A-LEVEL
MATHEMATICS

7357/3: Paper 3
Report on the Examination

7357
June 2019

Version: 1.0

Further copies of this Report are available from aqa.org.uk

General

This was the second examination for this specification and there was a significant increase in the number of entries in this series. It was pleasing to see that, although some students did not attempt parts of Question 17, very many students attempted all other questions on the paper. Students made good use of their calculators especially in the statistics section.

However, questions that require a qualitative response and evaluation were less well answered and this is clearly an area for focus. Students continue to make minor arithmetic errors involving negative and positive signs. Premature rounding was also common and as a result many failed to gain accuracy marks. It was evident that many students lacked rigour in their solutions or did not see a proof through to the end, thus not gaining all the marks.

Topics that were done well included:

- calculating the area of a sector and a triangle
- modelling using exponential functions
- partial fractions
- laws of logarithms
- implicit differentiation
- identifying correlation
- naming type of sampling
- identifying outliers
- binomial distribution
- probabilities in a two-way table.

Topics which students found challenging included:

- sketching graphs
- proof (including by contradiction)
- completing the square
- understanding sampling methods
- integration
- using the first derivative to solve problems
- understanding and carrying out hypothesis tests
- analysing the Large Data Set.

Question 1

Students generally scored highly on this question. There were few incorrect answers with most students able to deal confidently with domain of an inverse function. The two common incorrect responses were those involving π.

Question 2

This was not a familiar question to students but a good proportion of them still answered it correctly. The most frequent incorrect answer was evenly split between 50/147 and 161 700. It was evident that many students tried to find the answer using a calculator but were not able to.

Question 3

This was the least well answered of all the multiple-choice questions on the paper with just a third of students getting the correct answer. There was often evidence of a great deal of working and much crossing out. Successful answers were often accompanied by the first few terms of one or more sequences, which give the required information to select the correct response.

Question 4

Students usually identified that this is a quadratic graph with a maximum point and scored the first mark for drawing the correct curve. They went on to label the points of intersection with the axes correctly, although some missed the y-intercept or the positive root. It was common to see the vertex wrongly placed on the y-axis or in the first quadrant.

Very few students correctly drew the line $y - x = 3$ and shaded the correct region. At times the line drawn was not straight and some used dotted lines and shaded the wrong region. Some students failed to gain a mark because they did not make it clear that they were shading all of their corresponding region.

Question 5

Students performed well in this question and were able to complete the square to find the radius of the circle. A few students could not do this and used $\sqrt{264}$ as the radius and gained other available marks. Common errors involved the use of negative sign or simple arithmetic slips.

It was pleasing to see that most students could recall the formulae to find the area of a sector and of a triangle. Unfortunately, some students attempted to convert radians to degrees and prematurely rounded their values and the final accuracy mark was not gained or made an error by substituting in degrees into $\frac{1}{2}r^2\theta$. Several students tried to find the perpendicular height of triangle *OAB*, often by assuming the base *OB* was half the radius. However, there were a variety of correct approaches seen.

Question 6

Most students attempted all parts in this question. There were very few incorrect answers in part **(a)** with most students able to deal confidently with Pythagorean triple. The most common correct response was the short sides labelled 6 or 8 and the hypotenuse labelled 10. A few students did not give integer values or misinterpreted what $\mathbb{Z}$ represented. The common misconception was to state that the shorter sides were 2 and 4 followed by stating that the hypotenuse was equal to $\sqrt{20}$.

Students were caught out by the proof required in part **(b)**. Many realised a proof by contradiction was required and scored the first mark by assuming all three sides are odd but then defined their terms a and b with respect to a single variable, for example $(2n + 1)$ and, $(2n + 3)$, before considering Pythagoras' theorem. Those students who used two different variables went on to score at least two marks. The final R mark was awarded to few students and needed a statement such as 'so c^2 is even, so c is even' followed by a correct conclusion. It was disappointing to see some students could not find a correct expression for an odd number, with $a = n + 1$ and $b = n + 2$ commonly seen.

Question 7

Part **(a)** was poorly attempted. Many students scored no marks because they formed the wrong identity, with $4x + 3 = A(x - 1)^2 + B(x - 1)$ commonly seen. Those who obtained the correct identity often went on to get correct values for A and B.

Students showed some understanding of the integration process, but could not usually find the complete correct integral in part **(b)**. Many could correctly integrate their expression $\frac{A}{x-1}$ to form $A\ln(x - 1)$ but struggled when it came to integrating $\frac{B}{(x-1)^2}$ often obtaining $B\ln(x - 1)^2$. Fortunately, many correctly substituted the integral limits and used at least one law of logarithms correctly. It was common to see students use integration by substitution with little success. Incorrect use of laws of logarithms such as $ln3 - ln2 = \frac{ln3}{ln2}$ were seen frequently. The final mark was not often awarded, as the answer was not given in the required form.

Question 8

A high proportion of students made good progress in this question involving exponential modelling.

In part **(a)**, most students began by setting out the equation for the initial boundary condition and gaining the first two marks. Those who only scored the initial mark for setting up the equation failed to gain the mark when trying to evaluate λ because they incorrectly expanded the bracket in the expression $5(4 + \lambda e^0)$ to form either the expression $(20 + \lambda e^0)$ or $20 + \lambda e$. Most students did go on to gain credit by calculating their k value correctly and substituting this correctly when working out the final value for θ. It would be advisable for students to always show full working so that if they make errors in further working they are able to gain the other available marks.

Many students stated the correct room temperature in part **(b)(i)** but could not give a reason for their answer due to lack of understanding on the impact upon the model as t increases. Several students substituted for $t = \infty$ which gained no credit.

In part **(b)(ii)**, most students scored the first mark for a correct equation using their value of k.

Most students scored well in part **(c)** with the common explanation that the room temperature will be different. Those that were unsuccessful did not refer back to how the model is affected by the context, preferring instead to comment on the context in general.

Question 9

This question discriminated well between students.

In part **(a)**, very few students gained full marks because most found it difficult to write mathematical proofs. They substituted for either $x = 0$ or $y = 0$ without clearly evaluating their expressions to show that they equalled 0. Those who did correctly calculate the value of 0 went on to make a reference to 12 without explicitly stating that there was a contradiction between the LHS and RHS.

In part **(b)(i)**, a high proportion of students realised that implicit differentiation should be used and the question was answered well by most students, although the required step of factorising y and then cancelling was not often seen. Common mistakes were either to omit implicit differentiation entirely, or not making the RHS equal to zero after differentiation.

It was expected in **(b)(ii)** that students would use the given $\frac{dy}{dx}$ rather than use their incorrect version. Most students did use the correct version and set it equal to zero but made a mistake with the negative sign at the start of the fraction. Relatively few responses gained full marks, with many solutions gaining the first two marks before concluding with an argument, and not substituting back into the equation of the curve to consider the other coordinate. Many students got as far as $y^2 = -2x$, although quite a few ended up incorrectly with $y^2 = 2x$ but most ended there by saying this wasn't possible rather than trying to substitute this back into the original equation. Those that chose to substitute $y^2 = -2x$ in the equation leading to $x = \sqrt[3]{6}$ did not usually substitute this in to find the y value.

Part **(b)(iii)** was done better than other parts in the question with about a third of students getting full marks. Many students recognised that they needed to substitute $y = 1$ into the equation of the curve to find the correct quadratic and went on to factorise to solve the equation correctly. Some were able to choose the correct value to meet the condition that $x > 0$. Some were able to correctly substitute both $y = 1$ and $x = 3$ into their $\frac{dy}{dx}$ to find the gradient. A common mistake was again to misunderstand the negative sign when calculating the gradient of the tangent. Another mistake was to think that after correctly calculating the value of $\frac{dy}{dx}$, it was necessary to take the negative reciprocal as the gradient of the tangent.

Question 10

This question was well answered with approximately two thirds of students selecting the correct response. The most frequently chosen incorrect response was moderate negative.

Question 11

This was the most well answered of all the multiple-choice questions on the paper. However, a significant minority of students were confused in understanding the different types of sampling with stratified and systematic the common incorrect responses.

Question 12

Part **(a)** was well answered by most students. Some tried to work out the mean and standard deviation using formulas rather than the calculator; this is time consuming and often students obtained the wrong answers. Some students were careless when doing further work where they calculated x – one standard deviation, rather than performing the correct calculation or substituted the truncated value of 160 for their calculated mean. Most students who scored the first 3 marks went on to gain the final mark by making the correct comparison and stating the correct conclusion. Unfortunately, there was evidence that many did not read the question carefully and used a different definition of an outlier based on interquartile range.

Many students knew in part **(b)** the effect removing a lower outlier will have on the mean and standard deviation, but could not provide a valid reason. Most students did not state that by removing the lowest value from the distribution that the sample mean would increase. Various statements which did not compare the lowest value to the sample mean were provided. It was more common to score the second mark for referring to the removal of the outlier decreasing the value of the standard deviation. Students who calculated the new mean and standard deviation values upon removal of the outlier generally went on to score full marks.

Question 13

This question saw most students making excellent progress, with a full range of marks being awarded in all parts. Students could clearly use their calculators effectively in solving problems involving the binomial distribution.

Part **(a)(i)** was answered correctly by a very high proportion of students.

In part **(a)(ii)**, although most scored the mark, some calculated the standard deviation and stated this as their final answer. A few students missed out as they did not know how to use the formula for variance of binomial distribution shown in the formula booklet.

Part **(b)(i)** was completed using the calculator with very few students showing any working. Students who used the formula were less likely to be successful. Some missed off $\binom{30}{10}$ completely or used it as a fraction $\left(\frac{30}{10}\right)$. There were a small number of students who modelled the problem as a normal distribution and applied a continuity correction without stating why they used this model. (Note that the continuity correction is not required in this specification.)

Students did well in part **(b)(ii)**. Those who struggled appeared to be confused about how to evaluate $\mathrm{P}(X \geq 5)$ with many of them performing the calculation $1 - \mathrm{P}(X \leq 5)$

The most common mistake in part **(c)(i)** was to multiply the probability found in part **(b)(ii)** by 5 instead of raising to the power of 5. Otherwise, many scored full marks.

Part **(c)(ii)** was also well answered, although many students failed to gain the mark for not clearly stating what will happen to the **probability**.

Question 14

Most students scored the mark in part **(a)(i)**. The common error was $\frac{10}{65+28+21} = \frac{10}{114}$ by adding up totals in the table rather than using given information from the question. The wrong denominator was also used in part **(a)(ii)**. It was disappointing to see some students make an arithmetic error when adding the numbers 9, 2 and 1.

In part **(a)(iii)**, the common incorrect final answer was $\frac{38}{65}$ in which students gained the first mark only. It was pleasing to see that most students recognised the need to use the conditional probability formula.

Part **(b)** was poorly answered. Many students gave a descriptive answer without any quantitative analysis. Some students found the correct value of 14 + 38 = 52, but compared with the wrong value usually 65. Some added the probabilities correctly but then didn't compare the total to 1. A significant minority of students tried to show the events were independent instead of mutually exclusive.

Question 15

This question was not well answered. Many students started off by formally stating hypotheses, which was not required. Most students correctly identified the critical value from the table and correctly compared the value with 0.567. Very few students scored the final mark, often because they did not provide the required inference statement with reference to the two variables in context. It was common to see some students made a comparison with each of the possible values from the table without making a choice as to which was relevant. Students should be encouraged to use the correct language such as sufficient or significant evidence. Students who chose to refer to type of correlation often did not include the word positive.

Question 16

It was disappointing to see that many students are not familiar with the Large Data Set and so could not score well in part **(a)**. Very few students referred to the possible difference in the amount purchased compared to the amount consumed. Very few students recognised that the y-axis had no scale or did not start from 0. Many students gave random responses based on facts they knew from the Large Data Set which had no specific relevance to the context of this question. Note that the new Large Data Set about cars will be used for this paper in examinations from 2020.

Part **(b)** saw the majority of students score high marks. Most students were able to set up the correct hypotheses, though there were several who incorrectly set up their hypotheses with letters other than μ or with 80.4 rather than 78.9. The most successful students found the test statistic and compared with the critical value or compared the probability with 0.025. A small number of students chose to find the acceptance region but then forgot to compare with both ends of the region. Some students did not use a 2-tailed test so set up their hypotheses incorrectly or missed them off entirely. Most students did correctly refer to H_0 rather than H_1 for their conclusion. Students who did not gain the final mark did so because they either did not provide an inference statement or did not refer to the mean amount of sugar purchased.

In part **(c)**, an extremely small percentage of students gained both marks. Most students did not ‘appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis’, which appears in section O2 of the specification. Some students did not refer to the 10% specifically, instead using language such as ‘higher chance of’. Most students gave answers referring to the acceptance region being too large and that 5% would be better or referred to rounding errors.

Question 17

Most students were able to gain some marks in part **(a)** and nearly half of all students gained full marks. A common error was the omission of the negative sign in $z = -0.8416$. Some students used the probabilities 0.1 and 0.8 which were given in the question as their z values. Many formed simultaneous equations but wasted time solving them algebraically instead of using their calculators.

Many students did not attempt parts **(b)(i)**, **(b)(ii)** and **(c)**. Several students calculated P $(34.5 < X < 35.5)$, stated the probability was equal to 0 or tried to subtract a value away from 1 in answering part **(b)(i)**.

Students were able to gain the full marks for part **(b)(ii)** for correctly calculating $P(X < 35)$ using their μ and σ found in part **(a)**. A common mistake was to assume that $P(X < 35) = P(X \leq 34)$.

Most students recognised that they had to work with the binomial distribution in part **(c)** with many stating the distribution as part of their response. Many answers from follow through gave extremely small probabilities such as 5.8×10^{-16} and, on some occasions, probabilities given were greater than 1, which could not gain credit for the final mark.

Use of statistics

Statistics used in this report may be taken from incomplete processing data. However, this data still gives a true account on how students have performed for each question.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.