A-LEVEL
FURTHER MATHEMATICS

7367/1: Paper 1
Report on the Examination

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General Comments

This paper had questions on a variety of topics within the Further Mathematics specification, thus allowing students to show their aptitude across a wide range of skills. Not all students were able to attempt all questions, though a very large number were able to attempt nearly all of them and showed that they were well prepared for the exam.

Question 1

A quick sketch of the graphs of the functions allowed most students to select the correct answer, which could also be found on page 8 of the formulae booklet.

Question 2

With Maclaurin's and binomial series printed in the formulae booklet, most students chose the correct $f\left(x\right)$.

Question 3

The vast majority of students used their calculator successfully to find the correct mean value.

Question 4

This was a straightforward question and most students correctly found x and y. A few students then failed to gain a mark as they forgot to write down what z was.

Question 5

This was a standard question in which nearly all students scored well. The major reason for failing to gain marks was forgetting to take the angle between the line and the plane's normal vector from 90^{o} to get the angle between the line and the plane.

Question 6

Almost all students had a very good attempt at part (a). However, most failed to gain a mark as they didn't start with $\cosh^3 x + \sinh^3 x$ and end with $\frac{1}{4}e^{3x} + \frac{3}{4}e^{-x}$. Stating

$\therefore \cosh^3 x + \sinh^3 x = \frac{1}{4}e^{3x} + \frac{3}{4}e^{-x}$, after rigorous working would have secured the final mark.

Most students found part (b) to be very challenging. Those who used the approach in the typical solution easily got the first few marks, with most getting full marks.

Those students who attempted a full expansion of $\cosh^6 x$ and $\sinh^6 x$ usually fared poorly; they tended to get lost in all of the algebra.

Question 7

This question proved to be more challenging than expected.

In part (a), many students found $\mathbf{R}^{-1}$ by applying the standard method to find an inverse matrix, not recognising that the inverse of a rotation about the z–axis through an angle θ is a rotation about the z–axis through an angle $-\theta$. Of those students who followed the typical solution approach a surprisingly high number forgot to write a concluding comment to show that their argument was complete: "so $\mathbf{A}$ is independent of θ."

Those students who assumed that $\mathbf{A}$ was independent of θ in order to write $\mathbf{A} = \begin{pmatrix} a & b & c \\ d & e & f \\ h & h & i \end{pmatrix}$ and solve simultaneous equations, or found $\mathbf{A}$ by inspection, also had to explain why this was a unique solution in order to complete a rigorous argument.

Of those students who got the correct $\mathbf{A}$ in part (a), a very large number incorrectly said "reflection in x–axis" in part (b).

Question 8

Part (a) was generally well done. Where students failed to gain marks, it was due to forgetting to show the intermediate step (ie using de Moivre's theorem) in going from $\frac{1}{z^n}$ to $\cos n\theta - \mathrm{i}\sin n\theta$ or forgetting that, in this proof, working should start clearly from $z^n - \frac{1}{z^n}$ and end at $2\mathrm{i}\sin n\theta$.

In part (b), many students mistakenly used the technique "$\sin 5\theta$ in terms of powers of $\sin\theta$" and typically scored no marks.

Those students who used the correct approach in part (b) scored very well in part (c). Those students who used the incorrect approach in part (b) often left part (c) blank.

Question 9

Students were expected to use a calculator to find the modulus and argument of z^3, but many students incorrectly obtained $\arg\left(z^3\right) = +\frac{\pi}{3}$ which very quickly reduced the maximum score they could get. Others were unsure how to proceed, either forgetting to cube root $2\sqrt{2}$ or divide $\arg\left(z^3\right)$ by 3.

Those students who got $|z| = \sqrt{2}$ in part (a) could access all the marks in part (b).

Linking the area scale factor of a 2×2 matrix to the area formed by three complex numbers forming an equilateral triangle in the complex plane, is a challenging synoptic skill. Whilst some used the technique in the typical solution, others applied the matrix to the vertices and attempted to calculate the area of the image triangle directly. Such solutions typically achieved 3 marks, with the area seldom being calculated correctly.

Question 10

In part (a) the vast majority of students were able to construct the parametric equation of the line through *A* and *B*, but significantly fewer found the general vector from the line to the point. Most of the students getting to this point then took the scalar product with the correct direction vector, getting the correct values of $x, y \,\&\, z$. Surprisingly, however, a majority of these then wrote *D* as a position vector, rather than as a coordinate, thus failing to gain the last mark.
In part (b) everyone who had got a set of coordinates for *D* was then able to find the distance from their *D* to *C*, but it had to be in exact form. Some students forgot this, giving their answer as a decimal.

Question 11

This question required students to solve a first order differential equation, using an integrating factor and perform a challenging integration using an inverse trigonometrical function. Each part of this problem was marked largely as independently as possible.

Most students either dealt with the integrating factor part correctly or were very close to doing so. A minority recognised the inverse trigonometrical integral for what it was, but, once recognised, it was generally done very well.

Question 12

Part (a) was extremely well done by students, with the vast majority getting full marks.

In part (b) most dealt with the $k = 5$ case correctly, though some forgot to mention "consistent". In the $k = -4$ case a common error was to assume it was a prism. Some of those who recognised that two planes were parallel didn't indicate that they were distinct. A sketch showing the orientation of the planes helped some students obtain full marks.

A few students showed no working, stating the line of intersection from their calculators. This is not sufficient when an answer has to be fully justified.

Question 13

Part (a)(i) is a standard question that was generally very well done. Those students who mistakenly used $\alpha + \beta + \gamma = +k$ could still gain most of the marks.

In part (a)(ii) most students expanded correctly and then factorised to get $2\alpha\beta\gamma\left(\alpha + \beta + \gamma\right)$ and $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. Those who used $\alpha + \beta + \gamma = +k$ in part (a)(i) generally used $\alpha\beta\gamma = +9$ here and failed to gain the last couple of marks. Only a very few students tried using $\left(\alpha^2 + \beta^2 + \gamma^2\right)^2$, which quickly led nowhere.
In part (b)(i), most students clearly showed that the sum of the roots of the new equation equalled $\alpha + \beta + \gamma + \alpha\beta + \beta\gamma + \gamma\alpha$ then linked this to $\frac{40}{9}$. Using the results from part (a) they clearly showed the required result.

Relatively few students scored full marks in part (b)(ii). Common errors were putting $"s="$ rather than $"-\frac{s}{9}="$; not spotting that $\alpha^3\beta\gamma+\alpha\beta^3\gamma+\alpha\beta\gamma^3=\alpha\beta\gamma\left(\alpha^2+\beta^2+\gamma^2\right)$ and making slips substituting the value of k into their equation.

Question 14

In part (a) it was unusual to see the correct general force equation set up.
However, with the $6mv$ and $9m\varepsilon$ clearly signalled, nearly all students were able to set up a second order differential equation. Those with a non-zero right hand side were able to score good marks by demonstrating their skill at solving a second order differential equation.

Those with a zero right hand side were not able to score so well, but were still able to obtain reasonable marks.

With the overwhelming majority of auxiliary equations yielding repeated roots in part (a), even if slightly wrong, part (b) scored very highly indeed.

Question 15

There was evidence that a minority of students did not attempt this question. For those who did, most scored some marks in what was a challenging question.

In part (a) most students remembered that $x=r\cos\theta$ and $y=r\sin\theta$, with many demonstrating their understanding of implicit differentiation in this context. However, not all remembered to link their $\mathrm{d}y/\mathrm{d}x$ to the gradient of TPQ.

In part (b) many students were able to differentiate exponentials in polar coordinates and substitute into the equation given in part (a). Only a few were then able to get this into compound angle form and hence equal to $\tan\left(\theta+b\right)$. It was rare to see the geometrical argument showing that the angle between OP and the tangent TPQ does not depend on θ.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.