AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level
FURTHER MATHEMATICS

Paper 1

Monday 3 June 2019 Morning Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Answer **all** questions in the spaces provided.

1 Which one of these functions has the set $\{x : |x| < 1\}$ as its greatest possible domain?

Circle your answer.

[1 mark]

$\cosh x$ $\cosh^{-1} x$ $\tanh x$ $\tanh^{-1} x$

2 The first two non-zero terms of the Maclaurin series expansion of $f(x)$ are x and $-\frac{1}{2}x^3$

Which one of the following could be $f(x)$?

Circle your answer.

[1 mark]

$xe^{\frac{1}{2}x^2}$ $\frac{1}{2}\sin 2x$ $x\cos x$ $(1+x^3)^{-\frac{1}{2}}$

3 The function $f(x) = x^2 - 1$

Find the mean value of $f(x)$ from $x = -0.5$ to $x = 1.7$
Give your answer to three significant figures.

Circle your answer.

[1 mark]

-0.521 -0.434 -0.237 0.786

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4 Solve the equation $2z - 5iz^* = 12$

[4 marks]

Turn over for the next question

5 A plane has equation $\mathbf{r}.\begin{bmatrix}1\\1\\1\end{bmatrix} = 7$

A line has equation $\mathbf{r} = \begin{bmatrix}2\\0\\1\end{bmatrix} + \mu\begin{bmatrix}1\\0\\1\end{bmatrix}$

Calculate the acute angle between the line and the plane.

Give your answer to the nearest 0.1°

[3 marks]

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Turn over for the next question

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ANSWER IN THE SPACES PROVIDED

6 (a) Show that

$$\cosh^3 x + \sinh^3 x = \frac{1}{4}e^{mx} + \frac{3}{4}e^{nx}$$

where m and n are integers.

[3 marks]

6 (b) Hence find $\cosh^6 x - \sinh^6 x$ in the form

$$\frac{a\cosh(kx) + b}{8}$$

where a, b and k are integers.

[5 marks]

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Turn over for the next question

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7 Three non-singular square matrices, **A**, **B** and **R** are such that

$$\mathbf{AR} = \mathbf{B}$$

The matrix **R** represents a rotation about the z-axis through an angle θ and

$$\mathbf{B} = \begin{bmatrix} -\cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7 (a) Show that **A** is independent of the value of θ.

[3 marks]

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7 (b) Give a full description of the single transformation represented by the matrix **A**.

[1 mark]

_____

_____

_____

Turn over for the next question

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8 (a) If $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to prove that

$$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$$

[3 marks]

8 (b) Express $\sin^5\theta$ in terms of $\sin 5\theta$, $\sin 3\theta$ and $\sin\theta$

[4 marks]

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8 (c) Hence show that

$$\int_0^{\frac{\pi}{3}} \sin^5\theta \, \mathrm{d}\theta = \frac{53}{480}$$

[3 marks]

Turn over for the next question

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9 (a) Solve the equation $z^3 = \sqrt{2} - \sqrt{6}\mathrm{i}$, giving your answers in the form $re^{\mathrm{i}\theta}$ where $r > 0$ and $0 \leq \theta < 2\pi$

[5 marks]

9 (b) The transformation represented by the matrix $\mathbf{M} = \begin{bmatrix} 5 & 1 \\ 1 & 3 \end{bmatrix}$ acts on the points on an Argand Diagram which represent the roots of the equation in part **(a)**.

Find the exact area of the shape formed by joining the transformed points.

[4 marks]

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Turn over for the next question

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10 The points $A(5, -4, 6)$ and $B(6, -6, 8)$ lie on the line L. The point C is $(15, -5, 9)$.

10 (a) D is the point on L that is closest to C.

Find the coordinates of D.

[6 marks]

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10 (b) Hence find, in exact form, the shortest distance from C to L.

[2 marks]

Turn over for the next question

11 Find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = \frac{x^3}{\sqrt{4 - 2x - x^2}}$$

where $0 < x < \sqrt{5} - 1$

[7 marks]

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Turn over for the next question

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12 Three planes have equations

$$4x - 5y + z = 8$$
$$3x + 2y - kz = 6$$
$$(k - 2)x + ky - 8z = 6$$

where k is a real constant.

The planes do **not** meet at a unique point.

12 (a) Find the possible values of k.

[3 marks]

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12 (b) For each value of k found in part **(a)**, identify the configuration of the given planes.

Fully justify your answer, stating in each case whether or not the equations of the planes form a consistent system.

[5 marks]

Turn over for the next question

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13 The equation $z^3 + kz^2 + 9 = 0$ has roots α, β and γ.

13 (a) (i) Show that

$$\alpha^2 + \beta^2 + \gamma^2 = k^2$$

[3 marks]

13 (a) (ii) Show that

$$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = -18k$$

[4 marks]

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13 (b) The equation $9z^3 - 40z^2 + rz + s = 0$ has roots $\alpha\beta + \gamma$, $\beta\gamma + \alpha$ and $\gamma\alpha + \beta$.

13 (b) (i) Show that

$$k = -\frac{40}{9}$$

[1 mark]

Question 13 continues on the next page

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13 (b) (ii) Without calculating the values of α, β and γ, find the value of s.

Show working to justify your answer.

[6 marks]

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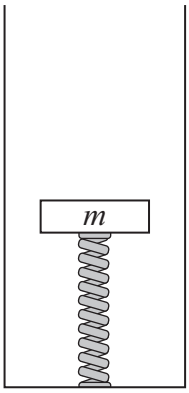
14 **In this question use $g = 10\,\text{m s}^{-2}$**

A light spring is attached to the base of a long tube and has a mass m attached to the other end, as shown in the diagram.

The tube is filled with oil.

When the compression of the spring is ε metres, the thrust in the spring is $9m\varepsilon$ newtons.



The mass is held at rest in a position where the compression of the spring is $\frac{20}{9}$ metres.

The mass is then released from rest. During the subsequent motion the oil causes a resistive force of $6mv$ newtons to act on the mass, where $v\,\text{m s}^{-1}$ is the speed of the mass.

At time t seconds after the mass is released, the displacement of the mass above its starting position is x metres.

14 (a) Find x in terms of t.

[10 marks]

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14 (b) State, giving a reason, the type of damping which occurs.

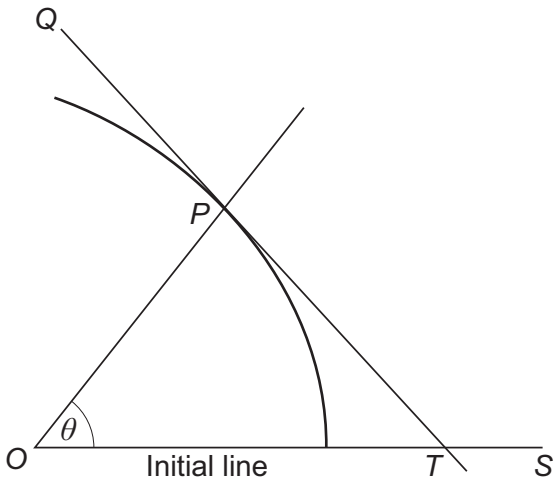
[1 mark]

15 The diagram shows part of a spiral curve.

The point P has polar coordinates (r, θ) where $0 \leq \theta \leq \frac{\pi}{2}$

The points T and S lie on the initial line and O is the pole.

TPQ is the tangent to the curve at P.



15 (a) Show that the gradient of TPQ is equal to

$$\frac{\frac{\mathrm{d}r}{\mathrm{d}\theta}\sin\theta + r\cos\theta}{\frac{\mathrm{d}r}{\mathrm{d}\theta}\cos\theta - r\sin\theta}$$

[4 marks]

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15 (b) The curve has polar equation

$$r = \mathrm{e}^{(\cot b)\theta}$$

where b is a constant such that $0 < b < \frac{\pi}{2}$

Use the result of part **(a)** to show that the angle between the line *OP* and the tangent *TPQ* does not depend on θ.

[7 marks]

END OF QUESTIONS

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