AQA

A-level
FURTHER MATHEMATICS
7367/2

Paper 2

Mark scheme

June 2019

Version: 1.0 Final

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

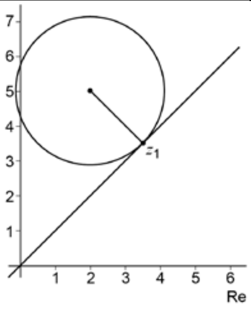
Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer.	AO2.2a	B1	$z - z^* = z^* - z$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer.	AO2.2a	B1	$y = ax + a$
	Total		**1**	

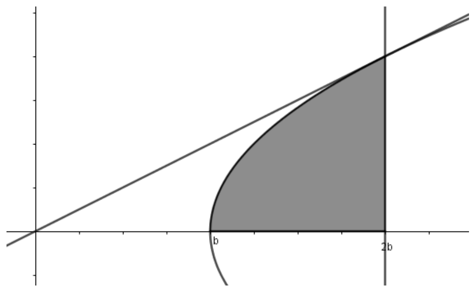
Q	Marking Instructions	AO	Marks	Typical Solution
3	Circles correct answer.	AO2.2a	B1	$\|x^2 - 1\| < 1$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Removes $\sum$ sign and creates an expression for the sum in terms of k. Allow max. 1 error	AO1.1a	M1	$\sum_{r=1}^{k}(3r - k) = \frac{3}{2}k(k+1) - k^2$ $\frac{1}{2}k^2 + \frac{3}{2}k = 90$ $k = 12$
	Forms a correct quadratic equation	AO1.1b	A1	
	Obtains the correct answer. NMS 3/3 Treat trial and improvement as NMS	AO1.1b	A1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Correctly substitutes the derivative of $\cosh x$ in the formula for arc length. Condone missing limits.	AO1.2	M1	$s = \int_a^b \left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)^{\frac{1}{2}} \mathrm{d}x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \sinh x$ $1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 1 + \sinh^2 x = \cosh^2 x$ $s = \int_a^b (\cosh^2 x)^{\frac{1}{2}} \mathrm{d}x = \int_a^b \cosh x \,\mathrm{d}x$ $= [\sinh x]_a^b$ $= \sinh b - \sinh a$
	Correctly uses the hyperbolic identity $1 + \sinh^2 x = \cosh^2 x$ to simplify the integrand. Condone $\sqrt{1 + \sinh^2 x} = \cosh x$	AO1.1a	M1	
	Correctly integrates $\cosh x$	AO1.1a	M1	
	Substitutes limits and uses a rigorous argument to show the required result	AO2.1	R1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
6	Sets as equal the real and imaginary parts of z_1 or uses the line $y = x$	AO2.2a	M1	 $z_1 = k + k\mathrm{i}$ Radius is perpendicular to tangent ∴ Gradient of line connecting (2,5) and $(k, k) = -1$ $\frac{5-k}{2-k} = -1$ $5 - k = k - 2$ $k = 3.5$ $a^2 = (5 - 3.5)^2 + (2 - 3.5)^2 = 4.5$ $a = \frac{3\sqrt{2}}{2}$
	Uses the fact that the half-line is a tangent to the circle	AO2.2a	M1	
	Forms an equation for the gradient of the line joining (2, 5) and (k, k) or the gradient of a tangent at any point on *C* *or* uses a right angled triangle containing the line joining (2, 5) and (k, k) or correctly substitutes $y = x$ into the equation of the circle	AO3.1a	M1	
	Forms a correct equation based on their method. PI by $k = 3.5$	AO2.2a	M1	
	Obtains the value of a from their equation	AO1.1b	A1	
	Produces a completely correct, rigorous proof leading to exact value of a. Must show all steps clearly	AO2.1	R1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical solution
7(a)	Forms two vectors from A, B and C, at least one correct	AO1.1a	M1	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} -7 \\ -3 \\ -6 \end{pmatrix} \times \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ -13 \end{pmatrix}$
	Obtains the correct vector product.	AO1.1b	A1	Area = $\frac{1}{2}\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\|$
	Uses their vector product correctly in a formula for the area of a triangle	AO1.2	M1	$= \frac{\sqrt{9^2+5^2+(-13)^2}}{2} = \frac{5\sqrt{11}}{2}$
	Uses a rigorous argument to show the required result, including stating “Area =” oe	AO2.1	R1	
(b)	States the correct equation	AO1.1b	B1	$\mathbf{r}.\begin{pmatrix} 9 \\ 5 \\ -13 \end{pmatrix} = 35$
(c)	Divides their non-zero k by the magnitude of their normal vector to obtain their exact value	AO3.1a	B1F	$\frac{7\sqrt{11}}{11}$
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Obtains the equation of P_2	AO1.2	B1	$y^2 = 4a(x-b)$ $m^2x^2 = 4a(x-b)$ $m^2x^2 - 4ax + 4ab = 0$ For equal roots $\Delta = 0$ $16a^2 - 4m^2(4ab) = 0$ $16a^2 = 16m^2ab$ $m = \pm\sqrt{\frac{a}{b}}$
	Combines equations to give a quadratic equation in x	AO1.1a	M1	
	Sets the discriminant to zero, with working	AO1.1a	M1	
	Completes a rigorous argument to show the required result.	AO2.1	R1	
(b)	Forms a quadratic equation in x	AO3.1a	M1	$\left(x\sqrt{\frac{a}{b}}\right)^2 = 4a(x-b)$ $x^2a = 4ab(x-b)$ $x^2 - 4bx + 4b^2 = 0$ $(x-2b)^2 = 0 \Rightarrow x = 2b$  $V = \pi\int_b^{2b} 4a(x-b)dx$ $V = 4a\pi\left[\frac{x^2}{2} - bx\right]_b^{2b}$ $= 4a\pi\left\{\left(\frac{4b^2}{2} - 2b^2\right) - \left(\frac{b^2}{2} - b^2\right)\right\}$ $= 2ab^2\pi$
	Finds correct x value at D	AO1.1b	A1	
	Writes a correct integral for V. Condone limits omitted	AO1.2	B1	
	Correctly integrates their two-term expression with lower limit b	AO1.1a	M1	
	Finds the correct answer [This can also be done by translating the curve by $\begin{pmatrix}-b\\0\end{pmatrix}$ and finding $\pi\int_0^b 4ax\,dx$, but this must be clearly explained to gain full marks.]	AO1.1b	A1	
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical solution
9(a)	Forms correct characteristic equation and solves. (PI) Condone one error	AO1.1a	M1	$\left(\frac{1}{5}-\lambda\right)\left(\frac{13}{10}-\lambda\right)+\frac{6}{25}=0$ $0.5-1.5\lambda+\lambda^2=0$ $\lambda=1$ & $\lambda=0.5$
	Obtains the correct eigenvalues	AO1.1b	A1	$0=\begin{bmatrix}\frac{-4}{5} & \frac{2}{5}\\ \frac{-3}{5} & \frac{3}{10}\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$
	Uses correct equation to find eigenvector for either $\lambda=1$ or $\lambda=0.5$ (PI)	AO1.1a	M1	or $0=\begin{bmatrix}\frac{-3}{10} & \frac{2}{5}\\ \frac{-3}{5} & \frac{4}{5}\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$
	Obtains correct eigenvector for $\lambda=1$ Allow any scalar multiple.	AO1.1b	A1	$\lambda=1$: $\begin{bmatrix}1\\ 2\end{bmatrix}$
	Obtains correct eigenvector for $\lambda=0.5$ Allow any scalar multiple.	AO1.1b	A1	$\lambda=0.5$: $\begin{bmatrix}4\\ 3\end{bmatrix}$
(b)	Finds their correct $\mathbf{U}$ with no zero column.	AO1.1b	B1F	$\mathbf{U}=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$ and $\mathbf{D}=\begin{bmatrix}\frac{1}{2} & 0\\ 0 & 1\end{bmatrix}$
	Finds their correct $\mathbf{D}$ (must be consistent with their $\mathbf{U}$)	AO1.1b	B1F	or $\mathbf{U}=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ and $\mathbf{D}=\begin{bmatrix}1 & 0\\ 0 & \frac{1}{2}\end{bmatrix}$
(c)	Finds correct $\mathbf{U}^{-1}$ – CAO	AO1.1b	B1	$\mathbf{U}^{-1}=\frac{1}{5}\begin{bmatrix}2 & -1\\ -3 & 4\end{bmatrix}$
	Multiplies their matrices in correct order with powers taken inside $\mathbf{D}^n$	AO1.1a	M1	$\mathbf{M}^n=\frac{1}{5}\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}\begin{bmatrix}\left(\frac{1}{2}\right)^n & 0\\ 0 & 1^n\end{bmatrix}\begin{bmatrix}2 & -1\\ -3 & 4\end{bmatrix}$
	Correctly takes $n\to\infty$ limit of their $\mathbf{D}^n$, ($\mathbf{D}$ must not contain only ones and zeros)	AO2.2a	M1	$\mathbf{L}=\frac{1}{5}\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}2 & -1\\ -3 & 4\end{bmatrix}$ $\mathbf{L}=\begin{bmatrix}-0.6 & 0.8\\ -1.2 & 1.6\end{bmatrix}$
	Obtains correct $\mathbf{L}$ Correct answer scores 4/4	AO2.1	R1	or $\mathbf{U}^{-1}=\frac{-1}{5}\begin{bmatrix}3 & -4\\ -2 & 1\end{bmatrix}$ $\mathbf{M}^n=\frac{-1}{5}\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}\begin{bmatrix}1^n & 0\\ 0 & \left(\frac{1}{2}\right)^n\end{bmatrix}\begin{bmatrix}3 & -4\\ -2 & 1\end{bmatrix}$ $\mathbf{L}=\frac{-1}{5}\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}3 & -4\\ -2 & 1\end{bmatrix}$ $\mathbf{L}=\begin{bmatrix}-0.6 & 0.8\\ -1.2 & 1.6\end{bmatrix}$

(d)	Obtains correct equations for image points from their **L** PI by $y=2x$	AO3.1a	M1	$x'=\frac{1}{5}(-3x+4y)$ $y'=\frac{2}{5}(-3x+4y)$ $y=2x$
	Obtains correct equation for the line Condone $y'=2x'$	AO3.2a	A1	
	Total		**13**	

Q	Marking Instructions	AO	Marks	Typical Solution
10	Demonstrates the result for $n=1$ and states that it is true for $n=1$	AO1.1b	B1	Let $n=1$ then $\mathrm{f}(1)=12=2\times 6$ so the result is true for $n=1$ Assume the result is true for $n=k$: Then $\mathrm{f}(k)=6m$ for some integer m
	Assumes the result true for $n=k$	AO2.4	M1	$\mathrm{f}(k+1)=(k+1)^3+3(k+1)^2+8(k+1)$ $=k^3+3k^2+3k+1+3(k^2+2k+1)+8(k+1)$ $=k^3+6k^2+17k+12$
	Obtains the difference between $\mathrm{f}(k+1)$ and $\mathrm{f}(k)$	AO3.1a	M1	$\mathrm{f}(k+1)=k^3+6k^2+17k+12-(k^3+3k^2+8k)$ $=6m+3k^2+9k+12$
	Calculates the difference between $\mathrm{f}(k+1)$ and $\mathrm{f}(k)$correctly	AO1.1b	A1	$3k^2+9k+12$ is a multiple of 3
	Deduces that the difference is a multiple of 3	AO2.2a	M1	$3k^2+9k+12=3(k^2+3k)+12$ $k^2+3k=k(k+3)$ and one of k or $k+3$ is even
	Deduces that the difference is a multiple of 2	AO2.2a	M1	$\therefore k^2+3k$ is even and $3(k^2+3k+4)$ is an even multiple of 3 and hence divisible by 6
	Completes a rigorous argument and explains how their argument proves the required result	AO2.1	R1	$\therefore \mathrm{f}(k+1)$ is divisible by 6 if $\mathrm{f}(k)$ is divisible by 6 We also know that $\mathrm{f}(1)$ is divisible by 6, so by induction this completes the proof.
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical solution
11	Obtains a position vector of a point on L_1.	AO2.5	B1	$\mathbf{r}_1 = \begin{pmatrix} 2 \\ -4 \\ 5/4 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 8 \\ 5/4 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 5/4 \end{pmatrix} + \mu \begin{pmatrix} 12 \\ 32 \\ 5 \end{pmatrix}$
	Obtains a direction vector for L_1 ISW	AO1.1b	B1	
	Obtains a position vector of a point on L_2 and a direction vector for L_2	AO2.5	B1	$\mathbf{r}_2 = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$
	Obtains a correct vector between the two lines. Follow through from their position vectors	AO1.1b	B1F	$\begin{pmatrix} 2 \\ -4 \\ 5/4 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ -7/4 \end{pmatrix}$
	Calculates the vector product of their two direction vectors or calculates the scalar product of the general vector between the lines with both direction vectors to obtain a pair of simultaneous equations	AO3.1a	M1	$\begin{pmatrix} 12 \\ 32 \\ 5 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 91 \\ -26 \\ -52 \end{pmatrix}$ $\dfrac{\begin{pmatrix} 4 \\ -4 \\ -\frac{7}{4} \end{pmatrix} \cdot \begin{pmatrix} 91 \\ -26 \\ -52 \end{pmatrix}}{13\sqrt{69}} = \dfrac{43}{\sqrt{69}}$ $= 5.17659... = 5.18$ 3sf
	Obtains the correct vector product or obtains a correct pair of simultaneous equations	AO1.1b	A1	
	Uses their vector product or the solutions to their simultaneous equations to calculate the shortest distance between the two lines	AO3.1a	M1	
	Obtains the correct shortest distance. Allow awrt 5.18 Accept exact answer	AO1.1b	A1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	States appropriate extra piece of information. Condone "m and p are integers"	AO2.3	R1	m and p are real numbers.
(b)	Finds one possible pair of values for m and p or finds the root ±7/2 in Abel's case or substitutes $z = 2 - 3\mathrm{i}$ in the cubic and expands or uses the sum of roots or the sum of pairwise products of roots (not using $z = 2 + 3\mathrm{i}$) to form an equation (condone sign errors)	AO2.1	R1	Product of roots = - 91/2 One possible solution to the cubic is given by (for example) $z = 2 - 3\mathrm{i}$, $z = 2 + 3\mathrm{i}$, $z = -\frac{7}{2}$ since $(2 - 3\mathrm{i})(2 + 3\mathrm{i})\left(-\frac{7}{2}\right) = -\frac{91}{2}$ This gives $m = -1$ and $p = -2$ Another solution to the cubic is (for example) $z = 2 - 3\mathrm{i}$, $z = 4 + 6\mathrm{i}$, $z = -\frac{7}{4}$ since $$(2 - 3\mathrm{i})(4 + 6\mathrm{i})\left(-\frac{7}{4}\right) = -\frac{91}{2}$$ This gives different values of m and p. Because there is more than one possible set of values of m and p, there is not sufficient information to solve the problem, and Bonnie is right.
	Uses product of roots = ±91/2 to find another possible pair of complex roots of the cubic or expresses m and p as complex numbers in their expansion or uses product of roots = ±91/2 to form another equation (not using $z = 2 + 3\mathrm{i}$)	AO3.1a	M1	
	Finds another possible correct pair of complex roots of the cubic (condone sign errors) or forms simultaneous equations for real and imaginary parts or reduces their system of equations to a system of linear equations	AO2.1	M1	
	Completes a rigorous argument to prove that Bonnie is correct; for example, explains that they have shown that there is more than one possible set of values for m and p or explains why their simultaneous linear equations have no unique solution	AO2.4	R1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	Explains that one of the limits is infinity (or that the interval of integration is infinite)	AO2.4	R1	The upper limit is infinity, so it is an improper integral.
(b)	Uses integration by parts twice	AO3.1a	M1	$u = x^2 \quad v' = \mathrm{e}^{-2x}$ $u' = 2x \quad v = \frac{-\mathrm{e}^{-2x}}{2}$
	Obtains the correct expressions for u' and v when integrating the first time	AO1.1b	B1	$\int x^2\mathrm{e}^{-2x}\,\mathrm{d}x = -\frac{x^2\mathrm{e}^{-2x}}{2} + \int x\mathrm{e}^{-2x}\,\mathrm{d}x$
	Correctly applies integration by parts formula the first time	AO1.1b	B1	$u = x \quad v' = \mathrm{e}^{-2x}$ $u' = 1 \quad v = \frac{-\mathrm{e}^{-2x}}{2}$
	Correctly applies integration by parts formula to an integral of the form $k\int x\mathrm{e}^{-2x}\,\mathrm{d}x$	AO1.1a	M1	$\int x\mathrm{e}^{-2x}\,\mathrm{d}x = \left(-\frac{x\mathrm{e}^{-2x}}{2}\right) + \int \frac{\mathrm{e}^{-2x}}{2}\,\mathrm{d}x$ $= -\frac{x\mathrm{e}^{-2x}}{2} - \frac{\mathrm{e}^{-2x}}{4}$
	Finds complete correct expression for integral with or without *c*. No limits needed at this stage. PI by later work	AO1.1b	A1	$\therefore \int x^2\mathrm{e}^{-2x}\,\mathrm{d}x = -\frac{x^2\mathrm{e}^{-2x}}{2} - \frac{x\mathrm{e}^{-2x}}{2} - \frac{\mathrm{e}^{-2x}}{4}$
	Defines the improper integral as a limit	AO2.4	E1	$\int_3^\infty x^2\mathrm{e}^{-2x}\,\mathrm{d}x = \lim_{n\to\infty}\int_3^n x^2\mathrm{e}^{-2x}\,\mathrm{d}x$
	Applies the limiting process correctly, using $\lim_{n\to\infty}(n^2\mathrm{e}^{-n}) = 0$ $\lim_{n\to\infty}(n\mathrm{e}^{-n}) = 0$ and $\lim_{n\to\infty}(\mathrm{e}^{-n}) = 0$	AO2.2a	M1	$= \lim_{n\to\infty}\left\{\left[-\frac{x^2\mathrm{e}^{-2x}}{2} - \frac{x\mathrm{e}^{-2x}}{2} - \frac{\mathrm{e}^{-2x}}{4}\right]_3^n\right\}$
	Substitutes correct lower limit correctly into their three-term expression	AO1.1a	M1	$\therefore \int_3^\infty x^2\mathrm{e}^{-2x}\,\mathrm{d}x = \frac{9\mathrm{e}^{-6}}{2} + \frac{3\mathrm{e}^{-6}}{2} + \frac{\mathrm{e}^{-6}}{4}$ $= \frac{25\mathrm{e}^{-6}}{4}$
	Obtains correct exact value or awrt 0.0155	AO1.1b	A1	
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Uses partial fractions	AO3.1a	M1	$\frac{1}{r+1}-\frac{1}{r+3}=\frac{2}{(r+1)(r+3)}$ $\therefore\ 2S_n=\sum_{r=1}^{n}\frac{1}{r+1}-\frac{1}{r+3}$ $=\frac{1}{2}-\frac{1}{4}$ $+\frac{1}{3}-\frac{1}{5}$ $+\frac{1}{4}-\frac{1}{6}$ $+\cdots$ $+\frac{1}{n-1}-\frac{1}{n+1}$ $+\frac{1}{n}-\frac{1}{n+2}$ $+\frac{1}{n+1}-\frac{1}{n+3}$ $2S_n=\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}$ $=\frac{5(n+2)(n+3)-6(n+3+n+2)}{6(n+2)(n+3)}$ $S_n=\frac{5n^2+13n}{12(n+2)(n+3)}$
	Correctly expresses the rational function as partial fractions	AO1.1b	A1	
	Uses the method of differences, showing at least the first three and last two terms (or vice versa) ("Term" here means one fraction minus another fraction)	AO2.5	M1	
	Correctly uses the method of differences to reduce the expression to four terms (oe)	AO1.1b	A1	
	Correctly expresses their three- or four-term answer with a common denominator	AO1.1a	M1	
	Completes fully correct working to reach the required result	AO2.1	R1	

(b)	Writes down a correct inequality. Condone $\frac{5}{12} - S_n < \frac{1}{k}$	AO3.1a	B1	$\frac{5}{12} - \frac{5n^2 + 13n}{12(n+2)(n+3)} < \frac{1}{k}$
	Simplifies the left-hand side of their inequality correctly	AO1.1a	M1	$\frac{5(n+2)(n+3) - (5n^2 + 13n)}{12(n+2)(n+3)} < \frac{1}{k}$ $\frac{12n + 30}{12(n+2)(n+3)} < \frac{1}{k}$
	Rearranges their inequality to remove fraction, explaining that denominators are positive	AO2.4	M1	$k(12n + 30) < 12(n+2)(n+3)$ since both denominators are positive.
	Writes their inequality or related equation in simplified quadratic form	AO1.1a	M1	$12n^2 + (60 - 12k)n + (72 - 30k) > 0$ $2n^2 + (10 - 2k)n + (12 - 5k) > 0$ The equation $2n^2 + (10 - 2k)n + (12 - 5k) = 0$ has a positive and a negative root (since $12 - 5k < 0$) Positive root $= \frac{2k - 10 + \sqrt{(10-2k)^2 - 8(12-5k)}}{4}$
	Obtains a correct root or roots of the quadratic equation in unsimplified form	AO1.1a	M1	$= \frac{2k - 10 + \sqrt{4k^2 + 4}}{4}$ $= \frac{k - 5 + \sqrt{k^2 + 1}}{2}$
	Completes a rigorous argument to show the required result. This must include a discussion of the signs of the roots, or other convincing reason that the inequality holds.	AO2.1	R1	Since the root is positive, if $12n^2 + (60 - 12k)n + (72 - 30k) > 0$ then $n > \frac{k - 5 + \sqrt{k^2 + 1}}{2}$
	Total		**12**	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	Uses the capacity of either tank and the coefficients of the equations to find r	AO3.4	M1	$r = 0.005 \times 800$ $r = 4$
	Obtains the correct answer	AO3.2a	A1	
(b)	Deduces that 12 L/min are flowing into A	AO3.3	M1	Flow into A = flow out of A (to keep volume of water constant) $= 16 - 4 = 12$ $12\mu = 36$ $\mu = 3$
	Divides 36 by their 12	AO2.2a	M1	
	Completes a rigorous argument to show the required result.	AO2.1	R1	
(c)	Differentiates one equation	AO3.1a	M1	(2) ... $0.02x = \dot{y} + 0.02y$ $x = 50\dot{y} + y$ and $\dot{x} = 50\ddot{y} + \dot{y}$ Sub in (1): $50\ddot{y} + \dot{y} = 36 - 0.02(50\dot{y} + y) + 0.005y$ $50\ddot{y} + 2\dot{y} + 0.015y = 36$ CF: $50m^2 + 2m + 0.015 = 0$ $m = -0.03, -0.01$ PI: $y = \frac{36}{0.015} = 2400$ $\therefore y = Ae^{-0.03t} + Be^{-0.01t} + 2400$ $\dot{y} = -0.03Ae^{-0.03t} - 0.01Be^{-0.01t}$ $x = 50\dot{y} + y$ so $x = -0.5Ae^{-0.03t} + 0.5Be^{-0.01t} + 2400$ When $t = 0$, $x = 0$ and $y = 0$ so $A + B + 2400 = 0$ and $-0.5A + 0.5B + 2400 = 0$ $\Rightarrow A = 1200$ and $B = -3600$ $x = -600e^{-0.03t} - 1800e^{-0.01t} + 2400$ $y = 1200e^{-0.03t} - 3600e^{-0.01t} + 2400$ or (1) $0.005y = \dot{x} + 0.02x - 36$ $y = 200\dot{x} + 4x - 7200$ and $\dot{y} = 200\ddot{x} + 4\dot{x}$ Sub in (2): $200\ddot{x} + 4\dot{x} = 0.02(200\dot{x} + 4x - 7200)$ $200\ddot{x} + 8\dot{x} + 0.06x = 144$ CF: $200m^2 + 8m + 0.06 = 0$ $m = -0.03, -0.01$ PI: $x = \frac{144}{0.006} = 2400$ $\therefore x = Ae^{-0.03t} + Be^{-0.01t} + 2400$ $\dot{x} = -0.03Ae^{-0.03t} - 0.01Be^{-0.01t}$ $y = 200\dot{x} + 4x - 7200$
	Substitutes both x and $\dot{x}$ or y and $\dot{y}$ in the other equation to eliminate one variable	AO3.1a	M1	
	Forms a correct second order differential equation	AO1.1b	A1	
	Obtains roots of their auxiliary equation	AO1.1a	M1	
	Uses a valid method to find a particular integral for their DE	AO2.2a	M1	
	States general solution for either x or y with their particular integral	AO1.1b	A1F	
	States general solutions for both x and y CAO	AO1.1b	A1	
	Uses initial conditions to find a value for each constant	AO3.4	M1	
	Writes correct solutions for both x and y	AO1.1b	A1	

				$y = -2Ae^{-0.03t} + 2Be^{-0.01t} + 2400$ When $t = 0$, $x = 0$ and $y = 0$ so $A + B + 2400 = 0$ and $-2A + 2B + 2400 = 0$ $\Longrightarrow A = -600$ and $B = -1800$ $x = -600e^{-0.03t} - 1800e^{-0.01t} + 2400$ $y = 1200e^{-0.03t} - 3600e^{-0.01t} + 2400$
	Total		**14**	