AS
FURTHER MATHEMATICS

7366/2D: Paper 2 Discrete
Report on the Examination

7366
June 2019

Version: 1.0

General Comments

There was a very good range of marks from 1 to 40. As in June 2018, students often failed to gain marks due to the lack of depth and clarity of their explanations, most notably where explanations were required such as in **4(b)(ii)**, **5(b)(ii)**, **6(c)** and **7(a)(ii)**.

Question 1 (Multiple choice)

The vast majority of students selected the correct answer in this question.

Question 2 (Multiple choice)

The vast majority of students selected the correct answer in this question.

Question 3

Many students scored well on this question. The most successful solutions had variables clearly defined as ‘number of apple cakes’ and ‘number of banana cakes’ and proceeded on to derive two non-trivial constraints. Where students did not receive full marks, the most common causes were not including an objective function or a statement that the objective function should be maximised, as well as not including the non-negativity constraints for the variables. We condoned responses which lacked the constraint that the variables should be integers, but students should be encouraged to state this constraint. There was a significant number of solutions which received zero marks due to misunderstanding the question and erroneously working in terms of variables related to the number of eggs and amount of flour.

Question 4

There were many good solutions to this question with students showing a very good understanding of bipartite graphs. Most students appreciated the idea that the vertices can be split into two sets in part **(a)**, but a common mistake was failing to state that vertices are not directly connected to vertices in the same set.
The vast majority of students were able to use the hint of part **(a)** to draw a bipartite graph for the situation in **(b)(i)**. The most effective and most clear solutions had two vertical columns of vertices labelled with *J*, *K*, *L* etc, with the vertices being connected with edges that were drawn using a ruler.
In part **(b)(ii)** many students did not refer to the graph they had drawn in part **(b)(i)**, or they did not use graph theory terminology in their response and so did not answer the question. The most successful solutions were clear and used accurate terminology, such as ‘the vertex *N* (which represents Nish) is not directly connected with an edge to the vertex *B* (which represents the bassoon), so there is no instrument available for Nish to play’.
The final part of the question was answered well, with nearly all students drawing at least one correct subgraph and most drawing the two correct subgraphs and no others. Mistakes which were made here included drawing the bipartite graph of **(b)(i)** with the ‘*N*’ and ‘*F*’ vertices removed, as well as any edges which were incident on these two vertices.

Question 5

Nearly all students produced a fully correct answer in part **(a)**, with the occasional answer mistakenly using addition modulo 4 rather than multiplication modulo 4.
As with part **(a)**, **(b)(i)** was answered extremely well with very few errors.
In part **(b)(ii)**, most students were able to gain at least one mark by setting up the test for associativity by considering the placement of brackets in a three-term combination. Many students considered $(a \bullet b) \bullet c$ and $a \bullet (b \bullet c)$, correctly showing them to equal a and c respectively. However, many students went on to state the operation was not associative as $a \neq c$ (rather than $(a \bullet b) \bullet c \neq a \bullet (b \bullet c)$), which did not fully justify the conclusion. Some students incorrectly responded that because the Cayley table in **(b)(i)** was symmetric about the leading diagonal then the operation was associative.

Question 6

Virtually all students recognised the situation in part **(a)** as a route inspection problem and proceeded to find all odd-degree nodes, the shortest distance between each pair of odd-degree nodes, and the three values for the possible pairings. A significant number of students did make a slip with the shortest distance between nodes *E* and *H*, with 275 being the most common error. Students who made this error were still able to score 4 out of 5. We condoned missing units on the answer as long as it was clear what the intended unit should have been, but students should be encouraged to include units in their answers.
For question **(b)(i)**, we required sight of the two shortest arcs from node *A*. A common mistake here was to use the two shortest arcs in the entire network. Part **(b)(ii)** was answered well by nearly all students, with the most effective responses including the Hamiltonian cycle found by using the nearest neighbour algorithm starting at *A*.
Most students dropped at least one mark in part **(c)(i)** for not stating that Brook will travel 615 m on his journey due to the lower bound and upper bound found in part **(b)** being equal, therefore not fully justifying the answer. The most effective solutions clearly showed the two contributions to Brook's travel time and compared this with Ashley's travel time, noting they were both about 15 minutes. Students who did not correctly answer part **(b)** could still gain marks in part **(c)(i)** by considering the time taken for both their lower bound and upper bound, with many doing this.
Part **(c)(ii)** received a variety of different answers, but many were not relevant or in the context of the question. The most effective answers centred on Brook not having to travel an extra distance in going from the path to the signs, or Ashley's mower not needing to be emptied.

Question 7

Part **(a)(i)** was answered extremely well, with nearly all students able to rewrite the pay-off matrix from the perspective of Bex. The most successful solutions included the strategies $\mathbf{A_1}$, $\mathbf{A_2}$, etc, in the columns and rows of the pay-off matrix.
Part **(a)(ii)** was answered less well, with some responses not stating anything to do with dominance, or getting the necessary language back-to-front, such as 'strategy $\mathbf{B_1}$ is dominated by strategy $\mathbf{B_3}$'. Many students were able to demonstrate their good understanding of dominance by realising that if the dominance between Bex's strategies was used first, then Ali's $\mathbf{A_2}$ strategy was dominated by $\mathbf{A_1}$, $\mathbf{A_3}$ and $\mathbf{A_4}$.
The final question on the paper, **7(b)**, provided a good test for the students. Many competently defined a probability variable for Bex and used the reduced pay-off matrix given in **(a)(ii)** to arrive at 3 correct gains for Bex. The most effective solutions then provided a sketch of a graph of the 3 expected gains and circled the optimal point of intersection. The value of the game for Bex was then found, and fully complete solutions then changed the sign on this value to arrive at the value

of the game for Ali. Incorrect solutions used a probability of $p - 1$ for Bex's $\mathbf{B_2}$ strategy or sign errors were made upon expanding brackets when finding the expected gains for Bex. Some students did not provide a graphical reason or sketch to justify why they had chosen to set two expected gains equal to each other and were therefore at risk of not gaining credit for their method.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.