AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

AS
FURTHER MATHEMATICS

Paper 1

Monday 13 May 2019 Afternoon Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

J U N 1 9 7 3 6 6 1 0 1

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Answer **all** questions in the spaces provided.

1 Which of the following matrices is an identity matrix?

Circle your answer.

[1 mark]

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2 Which of the following expressions is the determinant of the matrix $\begin{bmatrix} a & 2 \\ b & 5 \end{bmatrix}$?

Circle your answer.

[1 mark]

$5a - 2b$ $2a - 5b$ $5b - 2a$ $2b - 5a$

3 Point P has polar coordinates $\left(2, \frac{2\pi}{3}\right)$.

Which of the following are the Cartesian coordinates of P?

Circle your answer.

[1 mark]

$(1, -\sqrt{3})$ $(-\sqrt{3}, 1)$ $(\sqrt{3}, -1)$ $(-1, \sqrt{3})$

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4 The line L has polar equation

$$r = \frac{k}{\sin\theta}$$

where k is a positive constant.

4 (a) Sketch L.

[1 mark]

4 (b) State the minimum distance between L and the point O.

[1 mark]

Turn over for the next question

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5 A hyperbola H has the equation

$$\frac{x^2}{a^2} - \frac{y^2}{4a^2} = 1$$

where a is a positive constant.

5 (a) Write down the equations of the asymptotes of H.

[1 mark]

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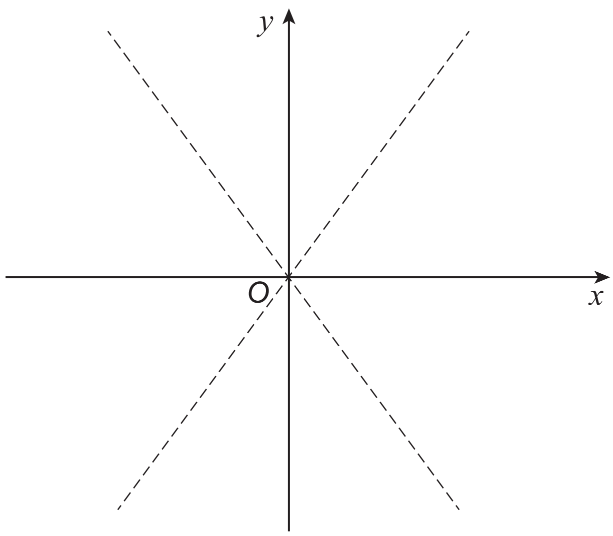
5 (b) Sketch the hyperbola H on the axes below, indicating the coordinates of any points of intersection with the coordinate axes.

The asymptotes have already been drawn.

[2 marks]



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5 (c) The finite region bounded by *H*, the positive x-axis, the positive y-axis and the line $y = a$ is rotated through 360° about the y-axis.

Show that the volume of the solid generated is ma^3, where $m = 3.40$ correct to three significant figures.

[5 marks]

Turn over for the next question

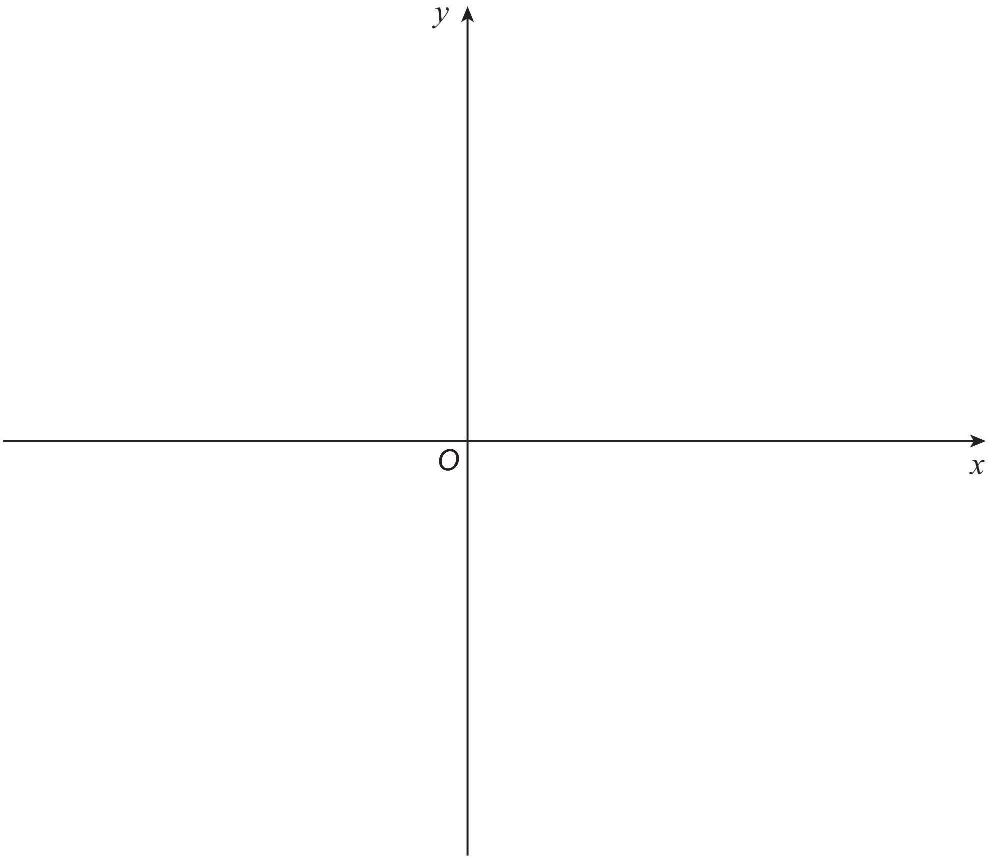
Do not write outside the box

6 (a) On the axes provided, sketch the graph of

$$x = \cosh(y + b)$$

where b is a positive constant.

[4 marks]



6 (b) Determine the minimum distance between the graph of $x = \cosh(y + b)$ and the y-axis.

[1 mark]

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Turn over for the next question

Do not write outside the box

7 (a) Show that

$$\frac{1}{r-1}-\frac{1}{r+1}\equiv\frac{A}{r^2-1}$$

where A is a constant to be found.

[1 mark]

7 (b) Hence use the method of differences to show that

$$\sum_{r=2}^{n}\frac{1}{r^2-1}\equiv\frac{an^2+bn+c}{4n(n+1)}$$

where a, b and c are integers to be found.

[4 marks]

Turn over for the next question

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8 Given that $z_1 = 2\left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)$ and $z_2 = 2\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$

8 (a) Find the value of $|z_1 z_2|$

[1 mark]

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8 (b) Find the value of $\arg\left(\frac{z_1}{z_2}\right)$

[1 mark]

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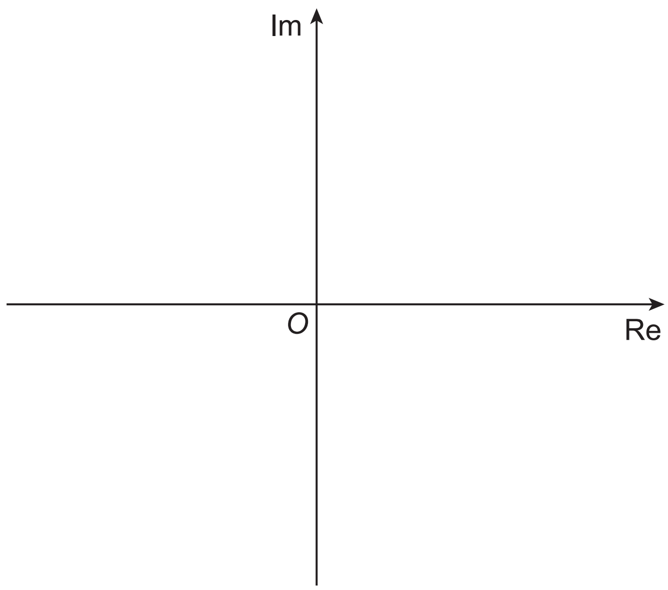
8 (c) Sketch z_1 and z_2 on the Argand diagram below, labelling the points as *P* and *Q* respectively.

[2 marks]



8 (d) A third complex number w satisfies both $|w| = 2$ and $-\pi < \arg w < 0$

Given that w is represented on the Argand diagram as the point *R*, find the angle $P\widehat{R}Q$.

Fully justify your answer.

[3 marks]

Turn over for the next question

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9 (a) Saul is solving the equation

$$2\cosh x + \sinh^2 x = 1$$

He writes his steps as follows:

$$2\cosh x + \sinh^2 x = 1$$

$$2\cosh x + 1 - \cosh^2 x = 1$$

$$2\cosh x - \cosh^2 x = 0$$

$$\cosh x \neq 0 \ \therefore\ 2 - \cosh x = 0$$

$$\cosh x = 2$$

$$x = \pm \cosh^{-1}(2)$$

Identify and explain the error in Saul's method.

[2 marks]

9 (b) Anna is solving the different equation

$$\sinh^2(2x) - 2\cosh(2x) = 1$$

and finds the correct answers in the form $x = \dfrac{1}{p}\cosh^{-1}(q + \sqrt{r})$, where p, q and r are integers.

Find the possible values of p, q and r.

Fully justify your answer.

[5 marks]

Turn over for the next question

10 (a) Using the definition of $\cosh x$ and the Maclaurin series expansion of e^x, find the first three non-zero terms in the Maclaurin series expansion of $\cosh x$.

[3 marks]

10 (b) Hence find a trigonometric function for which the first three terms of its Maclaurin series are the same as the first three terms of the Maclaurin series for $\cosh(ix)$.

[3 marks]

Turn over for the next question

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11 (a) Curve C has equation

$$y = \frac{x^2 + px - q}{x^2 - r}$$

where p, q and r are positive constants.

Write down the equations of its asymptotes.

[2 marks]

11 (b) Find the set of possible y-coordinates for the graph of

$$y = \frac{x^2 + x - 6}{x^2 - 1}, \qquad x \neq \pm 1$$

giving your answer in exact form.

No credit will be given for solutions based on differentiation.

[6 marks]

Turn over for the next question

12 The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

12 (a) Prove by induction that, for all integers $n \geq 1$,

$$\mathbf{A}^n = \begin{bmatrix} 1 & 3^n - 1 \\ 0 & 3^n \end{bmatrix}$$

[4 marks]

12 (b) Find all invariant lines under the transformation matrix **A**.

Fully justify your answer.

[6 marks]

12 (c) Find a line of invariant points under the transformation matrix **A**.

[2 marks]

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13 Line l_1 has Cartesian equation

$$x - 3 = \frac{2y + 2}{3} = 2 - z$$

13 (a) Write the equation of line l_1 in the form

$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$$

where λ is a parameter and **a** and **b** are vectors to be found.

[2 marks]

13 (b) Line l_2 passes through the points $P(3, 2, 0)$ and $Q(n, 5, n)$, where n is a constant.

13 (b) (i) Show that the lines l_1 and l_2 are **not** perpendicular.

[3 marks]

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13 (b) (ii) Explain briefly why lines l_1 and l_2 cannot be parallel.

[2 marks]

13 (b) (iii) Given that θ is the acute angle between lines l_1 and l_2, show that

$$\cos\theta = \frac{p}{\sqrt{34n^2 + qn + 306}}$$

where p and q are constants to be found.

[3 marks]

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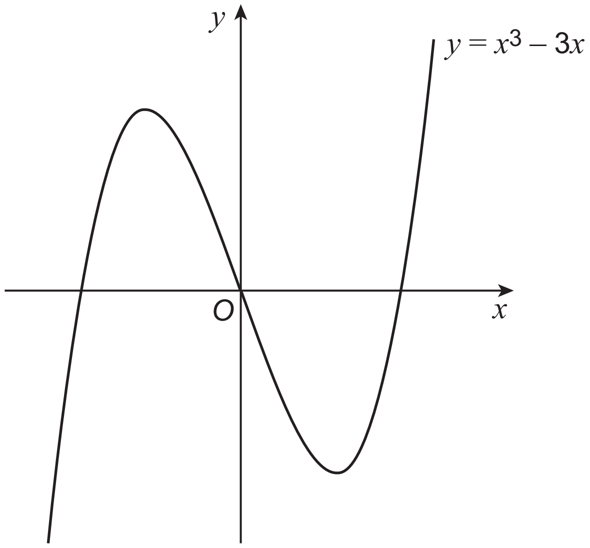
14 The graph of $y = x^3 - 3x$ is shown below.



The two stationary points have x-coordinates of -1 and 1

The cubic equation

$$x^3 - 3x + p = 0$$

where p is a real constant, has the roots α, β and γ.

The roots α and β are **not** real.

14 (a) Explain why $\alpha + \beta = -\gamma$

[1 mark]

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14 (b) Find the set of possible values for the real constant p.

[2 marks]

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14 (c) $f(x) = 0$ is a cubic equation with roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$

14 (c) (i) Show that the constant term of $f(x)$ is $p + 2$

[3 marks]

14 (c) (ii) Write down the x-coordinates of the stationary points of $y = f(x)$

[1 mark]

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