



Surname _____

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I declare this is my own work.

A-level MATHEMATICS

Paper 1

7357/1

Wednesday 3 June 2020 Afternoon

Time allowed: 2 hours

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

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INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question.
- Do NOT write on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO



SECTION A

Answer ALL questions in the spaces provided.

- 1 The first three terms, in ascending powers of x , of the binomial expansion of $(9 + 2x)^{\frac{1}{2}}$ are given by

$$(9 + 2x)^{\frac{1}{2}} \approx a + \frac{x}{3} - \frac{x^2}{54}$$

where a is a constant.

- 1 (a) State the range of values of x for which this expansion is valid.

Circle your answer. [1 mark]

$$|x| < \frac{2}{9}$$

$$|x| < \frac{2}{3}$$

$$|x| < 1$$

$$|x| < \frac{9}{2}$$



5

1 (b) Find the value of a .

Circle your answer. [1 mark]

1

2

3

9

[Turn over]



2 A student is searching for a solution to the equation $f(x) = 0$

He correctly evaluates

$$f(-1) = -1 \text{ and } f(1) = 1$$

and concludes that there must be a root between -1 and 1 due to the change of sign.

Select the function $f(x)$ for which the conclusion is **INCORRECT**.

Circle your answer. [1 mark]

$$f(x) = \frac{1}{x}$$

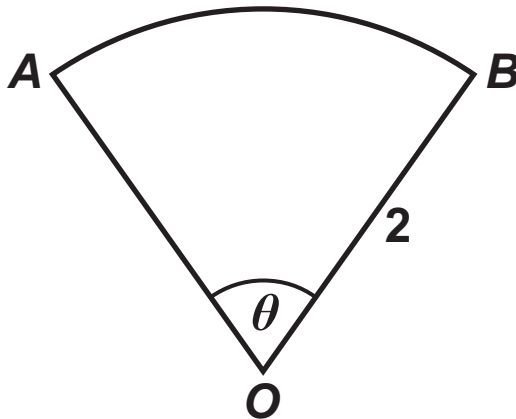
$$f(x) = x$$

$$f(x) = x^3$$

$$f(x) = \frac{2x + 1}{x + 2}$$



- 3 The diagram shows a sector OAB of a circle with centre O and radius 2



The angle AOB is θ radians and the perimeter of the sector is 6

Find the value of θ

Circle your answer. [1 mark]

1

$\sqrt{3}$

2

3

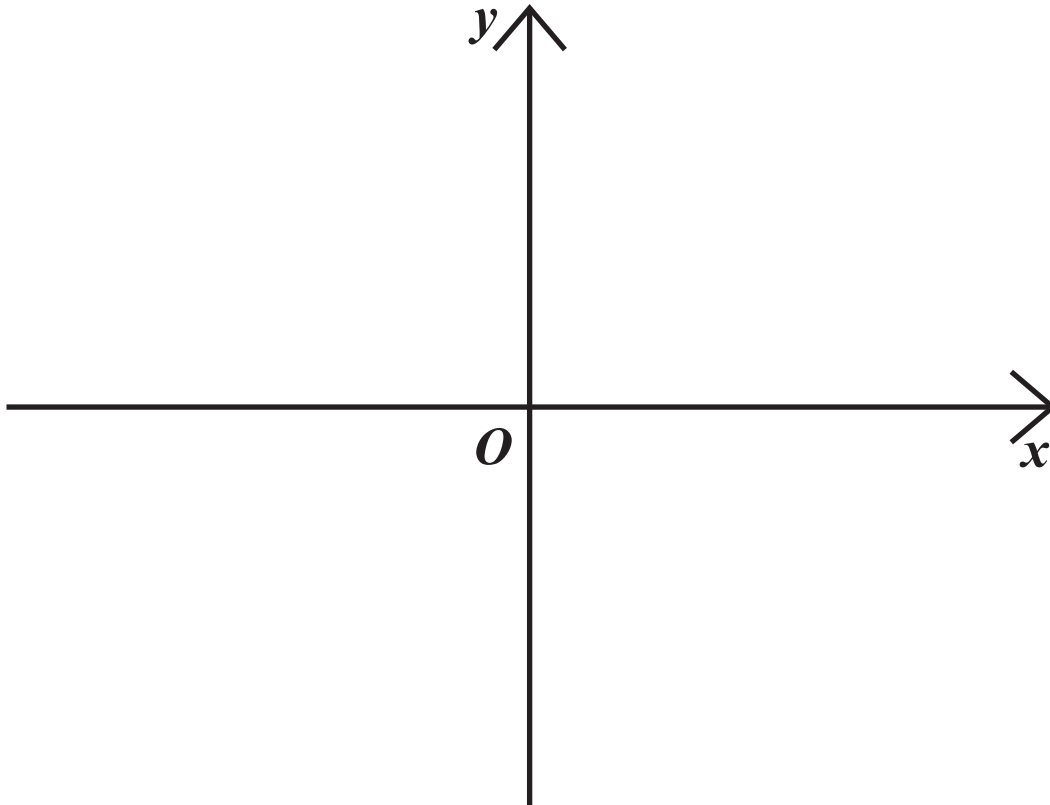
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4 (a) Sketch the graph of

$$y = 4 - |2x - 6|$$

[3 marks]



4 (b) Solve the inequality

$$4 - |2x - 6| > 2$$

[2 marks]



5 Prove that, for integer values of n such that $0 \leq n < 4$

$$2^{n+2} > 3^n$$

[2 marks]



[Turn over]



- 6 Four students, Tom, Josh, Floella and Georgia are attempting to complete the indefinite integral

$$\int \frac{1}{x} dx \quad \text{for } x > 0$$

Each of the students' solutions is shown below:

Tom $\int \frac{1}{x} dx = \ln x$

Josh $\int \frac{1}{x} dx = k \ln x$

Floella $\int \frac{1}{x} dx = \ln Ax$

Georgia $\int \frac{1}{x} dx = \ln x + c$



6 (a) (i) Explain what is wrong with Tom's answer.
[1 mark]

6 (a) (ii) Explain what is wrong with Josh's answer.
[1 mark]

[Turn over]

6 (b) Explain why Floella and Georgia's answers are equivalent. [2 marks]

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7 Consecutive terms of a sequence are related by

$$u_{n+1} = 3 - (u_n)^2$$

7 (a) In the case that $u_1 = 2$

7 (a) (i) Find u_3 [2 marks]

7 (a) (ii) Find u_{50} [1 mark]

7 (b) State a different value for u_1 which gives the same value for u_{50} as found in part (a)(ii). [1 mark]

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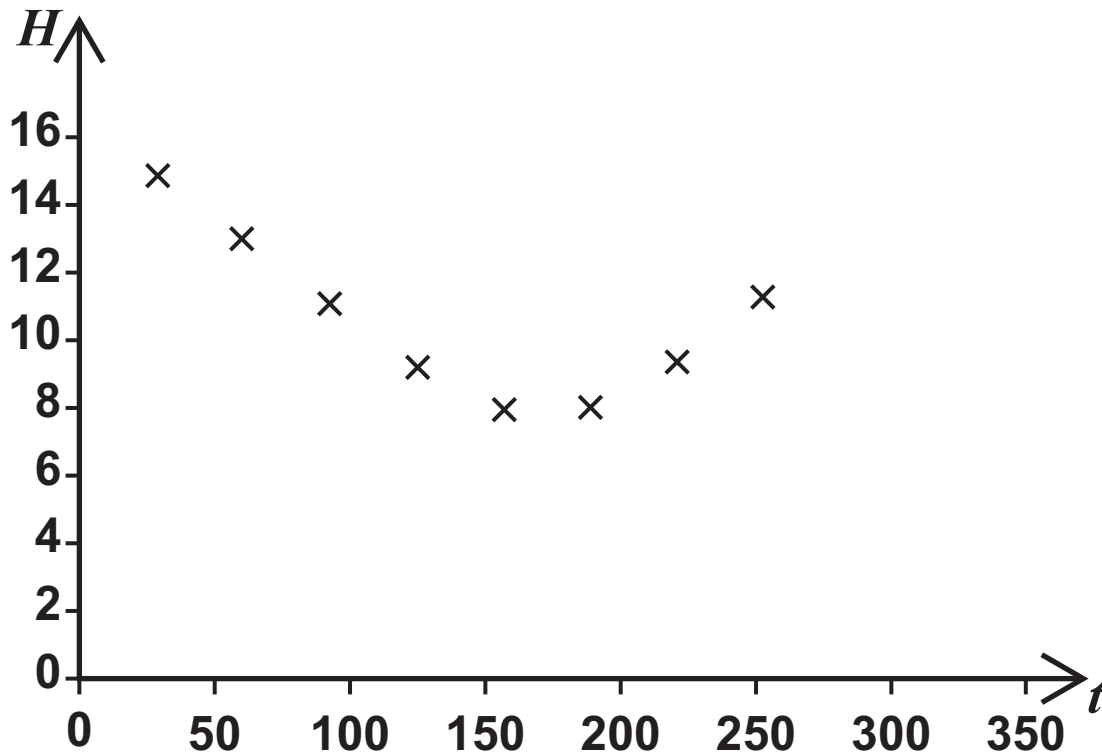


- 8 Mike, an amateur astronomer who lives in the South of England, wants to know how the number of hours of darkness changes through the year.

On various days between February and September he records the length of time, H hours, of darkness along with t , the number of days after 1 January.

His results are shown in FIGURE 1 below.

FIGURE 1



Mike models this data using the equation

$$H = 3.87 \sin\left(\frac{2\pi(t + 101.75)}{365}\right) + 11.7$$



- 8 (a) Find the minimum number of hours of darkness predicted by Mike's model. Give your answer to the nearest minute. [2 marks]

[Turn over]



8 (b)

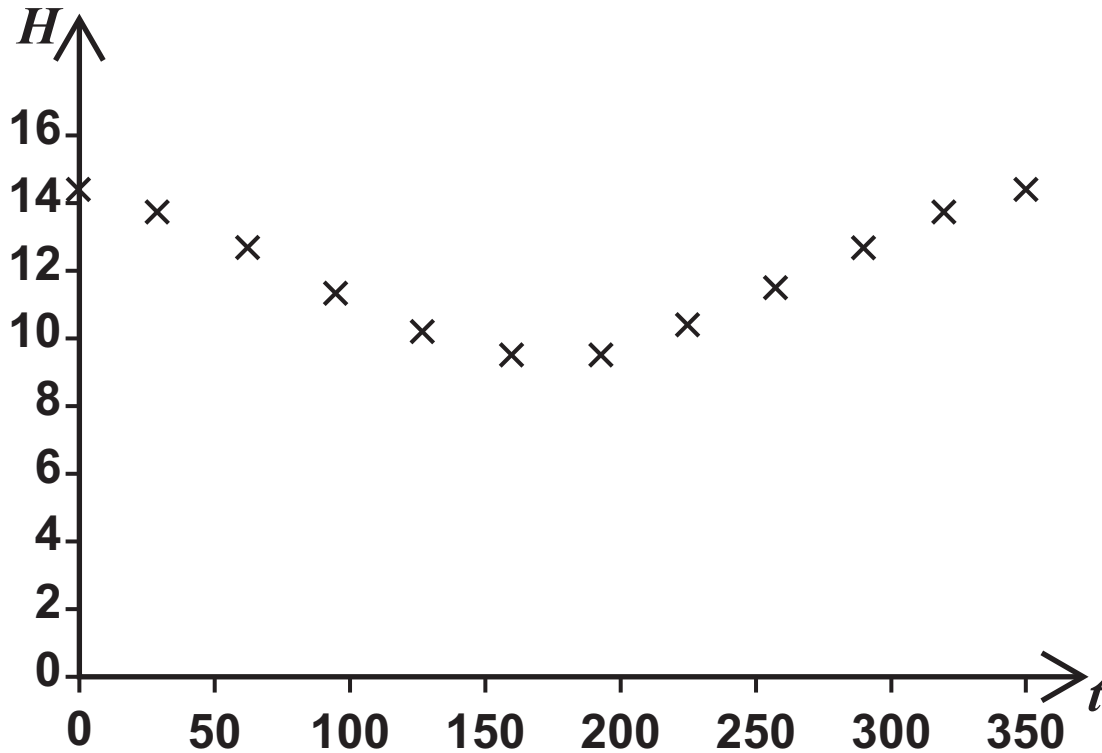
**Find the maximum number of consecutive days where the number of hours of darkness predicted by Mike's model exceeds 14
[3 marks]**



- 8 (c) Mike's friend Sofia, who lives in Spain, also records the number of hours of darkness on various days throughout the year.

Her results are shown in FIGURE 2 below.

FIGURE 2



Sofia attempts to model her data by refining Mike's model.

She decides to increase the 3.87 value, leaving everything else unchanged.

Explain whether Sofia's refinement is appropriate. [2 marks]



[Turn over]



- 9 Chloe is attempting to write $\frac{2x^2 + x}{(x + 1)(x + 2)^2}$ as partial fractions, with constant numerators.

Her incorrect attempt is shown below.

Step 1
$$\frac{2x^2 + x}{(x + 1)(x + 2)^2} \equiv \frac{A}{x + 1} + \frac{B}{(x + 2)^2}$$

Step 2
$$2x^2 + x \equiv A(x + 2)^2 + B(x + 1)$$

Step 3 Let $x = -1 \Rightarrow A = 1$
Let $x = -2 \Rightarrow B = -6$

Answer
$$\frac{2x^2 + x}{(x + 1)(x + 2)^2} \equiv \frac{1}{x + 1} - \frac{6}{(x + 2)^2}$$

- 9 (a) (i) By using a counter example, show that the answer obtained by Chloe cannot be correct. [2 marks]



9 (a) (ii) Explain her mistake in Step 1. [1 mark]

[Turn over]



9 (b) Write $\frac{2x^2 + x}{(x + 1)(x + 2)^2}$ as partial fractions, with constant numerators. [4 marks]



[Turn over]



10 (a) An arithmetic series is given by

$$\sum_{r=5}^{20} (4r + 1)$$

10 (a) (i) Write down the first term of the series. [1 mark]

10 (a) (ii) Write down the common difference of the series. [1 mark]



10 (a)(iii) Find the number of terms of the series.
[1 mark]

[Turn over]

10 (b) A DIFFERENT arithmetic series is given by

$$\sum_{r=10}^{100} (br + c)$$

where b and c are constants.

The sum of this series is 7735

10 (b) (i) Show that $55b + c = 85$ [4 marks]

10 (b) (ii) The 40th term of the series is 4 times the 2nd term.

Find the values of b and c . [4 marks]



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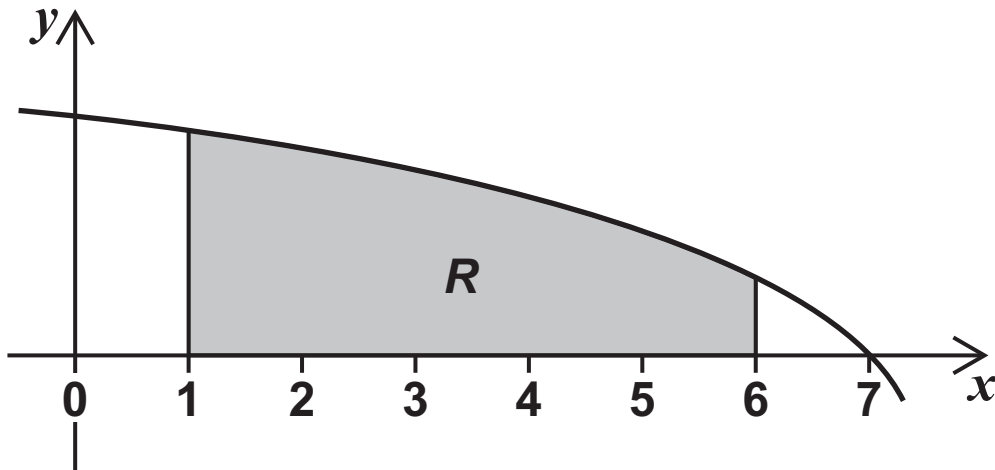


- 11 The region R enclosed by the lines $x = 1$, $x = 6$, $y = 0$ and the curve

$$y = \ln(8 - x)$$

is shown shaded in FIGURE 3 below.

FIGURE 3



All distances are measured in centimetres.

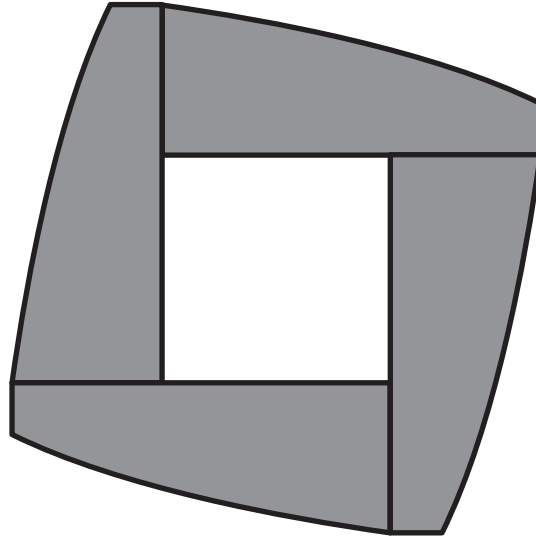
- 11 (a) Use a single trapezium to find an approximate value of the area of the shaded region, giving your answer in cm^2 to two decimal places.
[2 marks]

[Turn over]



- 11 (b) Shape *B* is made from four copies of region *R* as shown in FIGURE 4 below.

FIGURE 4



Shape *B* is cut from metal of thickness 2 mm

The metal has a density of 10.5 g/cm^3

Use the trapezium rule with SIX ordinates to calculate an approximate value of the mass of Shape *B*.

Give your answer to the nearest gram.
[5 marks]

11 (c) Without further calculation, give one reason why the mass found in part (b) may be:

11 (c) (i) an underestimate. [1 mark]

11 (c) (ii) an overestimate. [1 mark]

[Turn over]

12 A curve C has equation

$$x^3 \sin y + \cos y = Ax$$

where A is a constant.

C passes through the point $P\left(\sqrt{3}, \frac{\pi}{6}\right)$

12 (a) Show that $A = 2$ [2 marks]

12 (b) (ii) Hence, find the gradient of the curve at P .
[2 marks]

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13 (a) (ii) Write down an expression for $ff(x)$. [1 mark]

[Turn over]



[Turn over]

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14(b) (i) Show that

$$f'(x) = \frac{3^x(1 + x \ln 9)}{2\sqrt{x}}$$

[3 marks]

[Turn over]

14(b) (ii) Use the Newton–Raphson method with $x_1 = 1$ to find x_3 , an approximation for α .

Give your answer to five decimal places.
[2 marks]

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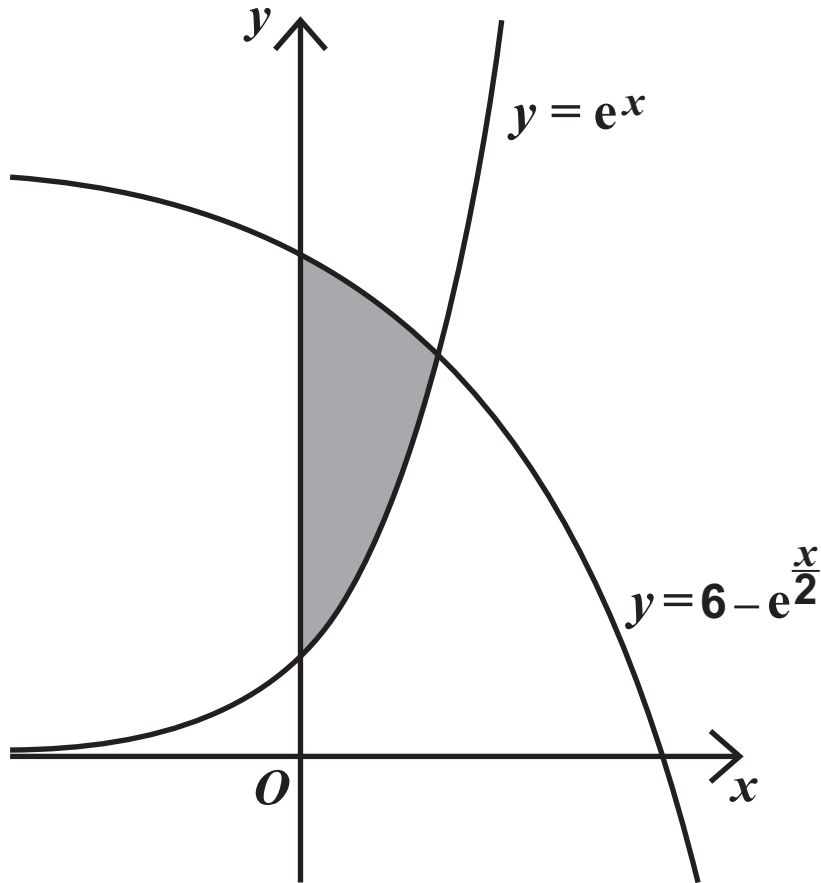


14(b)(iii) Explain why the Newton–Raphson method fails to find α with $x_1 = 0$ [2 marks]

[Turn over]



- 15 The region enclosed between the curves $y = e^x$, $y = 6 - e^{\frac{x}{2}}$ and the line $x = 0$ is shown shaded in the diagram below.



Show that the exact area of the shaded region is

$$6 \ln 4 - 5$$

Fully justify your answer. [10 marks]



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END OF QUESTIONS



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