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Centre Number _____

Candidate Number _____

Candidate Signature _____

A-level MATHEMATICS

Paper 2

7357/2

Wednesday 12 June 2019 Morning

Time allowed: 2 hours

For this paper:

- You must have the **AQA Formulae for A-level Mathematics** booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

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INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do NOT use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO

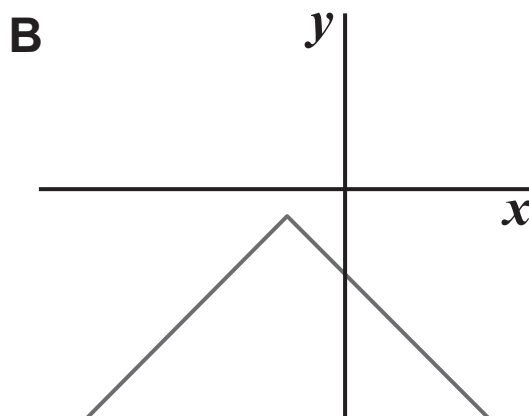
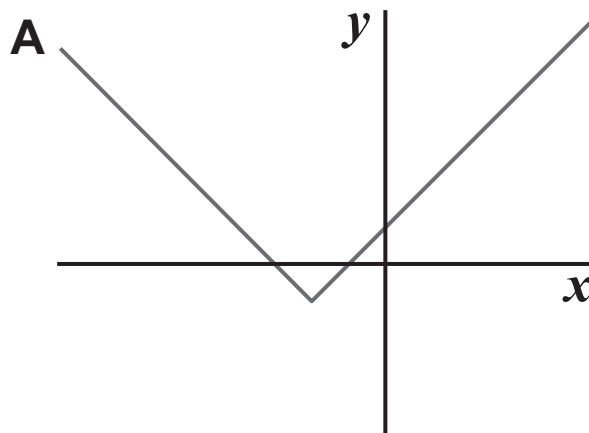


SECTION A

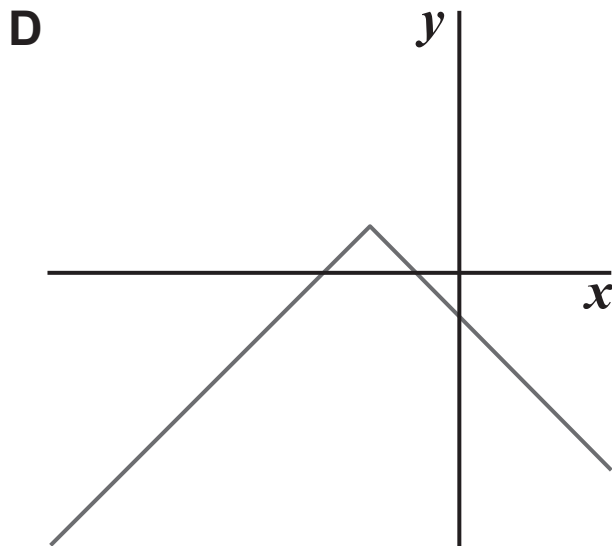
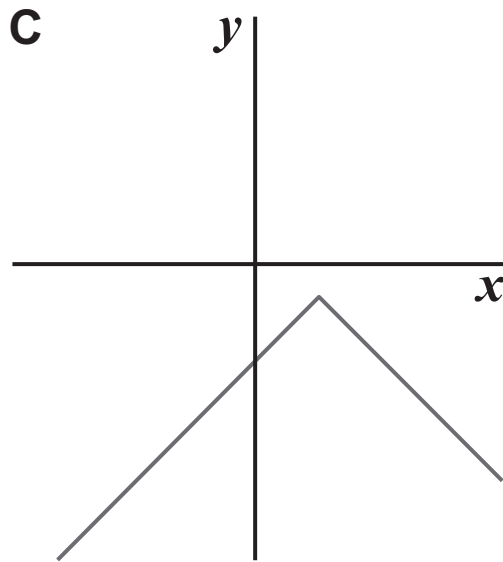
Answer ALL questions in the spaces provided.

- 1 Identify the graph of $y = 1 - |x + 2|$ from the options below.

Tick (✓) ONE box. [1 mark]



5



[Turn over]



2 Simplify $\sqrt{a^{\frac{2}{3}} \times a^{\frac{2}{5}}}$

Circle your answer. [1 mark]

$$a^{\frac{2}{15}}$$

$$a^{\frac{4}{15}}$$

$$a^{\frac{8}{15}}$$

$$a^{\frac{16}{15}}$$

3 Each of these functions has domain $x \in \mathbb{R}$

Which function does NOT have an inverse?

Circle your answer. [1 mark]

$$f(x) = x^3$$

$$f(x) = 2x + 1$$

$$f(x) = x^2$$

$$f(x) = e^x$$



4 $x^2 + bx + c$ and $x^2 + dx + e$ have a common factor $(x + 2)$

Show that $2(d - b) = e - c$

Fully justify your answer. [4 marks]

[Turn over]



5 Solve the differential equation

$$\frac{dt}{dx} = \frac{\ln x}{x^2 t} \quad \text{for } x > 0$$

given $x = 1$ when $t = 2$

Write your answer in the form $t^2 = f(x)$
[7 marks]

[Turn over]

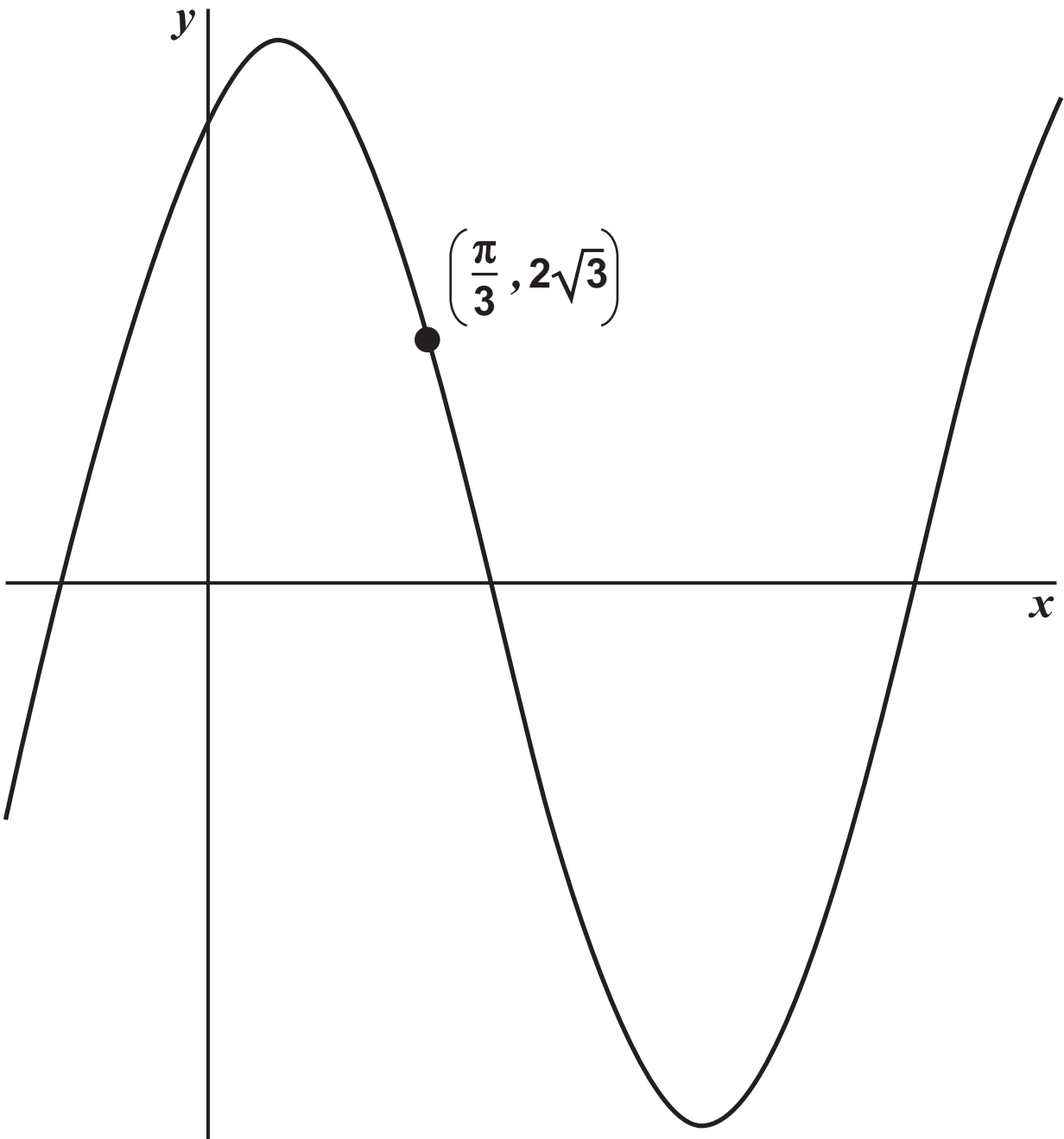


6 A curve has equation

$$y = a \sin x + b \cos x$$

where a and b are constants.

The maximum value of y is 4 and the curve passes through the point $\left(\frac{\pi}{3}, 2\sqrt{3}\right)$ as shown in the diagram.



11

Find the exact values of a and b . [6 marks]

[Turn over]



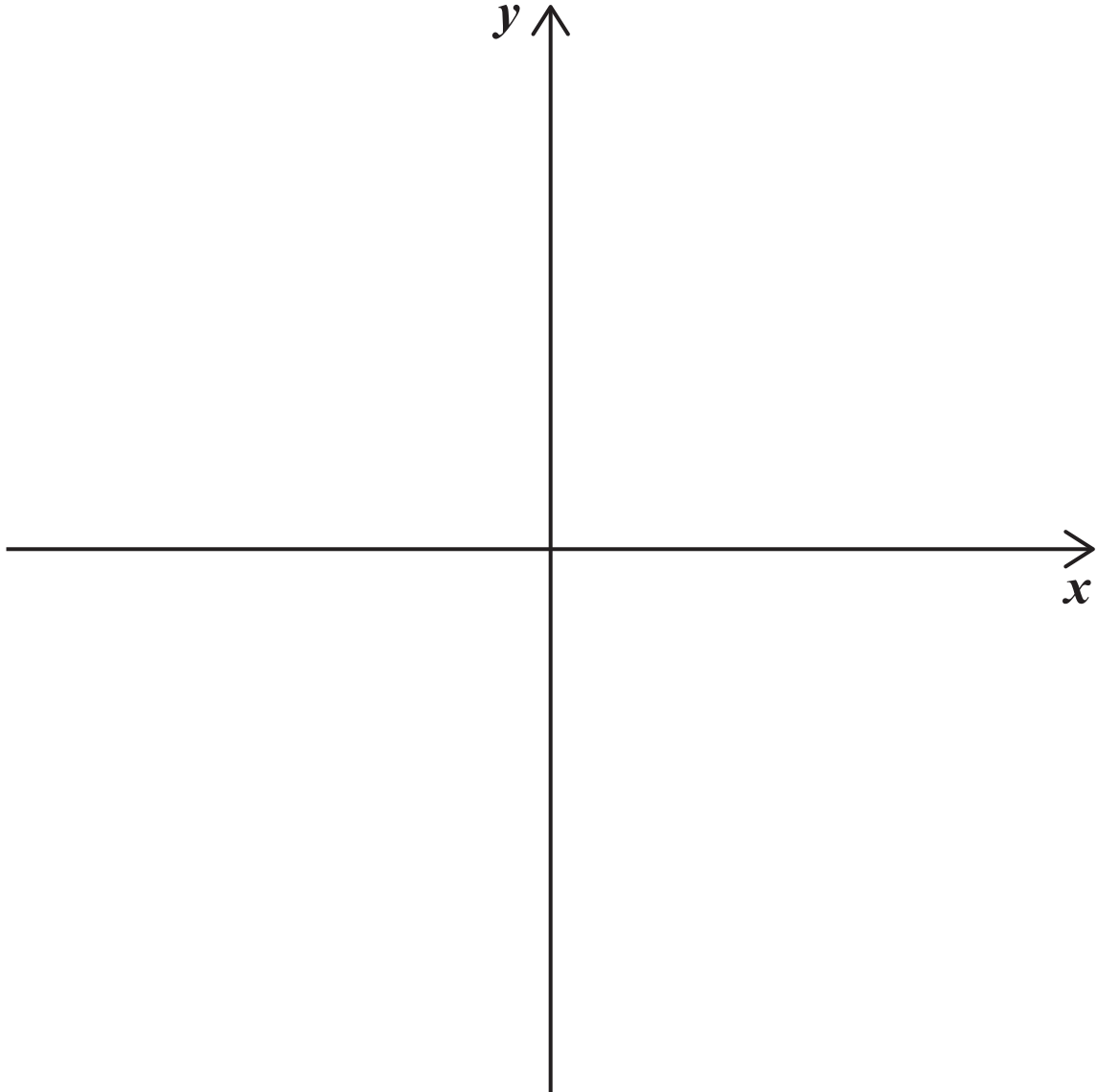


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[Turn over for the next question]



- 7 (a) Sketch the graph of any cubic function that has BOTH three distinct real roots AND a positive coefficient of x^3 [2 marks]



- 7 (b) The function $f(x)$ is defined by

$$f(x) = x^3 + 3px^2 + q$$

where p and q are constants and $p > 0$



7 (b) (i) Show that there is a turning point where the curve crosses the y -axis. [3 marks]

[Turn over]



- 7 (b) (ii) The equation $f(x) = 0$ has three distinct real roots.

By considering the positions of the turning points find, in terms of p , the range of possible values of q . [5 marks]



[Turn over]



- 8 Theresa bought a house on 2 January 1970 for £8000.

The house was valued by a local estate agent on the same date every 10 years up to 2010.

The valuations are shown in the following table.

YEAR	1970	1980	1990	2000	2010
VALUATION PRICE	£8 000	£19 000	£36 000	£82 000	£205 000

The valuation price of the house can be modelled by the equation

$$V = pq^t$$

where V pounds is the valuation price t years after 2 January 1970 and p and q are constants.

- 8 (a) Show that $V = pq^t$ can be written as $\log_{10} V = \log_{10} p + t \log_{10} q$ [2 marks]

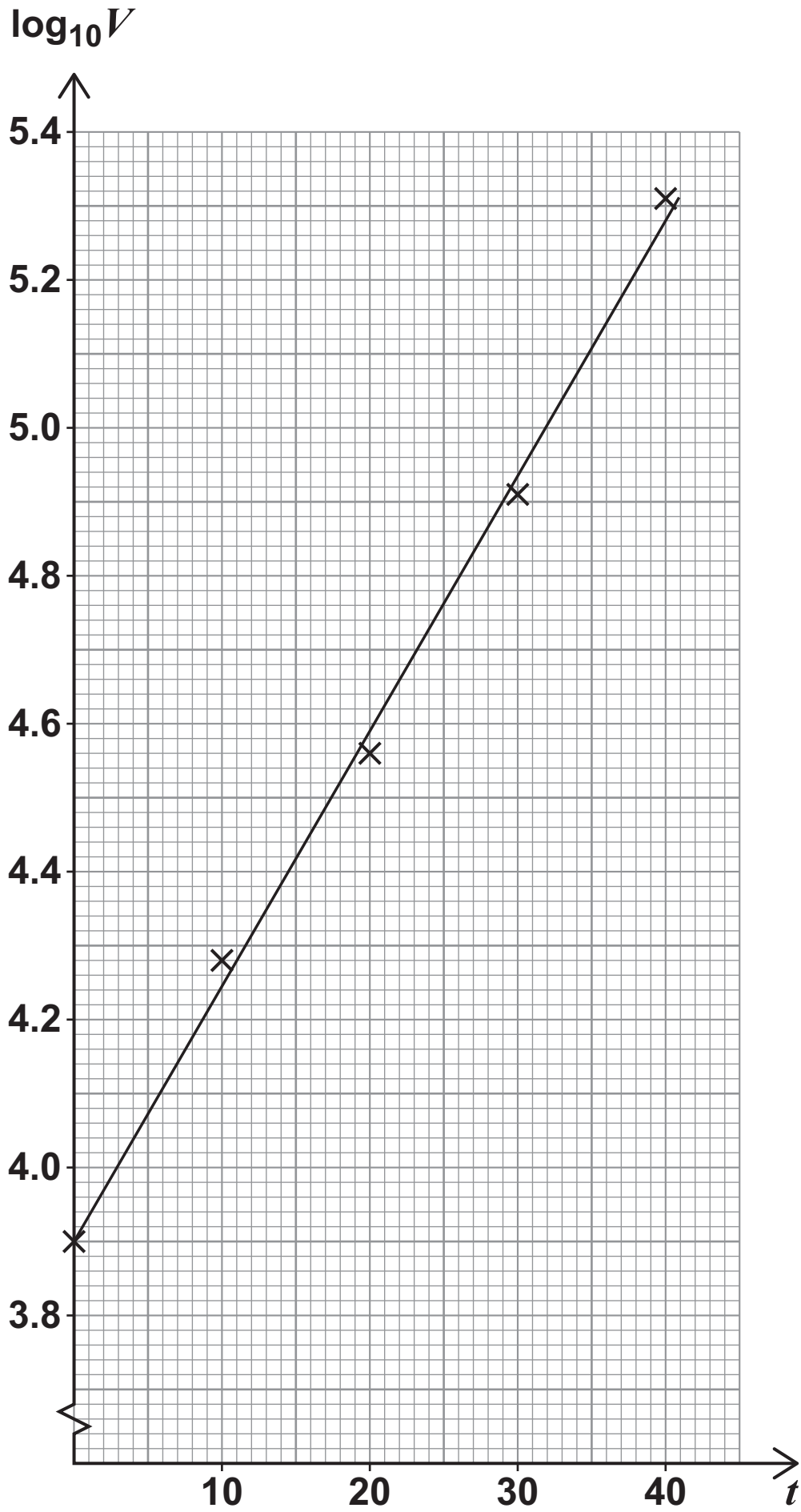


[Turn over]

- 8 (b) The values in the table of $\log_{10} V$ against t have been plotted and a line of best fit has been drawn on the graph opposite.

t	0	10	20	30	40
$\log_{10} V$	3.90	4.28	4.56	4.91	5.31





[Turn over]



Using the given line of best fit, find estimates for the values of p and q .

Give your answers correct to three significant figures. [4 marks]

[Turn over]



- 8 (c) Determine the year in which Theresa's house will first be worth half a million pounds.
[3 marks]**



8 (d) Explain whether your answer to part (c) is likely to be reliable. [2 marks]

[Turn over]

- 9 (a) Show that the first two terms of the binomial expansion of $\sqrt{4 - 2x^2}$ are

$$2 - \frac{x^2}{2}$$

[2 marks]

- 9 (b) State the range of values of x for which the expansion found in part (a) is valid. [2 marks]

[Turn over]



9 (c) Hence, find an approximation for

$$\int_0^{0.4} \sqrt{\cos x} \, dx$$

giving your answer to five decimal places.

Fully justify your answer. [4 marks]

- 9 (d) A student decides to use this method to find an approximation for

$$\int_0^{1.4} \sqrt{\cos x} \, dx$$

Explain why this may not be a suitable method.
[1 mark]

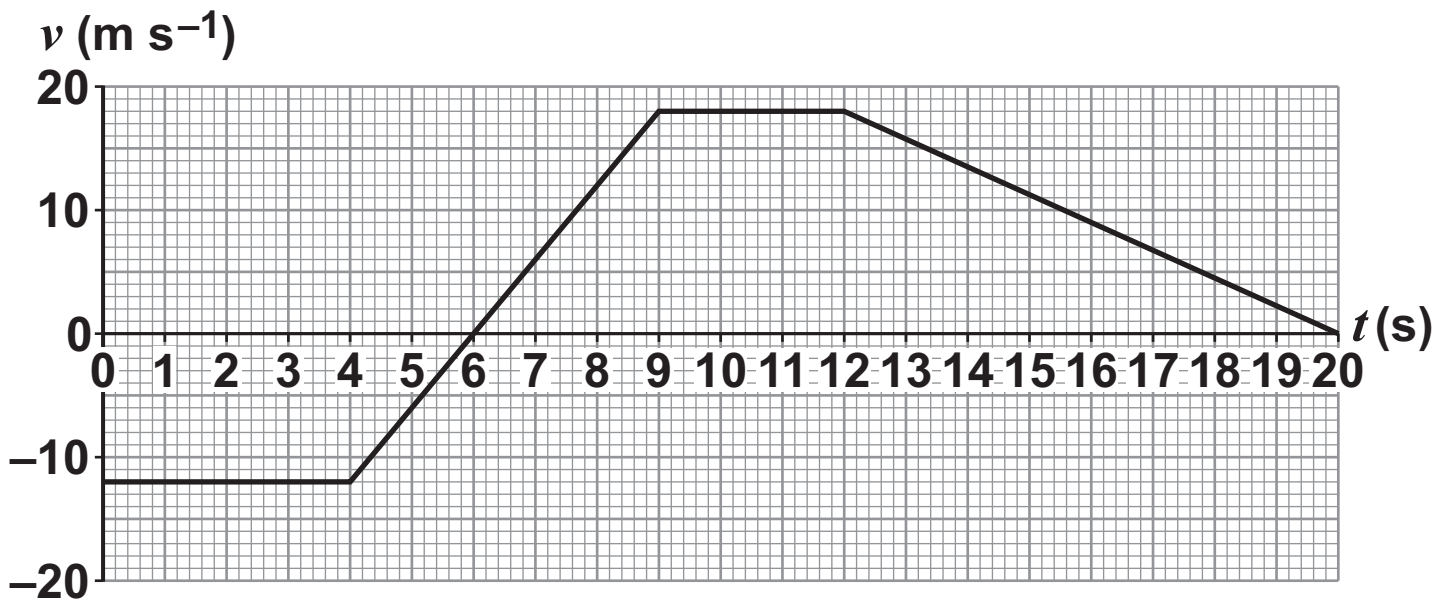
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SECTION B

Answer ALL questions in the spaces provided.

- 10 The diagram below shows a velocity-time graph for a particle moving with velocity $v \text{ m s}^{-1}$ at time t seconds.



Which statement is correct?

Tick (✓) ONE box. [1 mark]

The particle was stationary for
 $9 \leq t \leq 12$

The particle was decelerating for
 $12 \leq t \leq 20$

The particle had a displacement of zero
when $t = 6$

The particle's speed when $t = 4$ was
 -12 m s^{-1}

[Turn over]



11 A wooden crate rests on a rough horizontal surface.

The coefficient of friction between the crate and the surface is 0.6

A forward force acts on the crate, parallel to the surface.

When this force is 600 N, the crate is on the point of moving.

Find the weight of the crate.

Circle your answer. [1 mark]

1000 N

100 kg

360 N

36 kg



12 A particle, under the action of two constant forces, is moving across a perfectly smooth horizontal surface at a constant speed of 10 m s^{-1}

The first force acting on the particle is $(400\mathbf{i} + 180\mathbf{j}) \text{ N}$.

The second force acting on the particle is $(p\mathbf{i} - 180\mathbf{j}) \text{ N}$.

Find the value of p .

Circle your answer. [1 mark]

−400

−390

390

400

[Turn over]



- 13 In a school experiment, a particle, of mass m kilograms, is released from rest at a point h metres above the ground.

At the instant it reaches the ground, the particle has velocity $v \text{ m s}^{-1}$

- 13 (a) Show that

$$v = \sqrt{2gh}$$

[2 marks]

13 (b) A student correctly used $h = 18$ and measured v as 20

The student's teacher claims that the machine measuring the velocity must have been faulty.

Determine if the teacher's claim is correct.

Fully justify your answer. [3 marks]

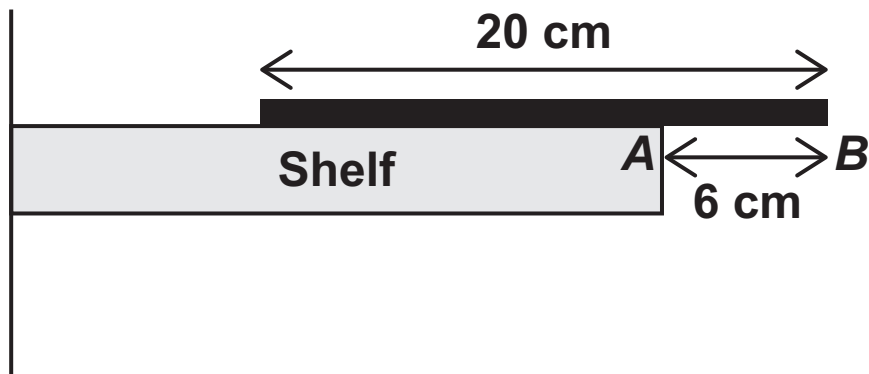
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14

A metal rod, of mass m kilograms and length 20 cm, lies at rest on a horizontal shelf.

The end of the rod, B , extends 6 cm beyond the edge of the shelf, A , as shown in the diagram below.



- 14 (a) The rod is in equilibrium when an object of mass 0.28 kilograms hangs from the midpoint of AB .

Show that $m = 0.21$ [3 marks]

[Turn over]



14 (b) The object of mass 0.28 kilograms is removed.

A number, n , of identical objects, each of mass 0.048 kg, are hung from the rod all at a distance of 1 cm from B .

Find the maximum value of n such that the rod remains horizontal. [4 marks]

14 (c) State one assumption you have made about the rod. [1 mark]

[Turn over]

- 15 Four buoys on the surface of a large, calm lake are located at A , B , C and D with position vectors given by

$$\vec{OA} = \begin{bmatrix} 410 \\ 710 \end{bmatrix}, \quad \vec{OB} = \begin{bmatrix} -210 \\ 530 \end{bmatrix}, \quad \vec{OC} = \begin{bmatrix} -340 \\ -310 \end{bmatrix} \quad \text{and} \quad \vec{OD} = \begin{bmatrix} 590 \\ -40 \end{bmatrix}$$

All values are in metres.

- 15 (a) Prove that the quadrilateral $ABCD$ is a trapezium but NOT a parallelogram. [5 marks]



[Turn over]



15 (b) A speed boat travels directly from *B* to *C* at a constant speed in 50 seconds.

**Find the speed of the boat between *B* and *C*.
[4 marks]**



[Turn over]



- 16 An elite athlete runs in a straight line to complete a 100-metre race.

During the race, the athlete's velocity, $v \text{ m s}^{-1}$, may be modelled by

$$v = 11.71 - 11.68e^{-0.9t} - 0.03e^{0.3t}$$

where t is the time, in seconds, after the starting pistol is fired.

- 16 (a) Find the maximum value of v , giving your answer to one decimal place.

Fully justify your answer. [8 marks]



Ruled lines for writing.

[Turn over]





- 16 (b) Find an expression for the distance run in terms of t . [6 marks]

[Turn over]



16 (c) The athlete's actual time for this race is 9.8 seconds.

Comment on the accuracy of the model.
[2 marks]

[Turn over]

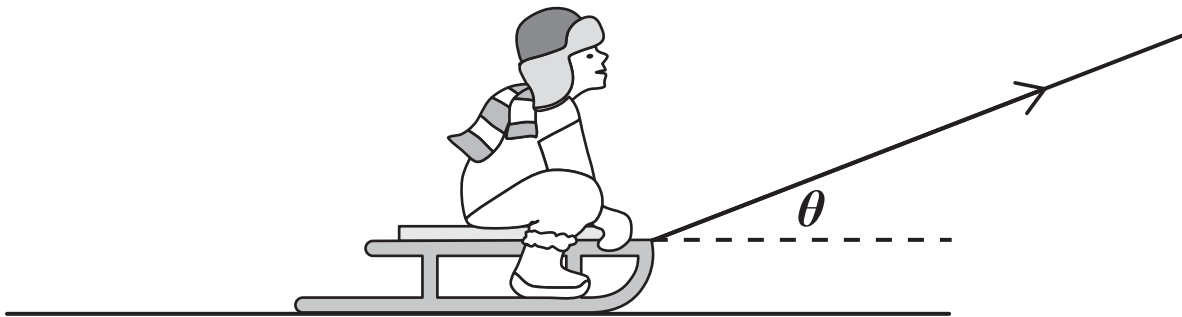
17

Lizzie is sat securely on a wooden sledge.

The combined mass of Lizzie and the sledge is M kilograms.

The sledge is being pulled forward in a straight line along a horizontal surface by means of a light inextensible rope, which is attached to the front of the sledge.

This rope stays inclined at an acute angle θ above the horizontal and remains taut as the sledge moves forward.



The sledge remains in contact with the surface throughout.

The coefficient of friction between the sledge and the surface is μ and there are no other resistance forces.

Lizzie and the sledge move forward with constant acceleration, $a \text{ m s}^{-2}$

The tension in the rope is a constant T Newtons.



17 (a) Show that

$$T = \frac{M(a + \mu g)}{\cos \theta + \mu \sin \theta}$$

[7 marks]

[Turn over]



- 17 (b) It is known that when $M = 30$, $\theta = 30^\circ$, and $T = 40$, the sledge remains at rest.

Lizzie uses these values with the relationship formed in part (a) to find the value for μ

Explain why her value for μ may be incorrect.
[2 marks]

END OF QUESTIONS



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For Examiner's Use	
Question	Mark
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