turn over]

12 (a) **Show that the equation**

$$2\cot^2 x + 2\,\text{cosec}^2 x = 1 + 4\,\text{cosec}\, x$$

can be written in the form

$$a\,\text{cosec}^2 x + b\,\text{cosec}\, x + c = 0$$

[2 marks]

[Turn over]

12 (b) **Hence, given x is obtuse and**

$$2\cot^2 x + 2\,\text{cosec}^2 x = 1 + 4\,\text{cosec}\,x$$

find the exact value of $\tan x$

Fully justify your answer.
[5 marks]

[Turn over]

13 **A curve, *C*, has equation**

$$y = \frac{e^{3x-5}}{x^2}$$

Show that *C* has exactly one stationary point.

Fully justify your answer.
[7 marks]

[Turn over]

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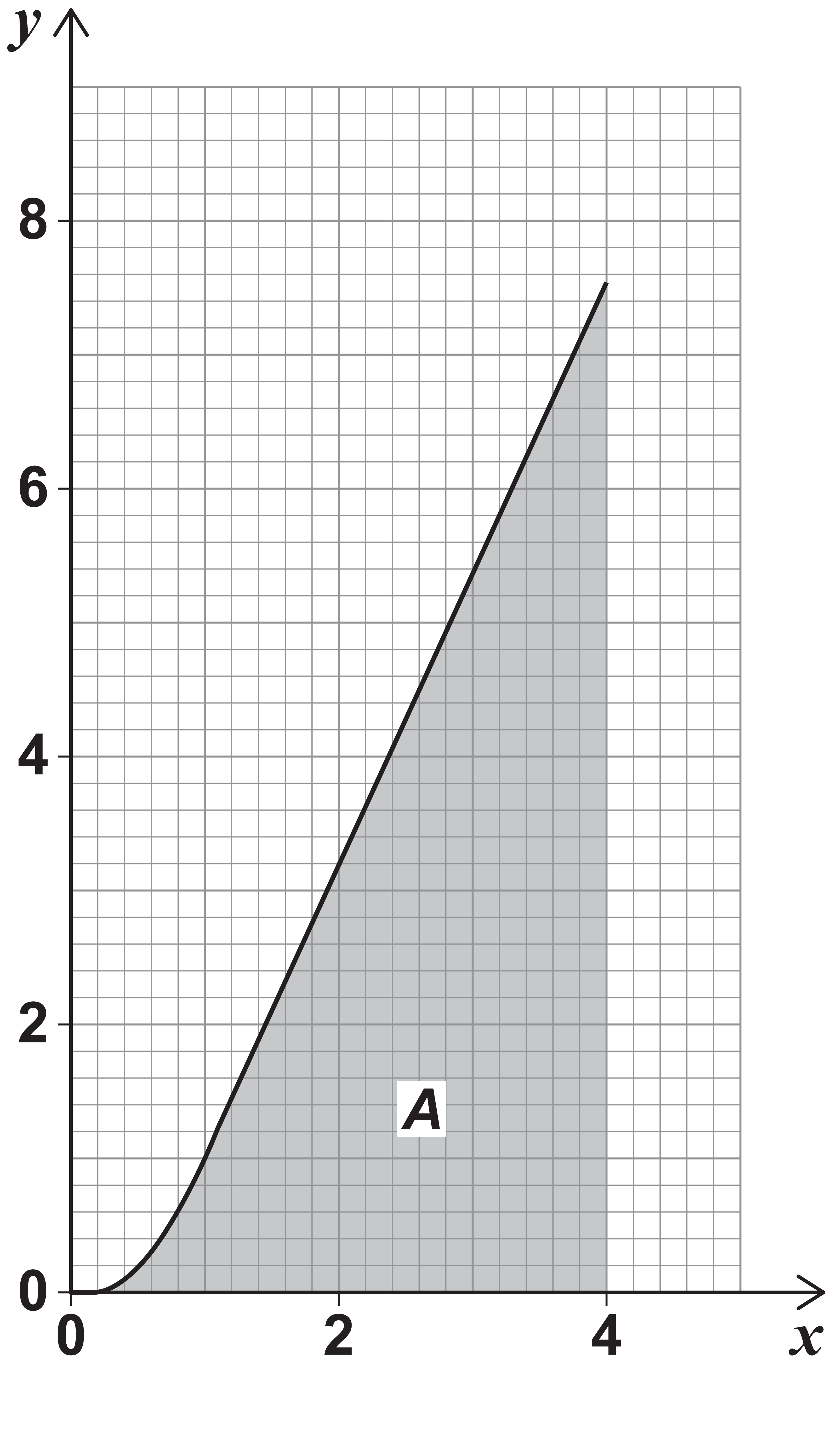
[Turn over for the next question]

14 **The graph of** $y = \dfrac{2x^3}{x^2 + 1}$ **is shown for** $0 \leq x \leq 4$



[Turn over]

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Caroline is attempting to approximate the shaded area, *A*, under the curve using the trapezium rule by splitting the area into n trapezia.

14 (a) When $n = 4$

14 (a) (i) State the number of ordinates that Caroline uses. [1 mark]

[Turn over]

14 (a) (ii) Calculate the area that Caroline should obtain using this method.

Give your answer correct to two decimal places. [3 marks]

[Turn over]

14 (b) **Show that the exact area of *A* is**

$16 - \ln 17$

Fully justify your answer.
[5 marks]

14 (c) **Explain what would happen to Caroline's answer to part (a)(ii) as $n \to \infty$ [1 mark]**

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[Turn over for the next question]

15 (a) At time t hours AFTER A HIGH TIDE, the height, h metres, of the tide and the velocity, v knots, of the tidal flow can be modelled using the parametric equations

$$v = 4 - \left(\frac{2t}{3} - 2\right)^2$$

$$h = 3 - 2\sqrt[3]{t - 3}$$

High tides and low tides occur alternately when the velocity of the tidal flow is zero.

A high tide occurs at 2 am.

15 (a) (i) Use the model to find the height of this high tide. [1 mark]

[Turn over]

15 (a) (ii) Find the time of the first LOW tide after 2 am. [3 marks]

[Turn over]

15(a)(iii) Find the height of this low tide. [1 mark]

15 (b) Use the model to find the height of the tide when it is flowing with maximum velocity. [3 marks]

[Turn over]

15 (c) **Comment on the validity of the model. [2 marks]**

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[Turn over for the next question]

16 (a) $y = e^{-x}(\sin x + \cos x)$

Find $\frac{dy}{dx}$

Simplify your answer. [3 marks]

[Turn over]

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16 (b) **Hence, show that**

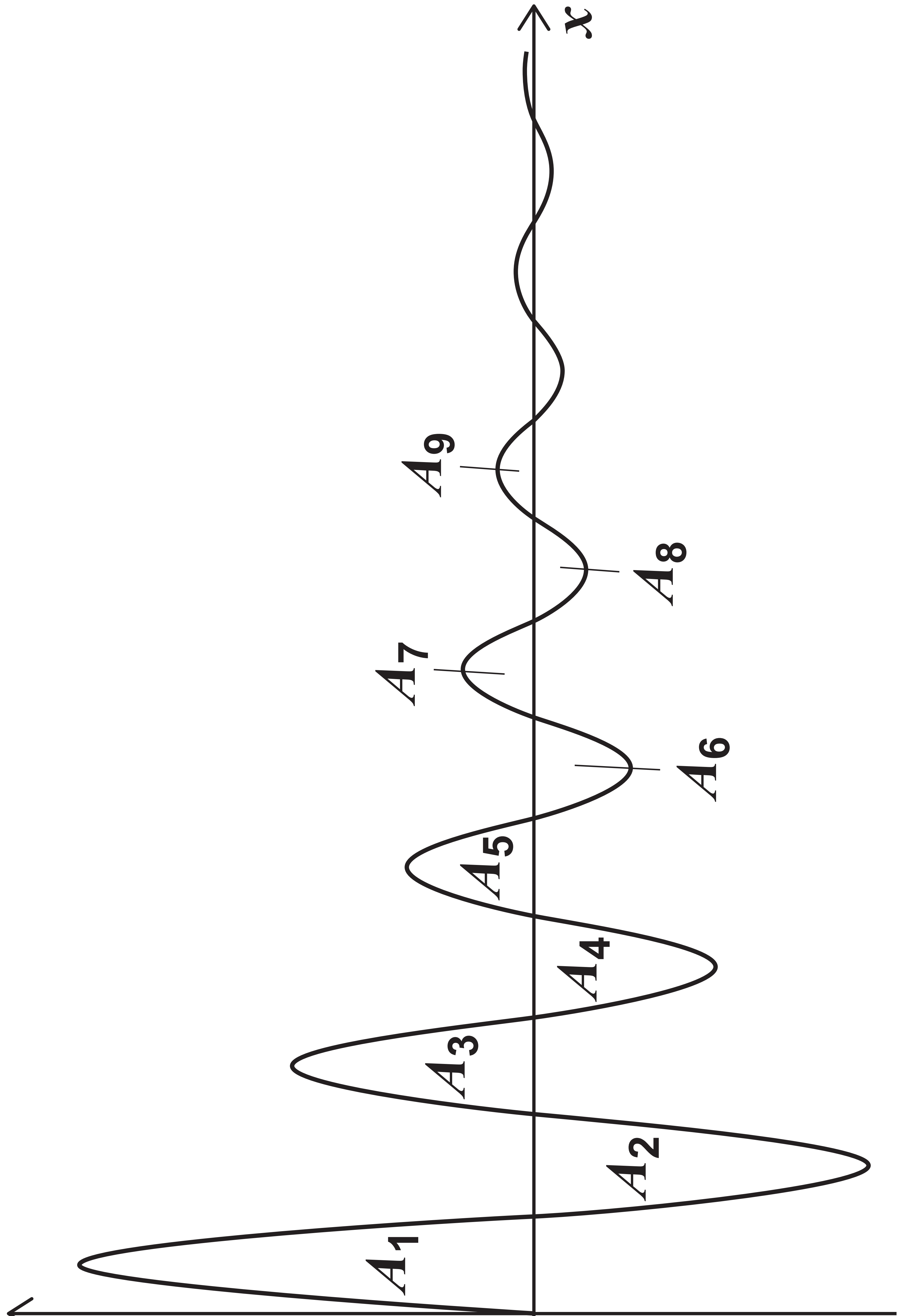
$$\int e^{-x} \sin x \, dx = a e^{-x}(\sin x + \cos x) + c$$

where a is a rational number.
[2 marks]

[Turn over]

16 (c) **A sketch of the graph of $y = \mathrm{e}^{-x} \sin x$ for $x \geq 0$ is shown on page 75.**

The areas of the finite regions bounded by the curve and the x-axis are denoted by $A_1, A_2, \ldots, A_n, \ldots$



[Turn over]

16 (c) (i) Find the exact value of the area A_1
[3 marks]

[Turn over]

16 (c) (ii) Show that

$$\frac{A_2}{A_1} = e^{-\pi}$$

[4 marks]

[Turn over]

16(c)(iii) Given that

$$\frac{A_{n+1}}{A_n} = e^{-\pi}$$

show that the exact value of the total area enclosed between the curve and the x-axis is

$$\frac{1 + e^{\pi}}{2(e^{\pi} - 1)}$$

[4 marks]

[Turn over]

END OF QUESTIONS

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For Examiner's Use	
Question	Mark
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