A-LEVEL
MATHEMATICS

7357/2: Paper 2
Report on the Examination

7357
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Overview

Students showed varying degrees of knowledge across different topics in the specification. However, there was a good balance of knowledge between the areas of pure mathematics and mechanics for the majority of students. The quality of algebraic manipulation varied considerably, with relatively few students able to answer a question concisely and accurately within a few lines. Understanding of mathematical applications was good and almost all students showed the ability to apply appropriate models, particularly in mechanics. Explanation marks proved challenging to most students in both sections.

Question 1

This question proved to be a successful starter with almost three quarters of students choosing the correct answer. The first two graphs proved to be equally successful distractors, with significantly fewer students choosing the third graph.

Question 2

This question proved to be very successful with around 85% of students choosing the correct answer, indicating a good strength of knowledge concerning laws of indices. Other incorrect choices were split reasonably evenly.

Question 3

This question was the most challenging multiple-choice question on the paper with only around 40% of students choosing the correct answer. The most popular choice for a function that was not one-to-one was e^x, chosen by about 45% of students, indicating that knowledge of types of function is poor.

Question 4

This question was meant to be an easy test of the factor theorem but proved more challenging than expected. A significant number of students did not quote or even attempt to use the factor theorem, attempting long division or comparison of coefficients instead. Many of these attempts proved fruitless with students getting lost in complex algebra of their own making.

One mark that many students failed to score was an explanation mark, which required them to clearly state the factor theorem or explain that there would be no remainder. It is worth noting that, when a question states 'Fully justify your answer,' then explanation marks will be awarded and full algebraic explanation is required. In the best solutions, students had clearly defined $\mathrm{f}(x) = x^2 + bx + c$ and $\mathrm{g}(x) = x^2 + dx + e$ and then stated 'As (x + 2) is a factor of both then f (-2) = 0 and g (-2) = 0 '

Less than a quarter of students scored full marks, with a further half of students scoring 3 marks, missing only the explanation mark. Where students scored 1 mark, they had often attempted to use long division or comparison of coefficients but failed to complete the algebraic steps required. When the factor theorem method was attempted, slips often occurred with signs. Approximately 22% of students failed to score anything on this test of basic knowledge.

Question 5

This question proved more challenging than expected and it was surprising to see that a significant number of students did not realise how to separate the variables correctly or identify that the method required after separation was integration by parts (even if substitution was used first). This resulted in a quarter of students failing to score a mark, mainly because there was still a mix of variables within each integrand. The most common error when separating variables was to obtain

$\int \frac{1}{x^2}\ln x\,\mathrm{d}x = \int \frac{1}{t}\,\mathrm{d}t$ and in this case an A1 follow through mark was allowed for noting that $\int \frac{1}{t}\,\mathrm{d}t = \ln t$.

Those students who realised that integration by parts should be used did attempt to obtain u' and v and to substitute their expressions into the correct formula. Sign errors occurred infrequently. A common error was to identify $u = x^{-2}$ and $v' = \ln x$ making a further error by incorrectly deducing $v = x^{-1}$ and $u' = -2x^{-3}$

A small number of students who scored the first 5 marks omitted the constant of integration and therefore could not obtain the final two marks.

A relatively small number of students did not gain the final mark because they rearranged incorrectly, even when the correct value of c was found. The most common answers seen were:

$t^2 = 3 - \frac{2}{x} - \frac{2\ln x}{x}$	multiplying by 2 but not doubling the constant
$t^2 = 6 - \frac{2}{x} + \frac{2\ln x}{x}$	making a sign error when removing brackets
$t^2 = 6 - \frac{2}{x} - \frac{\ln x}{x}$	multiplying by 2 but not doubling the second x term
$t^2 = 3 - \frac{1}{2x} - \frac{\ln x}{2x}$	multiplying by 2 but halving each term

Question 6

This question proved very challenging for students but only because they did not identify it as a question in which harmonic form should be used. Only around 12% of students completed this question fully correctly, whilst almost 50% of students scored no more than 2 marks. Students need to recognise that any expression of the form $a\sin x + b\cos x$ should suggest use of $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$. Furthermore, reference to maximum values does not always require use of calculus.

Those students who had identified the expected method mentioned above often scored 3 or 4 marks very quickly, even if an error was then made.

Those students who had chosen to use calculus often stopped after forming a single equation. If a second equation was obtained by using the coordinates of the given point, then it was rare to see any further progress. Some invalid equations were formed, the most common being:

$4 = a\frac{\sqrt{3}}{2} + \frac{b}{2}$ using 4 instead of $2\sqrt{3}$

$2\sqrt{3} = a\sqrt{3} + b$ multiplying the correct equation by 2 but not doubling $2\sqrt{3}$

$a^2 + b^2 = 16 \quad \Rightarrow \quad a + b = 4$ incorrectly taking square roots separately

Students who had made the last error could not obtain any further marks for reducing to a single variable equation, as the question had changed to a much simpler one. Many students who were pursuing the calculus method often produced pages of incorrect algebra.

In addition, some students made assumptions about the x value of the maximum point occurring at $\frac{\pi}{6}$ and scored no further marks since this assumption could not be justified.

Note that the values $a = 4$ and $b = 0$ were not accepted as a valid answer, although this did not prevent students from gaining marks if these values were stated alongside the correct non-zero values.

Question 7

This question differentiated well and was structured to enable all students to have some success. For part 7(a), almost 90% scored both marks, and it is pleasing to see that sketches have improved significantly since last year. A few students incorrectly sketched a graph with a negative coefficient of x^3 , whilst some only had a curve with one root.

In part 7(b)(i), about 90% of students scored at least one mark by differentiating at least one term in the given expression correctly. For the second mark, students needed to solve their quadratic to obtain one of the correct roots, either $x = 0$ or $x = -2p$, but sometimes incorrect roots appeared from a correct equation: it was common to see $x = 2p, 6p$ or $-6p$.

Only around 15% of students scored the final mark which required both correct roots to be found and a comment made about $x = 0$ being on the y-axis, but it was often having an incorrect root that stopped students from gaining this mark. A small number of students verified the result by substituting $x = 0$ into $\mathrm{f}'(x) = 3x^2 + 6px$, but could then only score a maximum of two marks.

Part 7(b)(ii) proved very challenging, with over a half of students scoring 0 and only around 10% scoring 3 or more marks. There was 1 ‘easy’ mark here for evaluating f(x) at their non-zero value of x from the previous part. Students who sketched the curve, showing the relative positions of the turning points, were more successful than those who did not attempt any sketch.

Question 8

This question was answered well by all students, showing confidence with modelling questions. Unsurprisingly, latter parts differentiated more than earlier parts.

The proof in part 8(a) was completed fully correctly by about 90% of students, showing good understanding of logarithms.

For part 8(b), more than a half of students scored full marks with approximately 80% obtaining 2 or more marks. It was crucial here that students followed the instruction given in the question and used points on the line of best fit, not points that were only in the table. This often resulted in students failing to gain two marks if a point was used that was not on the given line. It was expected that a calculator would be used to solve the equations.

For part 8(c) around 82% of students could form an appropriate equation, although some students thought that half a million pounds was £250,000 or £100,000, or even £50,000.

Solving their equation and interpreting the result appropriately proved more challenging, with around 43% obtaining full marks. It was expected that students would use a calculator to solve such an equation. The last mark was often dropped for rounding the decimal value of t up rather than using the integer part of it. The answer 2024 was commonly seen.

Part 8(d) proved the most challenging part, with only a quarter of students scoring both marks. These marks were independent and students needed to comment on the fact that 2023 was out of the range of data given along with a contextual reason relating to changes in house prices. Some students showed good insight here by referring to Brexit, home improvements or market crashes.

Question 9

Part 9(a) was done well, with around 82% of students scoring the first mark and almost 70% of students completing the question fully correctly. The most common errors that occurred were in rewriting the expression appropriately to use the binomial theorem. The following errors were seen on several occasions:

$$4\left(1-\frac{x^2}{2}\right)^{\frac{1}{2}}$$

$$2\left(1-\frac{x^2}{4}\right)^{\frac{1}{2}}$$

$$4\left(1-2x^2\right)^{\frac{1}{2}}$$

Part 9(b) was poorly done, with many students not realising or using the correct formula for the range of validity of this expansion, which is given in the formulae booklet. Some that did correctly write $\left|-\frac{x^2}{2}\right|<1$ or $\left|\frac{x^2}{2}\right|<1$ then simplified it incorrectly to obtain either $|x|<2$ or $|x|<\frac{1}{2}$.Many students attempted to argue by stating that $4-2x^2>0$ and rearranging, which relates to the function and not the expansion, therefore scoring 0. Other poor rearrangements involved statements such as $x^2<-2$.

Part 9(c) was completed fully correctly by only around 2% of students, with fewer than 30% of students scoring a method mark for integrating one term in their expression correctly. 65% of students scored 0 for this part of the question. The key factor here was to realise that small angle approximations would be required, and these expansions are given in the formula booklet. Once again 1 mark related to explaining why a small angle approximation could be used and required students to state that 0.4 radians is small. For those that did realise that small angle approximations could be used, it was often the case that they integrated $2-\frac{x^2}{2}$ rather than $\frac{1}{2}\left(2-\frac{x^2}{2}\right)$. A handful of students attempted to use the trapezium rule scoring 0 marks.

Part 9(d) was very challenging, with only about 6% of students scoring a mark. The most common incorrect explanation here related to 1.4 being close to $\sqrt{2}$. For the mark to be awarded, students had to state that 1.4 radians was not a small angle and state that the small angle approximation could not then be used.

Question 10

This question proved to be the most successful multiple-choice question in the mechanics section, with around 80% of students choosing the correct answer. Only very small proportions of students chose each of the incorrect options, with the most common incorrect answer being 'The particle's speed when t = 4 was -12 ms^{-1}', showing students' confusion between speed and velocity.

Question 11

This question proved to be almost as successful as Q10 with three quarters of students choosing the correct answer. Only relatively small proportions of students chose incorrect options, with the most common incorrect answer being '100 kg', showing students' confusion between weight and mass.

Question 12

Surprisingly, this question proved to be the least successful mechanics multiple-choice question, with just over a half of students choosing the correct answer. Almost 30% of students chose the incorrect option of '-390', suggesting that they had been tempted to add the velocity of 10 ms^{-1} to the **i** component of the force. It is worth noting that it is not always necessary to use all the numbers given.

Question 13

This question was done really well, with around 90% of students scoring both marks in part (a) and at least 2 marks in part (b). When students failed to gain a mark in part 13(a), it was usually because they did not show clear substitutions or did not clearly state the values for each variable. Almost all students used a correctly stated constant acceleration equation.

In part 13(b) a variety of approaches was seen, which involved substituting two of the variables to find the third one and then comparing with the recorded or known value. Finding v was most common, closely followed by those who chose to find a value for g (11.1) and then compare with 9.8, 9.81 or even 10. For those students who chose to argue that the value of 18.8 would round to 20 and that the teacher was incorrect, they could only obtain the third mark if the value of g had been used as 10, correct to one significant figure, matching the accuracy they were referring to.

Question 14

In this question, students had to choose and apply an appropriate model to satisfy the demands of the relevant assessment objective. It was expected that students would assume that the object was on the point of slipping and take moments about *A*. Some students took moments about other points and then formed a second expression for the normal reaction at *A*. Many did produce fully correct answers.

Common errors that occurred in both parts 14(a) and 14(b) were:

- omitting g when taking moments – this scored 0 in part 14(a) but was condoned in part 14(b) giving students the benefit of the doubt that they had cancelled the g. It is important that students realise that any moment of a force requires a force and not a mass
- mixing the units used – cm and m when forming equations
- omitting a term when taking moments about *B* or *C*
- using incorrect distances by making simple errors involving subtraction.

In part 14(b), many students who solved the moments equation to obtain 3.5 then incorrectly rounded up to get 4. A handful of students considered the moments involved when 1, 2, 3, 4…. items were placed and correctly deduced the answer was 3 – a nice, simple approach.

Both the above parts worked quite well, with around half of all students scoring full marks in 14(a) and similarly in 14(b), although around 12% made no attempt for 14(b).

Part 14(c) was done extremely well, with more than three quarters of students stating a valid assumption.

Question 15

This question differentiated well with students more successful at part 15(b) than part 15(a).

In part 15(a). many different approaches were seen here involving vectors, coordinate geometry or ratios. It was disappointing to find that vectors were least used, especially when those students who did use vectors were the most successful. Many different errors were seen; dependent on the method chosen, these included:

- incorrect labelling of vectors with letters reversed
- inverted gradients referred to as ‘gradient of the line’
- numerical errors particularly when subtracting
- using only lengths of the sides and not considering whether sides were parallel or not
- failing to show that sides *BC* and *DA* were not parallel.

Any numerical error meant that the most marks awarded would be 3 out of 5 and any labelling error would be awarded at most 2 out of 5.

In part 15(b), over three times as many students scored full marks when compared with part 15(a). Most students found the length of *BC* first before using the time period. Only a very small minority of students failed to state the correct units.

Question 16

This question differentiated well in both parts 16(a) and 16(b).

In part 16(a), it was very pleasing to see virtually no student attempt to use constant acceleration equations. It was also very pleasing to see most students score the E1 mark for explaining that the maximum velocity occurred when $\frac{dv}{dt} = 0$.

Most students knew how to differentiate e^{kt} correctly. Having set up the correct equation, there were many invalid attempts to solve the equation for t; however, it was expected that students would have used their calculators to solve such an equation. This clearly should be encouraged. However, where a student had made invalid attempts to obtain the derivative but then obtained the correct answer out of the blue using a calculator, this was treated as two different attempts and marked accordingly if nothing was crossed out. An answer from a calculator here, with no attempt to obtain the derivative, giving the maximum velocity as 11.5 ms^{-1} would have scored a maximum of 5/8.

The final R1 mark could have been awarded for observing that this had to be the maximum value given the context and the only value where $\frac{dv}{dt} = 0$. All other valid methods were credited, but had to be fully correct.

A handful of students attempted to use integration instead of differentiation.

Part 16(b) was more successful, with nearly a third of students obtaining full marks. Students showed that they were more confident with integration than differentiation. When errors occurred, they often related to sign errors or incorrect evaluation of coefficients. A significant number of students assumed $c = 0$ rather than substitute to find it was non-zero. A few students failed to consider c at all and scored only 2 marks.

Pert 16(c) proved challenging, with only a third of students scoring 1 mark and a fifth of students scoring both marks. To score both marks here, part 16(b) had to be correct. However, some students failed to gain the M1 mark for not clearly showing the substitution required.

Question 17

In this question, attempts at part 17(a) were very pleasing, with students able to form appropriate equations and manipulate them. Errors tended to occur with signs and mixing sine/cosine components. Half of the students scored 6 out of 7, failing to gain the last mark for not clearly showing the line:

$$T(\cos\theta + \mu\sin\theta) = Ma + \mu Mg.$$

Part 17(b) proved much more challenging, with many students substituting the given values, which was not required. Some even argued that μ had to be less than 1. This was a subtle test of knowledge about friction and required two points to be commented on:

- the sledge was at rest
- the relationship used in part 17(a) might not be valid as limiting friction had not been reached.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.