AQA

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FURTHER MATHEMATICS

7366/1: Paper 1

Report on the Examination

7366

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General Comments

This question paper was accessible to all students, whilst providing challenges through higher demand questions. There was no evidence of students running out of time. Poor presentation hampered a number of students, for example when illustrating the behaviour of a graph near an asymptote. Indices written in line with the base sometimes caused problems, particularly in question 12, with some students misreading their own writing. In question 11, an answer of $\frac{7\pm2\sqrt{6}}{2}$ was sometimes written down as $7\pm\frac{2\sqrt{6}}{2}$.

Question 1

This question was answered very well, with almost all students selecting the correct answer of $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$. A small minority of students selected the third option with 1s on the non-leading diagonal.

Question 2

The majority of students selected the correct determinant. The incorrect responses were distributed evenly amongst the other three options.

Question 3

The correct answer of $\left(-1,\sqrt{3}\right)$ was correctly chosen by most students, usually accompanied by a small sketch. Once again, the minority of incorrect responses were distributed evenly amongst the other three options.

Question 4

Most students coped well with this question. Correct answers were usually accompanied by a rearrangement of the given equation to $r\sin\theta = k$, often followed by $y = k$. Many students added a vertical axis through the pole and labelled it y. A common error was to draw the required graph as a vertical line passing through the initial line. Most of the students who sketched the graph correctly also wrote down the correct minimum distance in part (b). A minority of students who did not draw the correct graph in part (a) were still able to find the minimum distance using their knowledge of the range of values of $\sin\theta$.

Question 5

Part (a) was not answered well, despite the formulae provided in the accompanying booklet. Incorrect responses included $y = \pm\frac{1}{2}x$ or just one asymptote, typically $y = 2x$. Some also left a in the equation, failing to realise it had no effect on the asymptotes.
Part (b) was answered well by most students. However, some omitted the x-intercepts whilst others did not illustrate the behaviour of the graph correctly at the asymptotes, often drawing two parabolic shapes despite the hints in the question.
Part (c) was a good discriminator. The majority of students were able to gain some credit for their response, but only a few achieved full marks. A common error was to rotate the graph around the x-axis instead of the y-axis. A minority of students achieved a correct solution but failed to realise that as the question was a 'show that', then correct notation must be used and each step shown clearly.

Question 6

The majority of students knew the shape of the cosh curve, but only a minority were able to transform it correctly. Most students realised that the graph should be reflected in the line $y = x$, but then either did not apply a translation or translated in the wrong direction. It was not uncommon for the x-intercept to be omitted.
In part (b) only half of the students gave the correct minimum distance.

Question 7

Part (a) was answered very well. Most students understood how to use the method of differences in part (b), but only a minority were able to write their answer in the required form. Common errors included starting from $r = 1$ even though it produced the fractional pair $\frac{1}{0} - \frac{1}{1}$. Neglecting to multiply by $\frac{1}{2}$ was another common problem.

Question 8

The first three parts were answered well, particularly part (c). A common error seen in parts (a) and (b) was to give the answer in the form $r(\cos\theta + \sin\theta)$ instead of identifying the modulus or the argument. Another common error in part (b) was to divide rather than subtract.
The majority of students were unable to make any progress in part (d). A minority realised that the equation $|w| = 2$ represented a circle which includes all three points, but only some students managed to determine the correct angle and to justify their answer. A well-drawn diagram usually accompanied the successful attempts.

Question 9

The majority of students were able to spot the error in part (a) and included a satisfactory explanation.
Part (b) was a good discriminator. Students who opted to rewrite the equation in terms of exponentials rarely made much progress beyond forming an equation in terms of a single function. This method produced an octic equation which could be reduced to a quartic, but there was little else which could be done other than to use a calculator to find approximate roots. A small minority used identities, to change $\sinh 2x$ and $\cosh 2x$ into a quadratic equation in terms of just $\cosh x$ or $\sinh x$. Students were able to progress much further using this method, but the required form was not possible unless converted back to $\cosh 2x$. Of those students who solved the equation and achieved the required form, usually by forming a quadratic in $\cosh 2x$, many neglected to reject an invalid answer or omitted a possible answer.

Question 10

This question was a good discriminator. The majority of students correctly wrote $\cosh x$ in terms of e^x, and many of these went on to write a correct expansion. A minority of students found the expansions of e^x and then e^{-x} by differentiating each function separately, instead of simply quoting the expansion of e^x from the formulae booklet and adapting it for e^{-x}. Although a perfectly valid approach, it penalised students in terms of time. However, such students usually made good progress and went on to find the correct Maclaurin expansion of $\cosh x$. A common error was to write each term of the e^x expansion as its reciprocal.
In part (b), a significant number of students made little or no progress. Of those who realised they could substitute ix for x, most simplified correctly and noticed that it was the same as the expansion of $\cos x$.

Question 11

Most students were able to identify at least one of the asymptotes, but only about half were able to write down all three correctly. Common incorrect asymptotes included $x = \pm r$. Of the students who realised that one of the asymptotes was $x = \sqrt{r}$, many failed to include $x = -\sqrt{r}$.
A significant minority of students did not know how to tackle part (b) despite the clue not to use differentiation. Some used graphic calculators and made incorrect assumptions about the curve, failing to notice the local maximum to the right-hand side of the graph. Consequently, many students assumed that the horizontal asymptote was a critical value and formed part of the answer. Also, some assumed the graph was symmetrical about the y-axis and treated the y-intercept as a local minimum. The students who formed a quadratic in x and used the discriminant to determine the critical points were usually able to do so successfully. However, arithmetic slips were not uncommon. Some used approximations in their range, instead of an exact form as directed.

Question 12

This question was a good discriminator. The majority of students were able to make good progress in the proof by induction. Only a few, however, were able to complete it with a correct conclusion. Some poor presentation led to indices being confused with non-indices, although this only affected a minority of students.
A very common error in part (b) was to assume that all invariant lines pass through the origin. A minority of students found a line of invariant points instead. Of the students who included a non-zero y-intercept, most were able to write down suitable equations linking their variables. Many of these students, however, became bogged down with the algebra and failed to realise that, as the equations had to be true for all values of x, then they could compare coefficients.
Most students were able to find the equation of the line of invariant points in part (c). There were very few errors made after applying $\mathbf{Av} = \mathbf{v}$.

Question 13

This question was a good discriminator. A significant number of students confused position vectors with direction vectors, significantly limiting their success.
The majority of students were able to find the direction vector of line l_1, usually by rearranging the equation into the form given in the formulae booklet. An alternative, less popular method, was to equate the given equation to λ and then rearrange into three expressions in terms of λ.
In part (b)(i), a significant minority compared the position vector $\begin{pmatrix} n \\ 5 \\ n \end{pmatrix}$ with their direction vector from part (a) and consequently scored no marks. Most students understood that they were required to show that the scalar product was non-zero. A common error was to omit a conclusion despite the question being a 'show that'.
Most students explained that two lines are parallel if their direction vectors are scalar multiples of each other. Some, however, could not then use algebra successfully to explain why the two lines in question could not be parallel. A minority of students thought that the direction vectors had to be equal.
The majority of students knew that a scalar product was required in part (b)(iii), but only a minority successfully found the required expression. A significant minority did not realise that, as the question is a 'show that', then $\cos\theta$ would have to appear at some stage in their working.

Question 14

The majority of students had no problem with part (a). Of the unsuccessful attempts, some common errors included $\alpha + \beta + \gamma = 3$ or -3, presumably from the coefficient of x rather than x^2. Also, some correctly explained that the sum of α and β must be real as they were complex conjugates, but were unable to progress further.

Part (b) assessed students' understanding of how complex roots relate to the graph of an equation. Unfortunately, only a minority of students were able to give the correct set of values for p.

The most popular method for answering part (c)(i) was by substitution. A less successful method was to find the product of the roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$. The difficult algebra involved in this latter method often led to errors. Some students correctly showed that the constant term could be a multiple of $p + 2$ if the coefficient of x^3 in $\mathrm{f}(x)$ was not 1.

Only a minority of students were able to give the correct x-coordinates in part (c)(ii). Common incorrect answers included -2 and 0, or -1 and 1.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.