A-LEVEL
FURTHER MATHEMATICS

7367/2: Paper 2
Report on the Examination

7367
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General

This was the first exam series available for this paper and it was an encouraging performance in this new specification by students who may not have been sure what to expect. Apart from questions 2, 6 and 15, the mean score on the first part of each question was over 50%. The mean score for the paper was also significantly over 50%.

For most question parts, the number of students not attempting the question was low. For example, under 2% of students did not attempt 13(b); and around 85% attempted 15(c), suggesting that most students had sufficient time to attempt the whole of the paper.

It was expected that those items requiring students to demonstrate a rigorous proof would be the most challenging ones, and this did turn out to be the case. Few students gained full marks on questions 6, 10, 12(b), 14(b) and 15(b).

The other question parts where few students gained full marks were 8(b), which was a synoptic question combining conic sections and volume of revolution, and 15(c), which covered coupled differential equations, a new topic for A-level Further Maths.

On questions 1, 3, 4, 5, 7(a), 7(b), 9(a), 9(b), 12(a) and 13(a) most students scored full marks. Just under half scored full marks on questions 8(a), 9(c) and 14(a). This shows that students came into the exam well-prepared to answer the more straightforward questions across the whole range of the specification.

Question 1

Around three-quarters of students answered correctly, with a small number choosing the second or third option and very few selecting the first option.

Question 2

This proved quite challenging; nearly as many students selected the second option as selected the correct answer.

Question 3

Two-thirds got this question right, with similar numbers of students selecting each of the incorrect options.

Question 4

Most students provided a completely correct solution.

Some common errors were:

- making the sum of k from 1 to k equal k instead of k^2
- using a minus sign instead of a plus in the expression for the sum of integers from 1 to k.

Question 5

This question was done well, with around 80% gaining 4 or 5 marks. Some did not gain the final mark for incorrect notation such as omitting the '$\mathrm{d}x$' from an integral, or not at any stage stating 's =' or 'arc length ='.

Question 6

Several different correct methods were used here, in many cases with complete success.

Nearly all students attempted this question, but some neither stated that the real and imaginary parts of z_1 were equal, nor used the fact that the half-line is a tangent to the circle. These students generally scored no marks.

Those who used a calculator to find the inverse sine of the difference between two angles, and then obtained a result close to the one required, generally only gained 5 marks out of 6. A few students used a compound angle formula to obtain the exact answer, and were able to gain full marks.

Question 7

Around 70% of students scored 3 or 4 marks in part (a). Some missed out on a mark by not stating 'Area =' or equivalent.

Over half obtained the correct answer in part (b), but this went down to a quarter in part (c), despite the fact that correct follow-through from a previous incorrect result could gain the mark.

Students need to be aware that the instruction 'Use a vector product' requires the use of the cross product, not the dot product.

Question 8

Nearly all students gained the first mark, either for the correct equation of P_2 or for forming a quadratic equation. However, under half went on to gain full marks for part (a).

Errors included:

- Using the equation of P_1 instead of P_2
- Not stating '$b^2 - 4ac = 0$' or equivalent

Most students made a serious attempt at part (b), but few managed to get everything right.

Some common errors were:

- Using the equation for P_1 in their integral
- Multiplying by 2π instead of π
- Using zero as the lower limit instead of b
- Not finding the upper limit of $2b$

Question 9

Part (a) was answered very competently, with two-thirds of students gaining full marks.

Part (b) was also done extremely well.

Students needed to show the limiting process, with powers taken inside $\mathbf{D}^n$, to do part (c) correctly. It was easier for students to take the limit before multiplying the matrices, although it was also possible to multiply the matrices first and then take the limit.

Many students appeared unclear about how to approach part (d). Some used techniques which would be more suitable for finding an invariant line, or a line of invariant points.

Question 10

Most students showed a reasonably good understanding of the steps required to carry out a proof by induction, with around three-quarters gaining 4 or more marks out of 7.

However, many students, having found a correct expression for the difference between $\mathrm{f}(k + 1)$ and $\mathrm{f}(k)$, then stated that this difference was divisible by 6, without giving any evidence for this.

Some students also missed out on a mark for not stating explicitly that the result was true for $n = 1$, or for not stating that they were assuming the result to be true for $n = k$; so those few (under 15%) who gained full marks had to be very accurate and thorough.

Question 11

Most students were able to find a position vector and a direction vector for L_1, though there were some sign errors, and quite a few students used 5 instead of 5/4 as the k-component of either the position vector or the direction vector, or both.

Some students tried to use the vector product form of L_2 directly, but this was not usually successful.

There were several different correct methods used for the next stage. The ‘typical solution’ shown on the mark scheme was a reasonably direct method compared with those chosen by some students.

In particular, the method of finding a general vector between the lines and then equating the scalar product of this vector and each of the direction vectors in turn to zero, required a lot of calculation and thus gave more opportunities for errors. Nevertheless, several students obtained the correct answer by this method.

Question 12

Part (a) was done well, with around 80% answering correctly.

Part (b) proved very challenging. Not many students saw that one way of approaching the problem was to find two different possible solution sets.

Those who set up systems of simultaneous equations and then claimed that these must have multiple solutions were only able to gain full marks if their equations were linear.

Question 13

In part (a) many responses showed a lack of understanding of the definition of an improper integral, discussing, for example, the limit of the integrand as x tends to infinity, or claiming that the integral was not defined.

Most students showed very good technique in part (b); nearly all integrated by parts twice. This was mainly done correctly, though there was some evidence of sign errors.

Most students correctly defined the integral as a limit, then calculated the limit correctly. A few incorrectly used a lower limit of zero instead of 3.

Question 14

In this question, students were required to work with algebraic fractions. This was not always done in the most efficient way. Students who simplified as they went along were less likely to make errors than those who carried on working with unnecessarily complicated expressions – for example, a denominator of $(2n + 4)$ instead of $(n + 2)$.

Part (a) was done reasonably well. Most students used partial fractions and the method of differences accurately, though some did not include enough terms in their proof.

Some students missed out on a mark by not stating '$S_n =$' or equivalent.

In part (b), students found it difficult to pick up many marks. Some gave the inequality the wrong way round. Many did not justify clearing the fractions with reference to the positive denominators.

Because the final result was given in the question, it was important for students to show every step and not simplify prematurely.

Question 15

As this topic is a new one, the question was quite challenging. Encouragingly, most made a genuine attempt to solve the coupled differential equations in part (c).

A quarter of students were able to successfully analyse the system and answer part (a) correctly.

Under 15% of students gained any marks in part (b), the lowest number of any question part on the paper.

In contrast, nearly two-thirds of students gained at least one mark on part (c), with over a quarter gaining 6 or more marks out of 9.

Many started correctly by differentiating one of the differential equations, but then did not proceed by the most direct route to the elimination of x or y. Nevertheless, even those who had made errors at the first stage were able to gain method marks and a follow-through mark by correctly solving their differential equations.

All in all, students are to be commended for their approach to this question.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.