A

**Surname** _____

**Other Names** _____

**Centre Number** _____

**Candidate Number** _____

**Candidate Signature** _____

**I declare this is my own work.**

# A-level
# MATHEMATICS

## Paper 3

**7357/3**

**Friday 12 June 2020 Afternoon**

**Time allowed: 2 hours**

**At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.**

**[Turn over]**

JUN207357301

- **You must have the AQA Formulae for A-level Mathematics booklet.**
- **You should have a graphical or scientific calculator that meets the requirements of the specification.**

## INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question.**
- **Do NOT write on blank pages.**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work that you do not want to be marked.**

## INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

## ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

**DO NOT TURN OVER UNTIL TOLD TO DO SO**

## SECTION A

**Answer ALL questions in the spaces provided.**

**1** Given that

$$\int_0^{10} f(x)\,dx = 7$$

deduce the value of

$$\int_0^{10} \left( f(x) + 1 \right) dx$$

Circle your answer. [1 mark]

−3 7 8 17

2 Given that

$$6\cos\theta + 8\sin\theta \equiv R\cos(\theta + \alpha)$$

find the value of $R$.

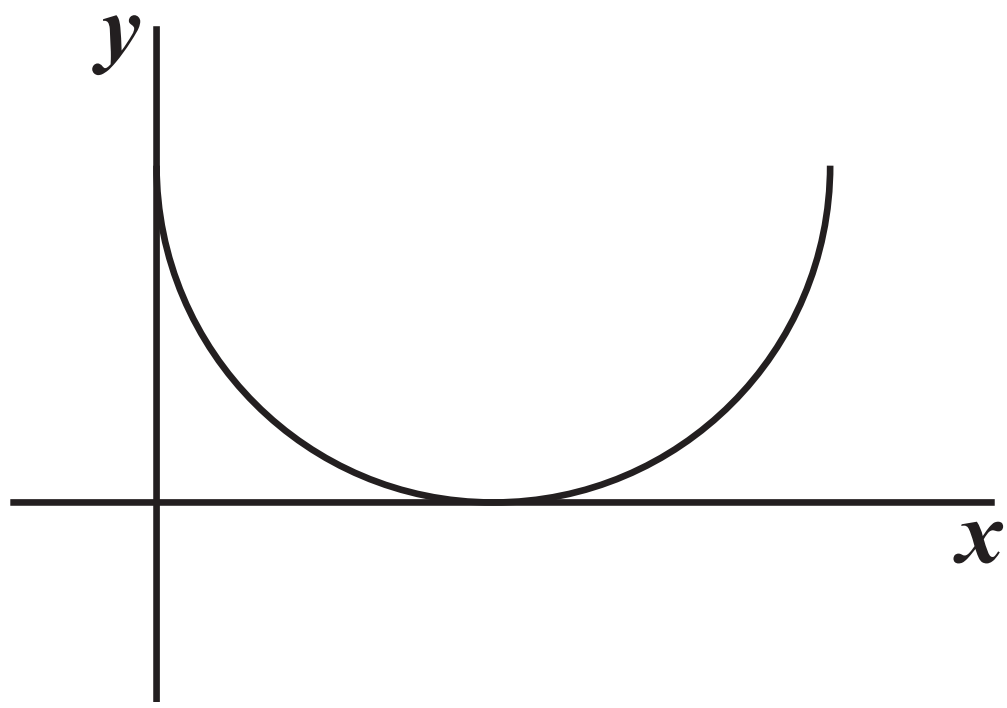
Circle your answer. [1 mark]

6 8 10 14

**3** **Determine which one of these graphs does NOT represent $y$ as a function of $x$.**

**Tick (✓) ONE box. [1 mark]**

☐

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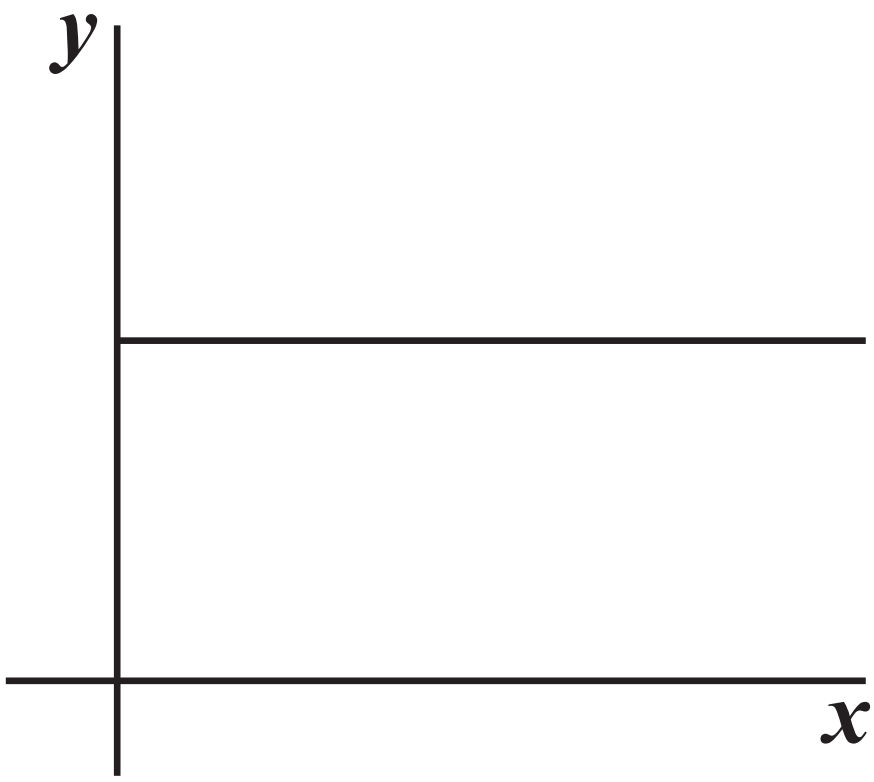


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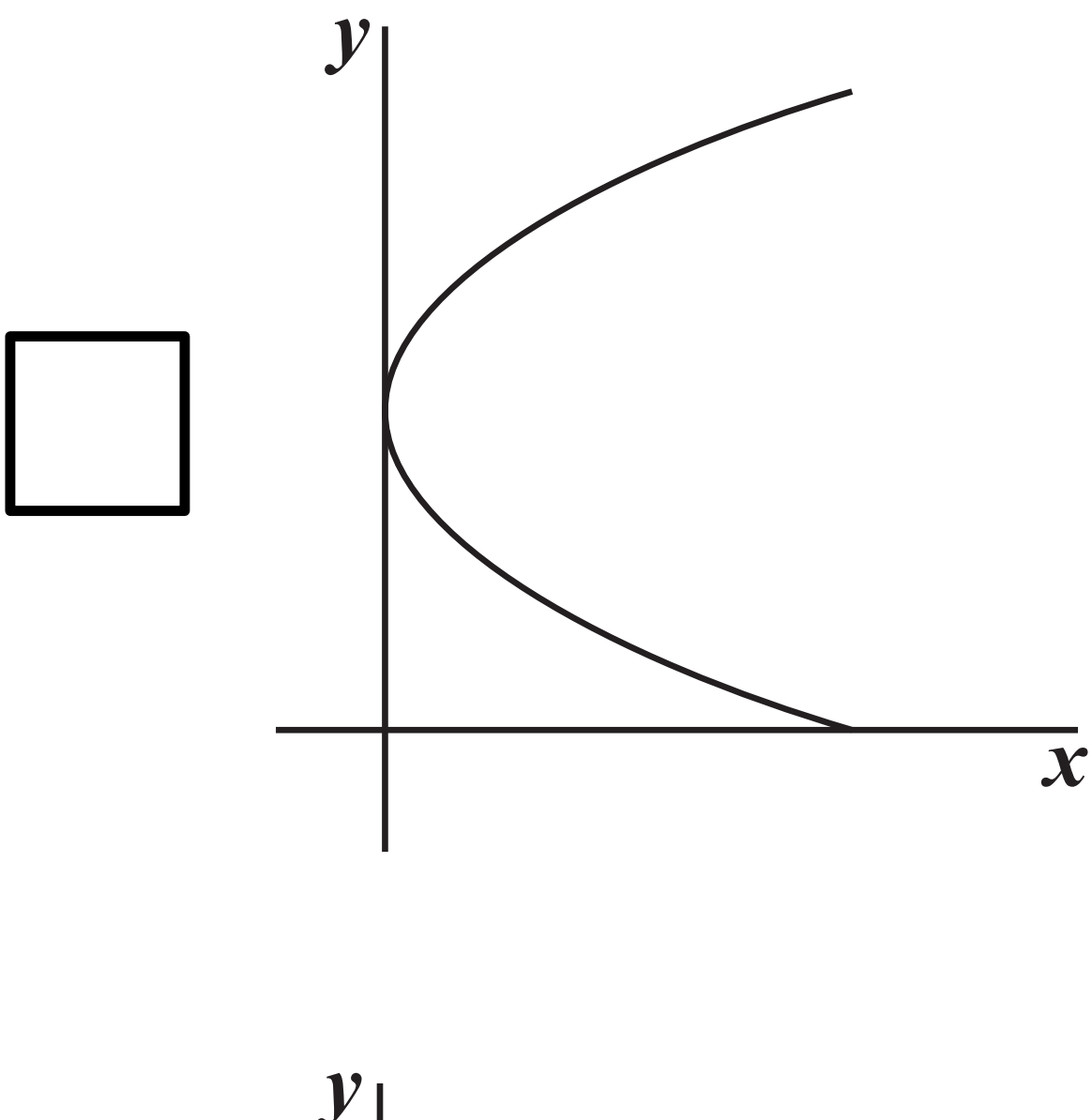
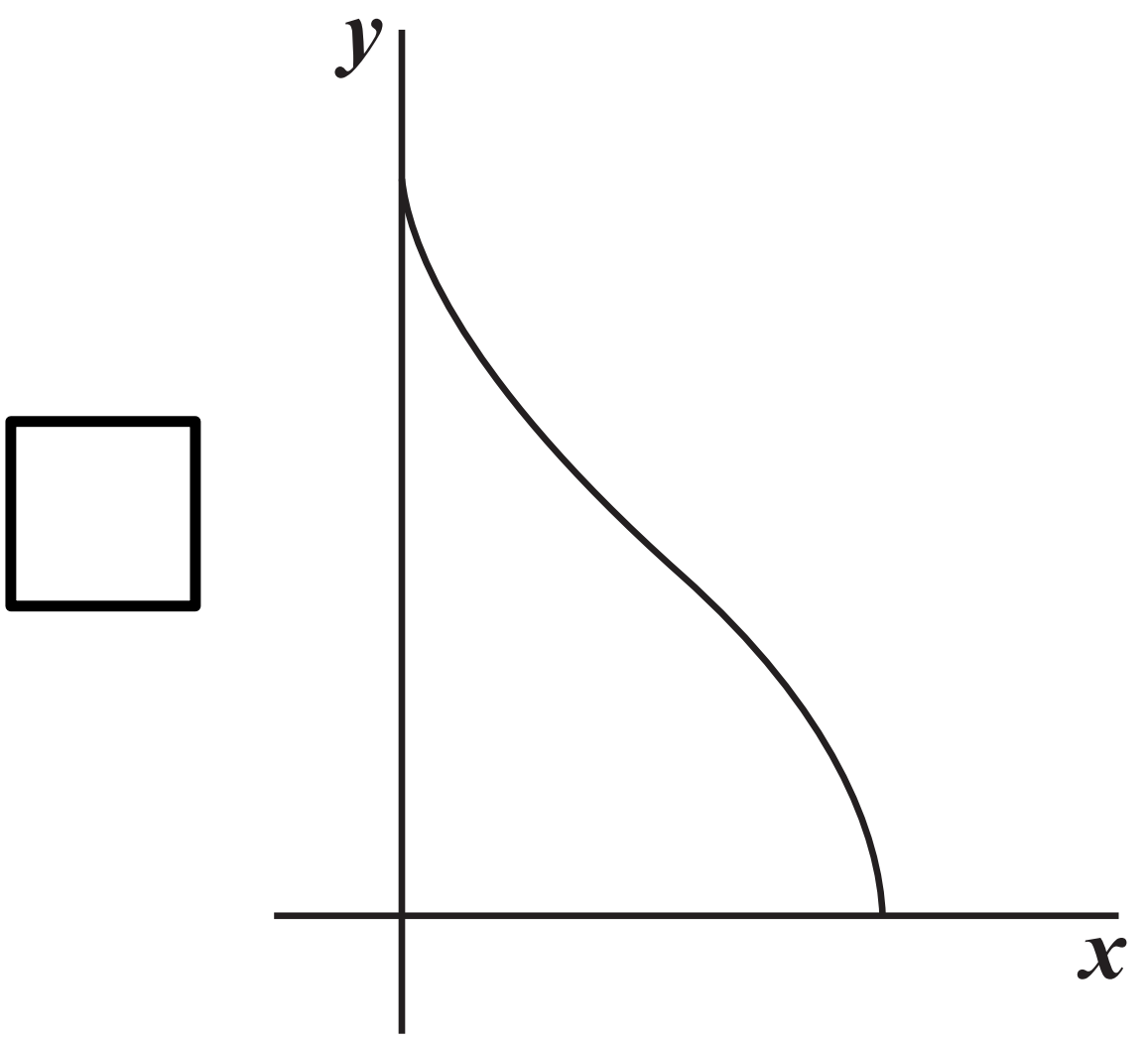



[Turn over]

**4** $\mathrm{p}(x) = 4x^3 - 15x^2 - 48x - 36$

**4 (a)** **Use the factor theorem to prove that $x - 6$ is a factor of $\mathrm{p}(x)$. [2 marks]**

[Turn over]

**4(b) (i) Prove that the graph of $y = \mathbf{p}(x)$ intersects the $x$-axis at exactly one point. [4 marks]**

[Turn over]

12

4(b) (ii) State the coordinates of this point of intersection. [1 mark]

5 The number of radioactive atoms, $N$, in a sample of a sodium isotope after time $t$ hours can be modelled by

$$N = N_0 e^{-kt}$$

where $N_0$ is the initial number of radioactive atoms in the sample and $k$ is a positive constant.

The model remains valid for large numbers of atoms.

5 (a) It takes 15.9 hours for half of the sodium atoms to decay.

Determine the number of days required for at least 90% of the number of atoms in the original sample to decay. [5 marks]

[Turn over]

16

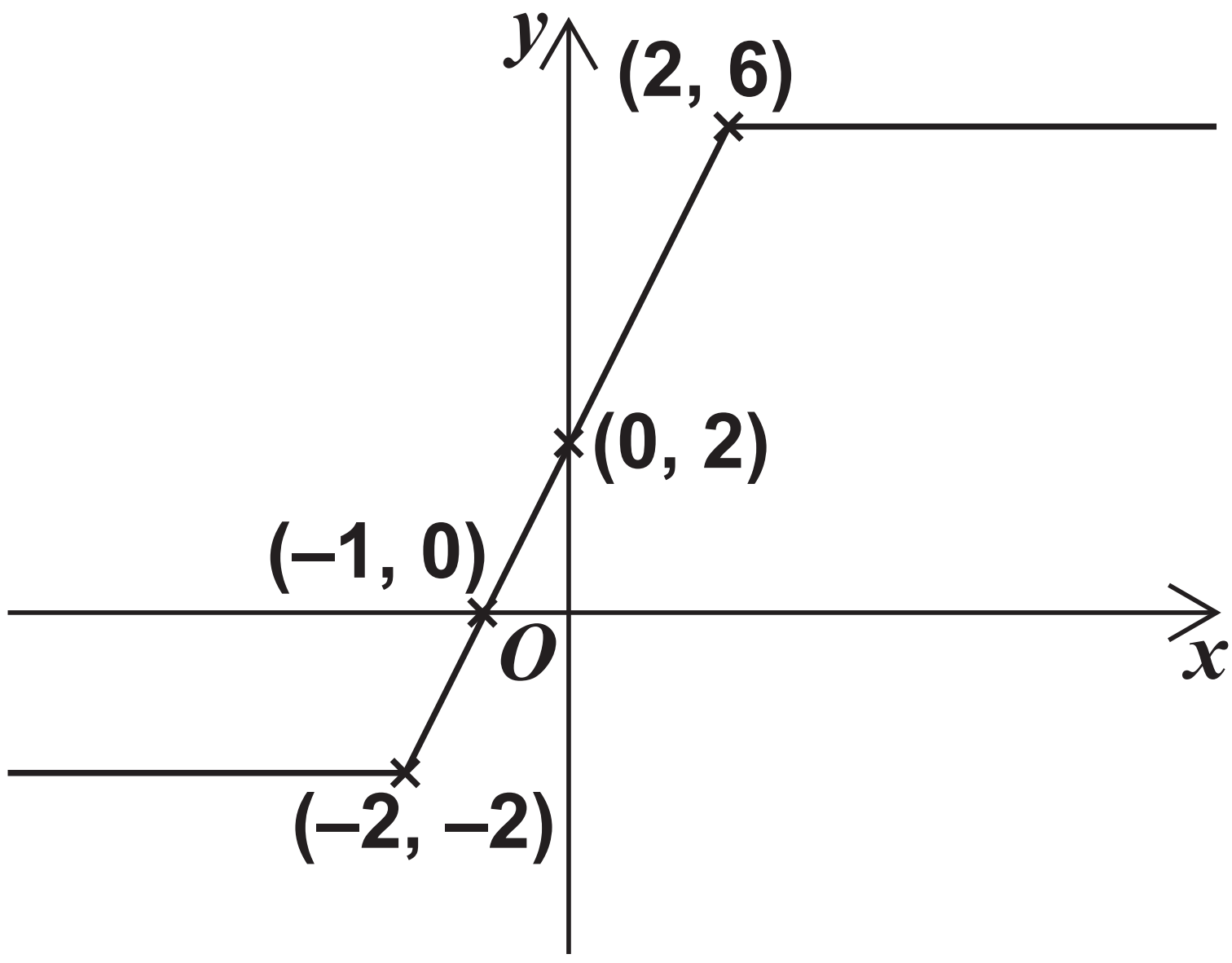
5(b) Find the percentage of the atoms remaining after the first week.

Give your answer to two significant figures. [2 marks]

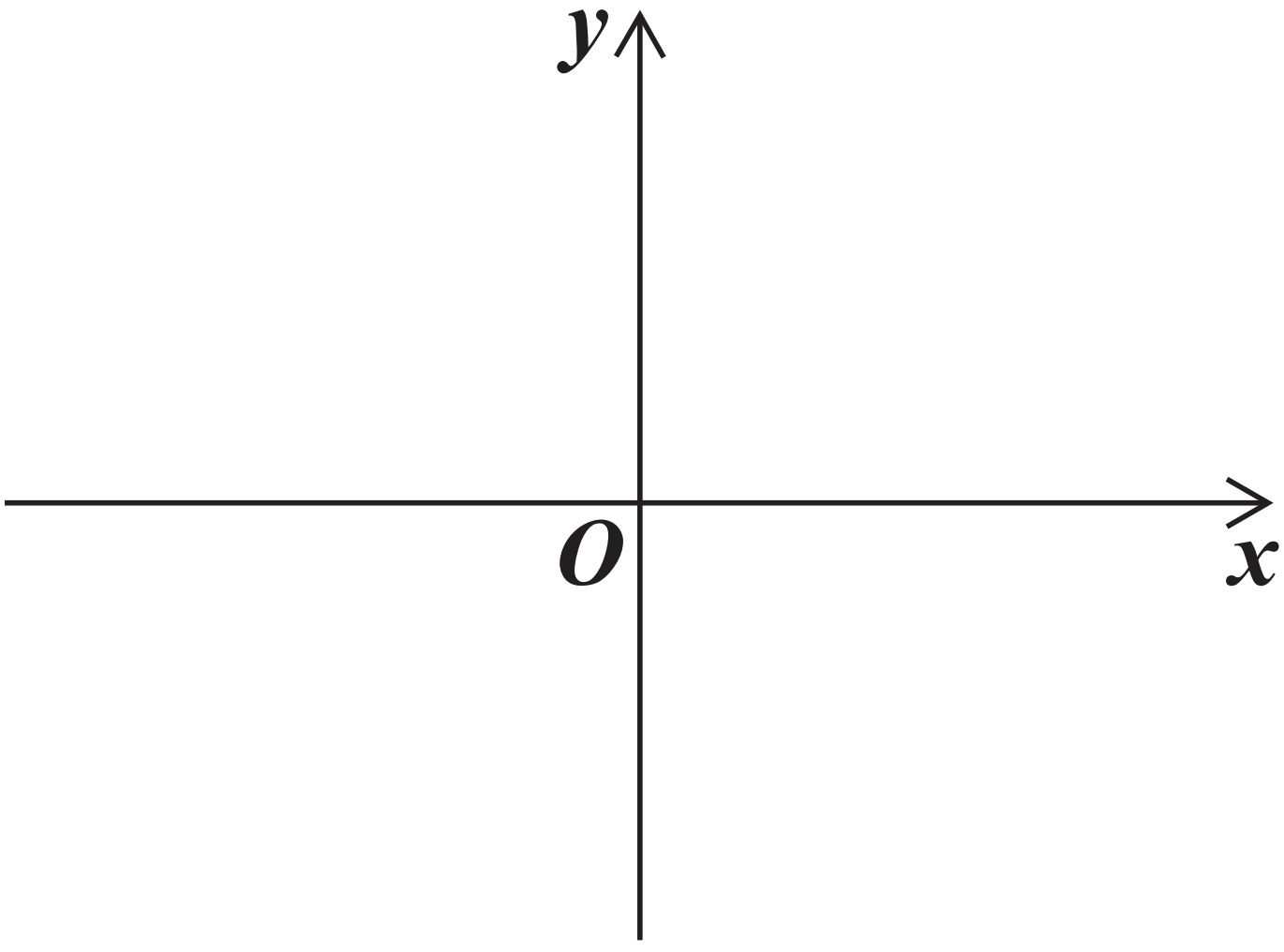
5 (c) Explain why the model can only provide an estimate for the number of remaining atoms.
[1 mark]

5(d) Explain why the model is invalid in the long run. [1 mark]

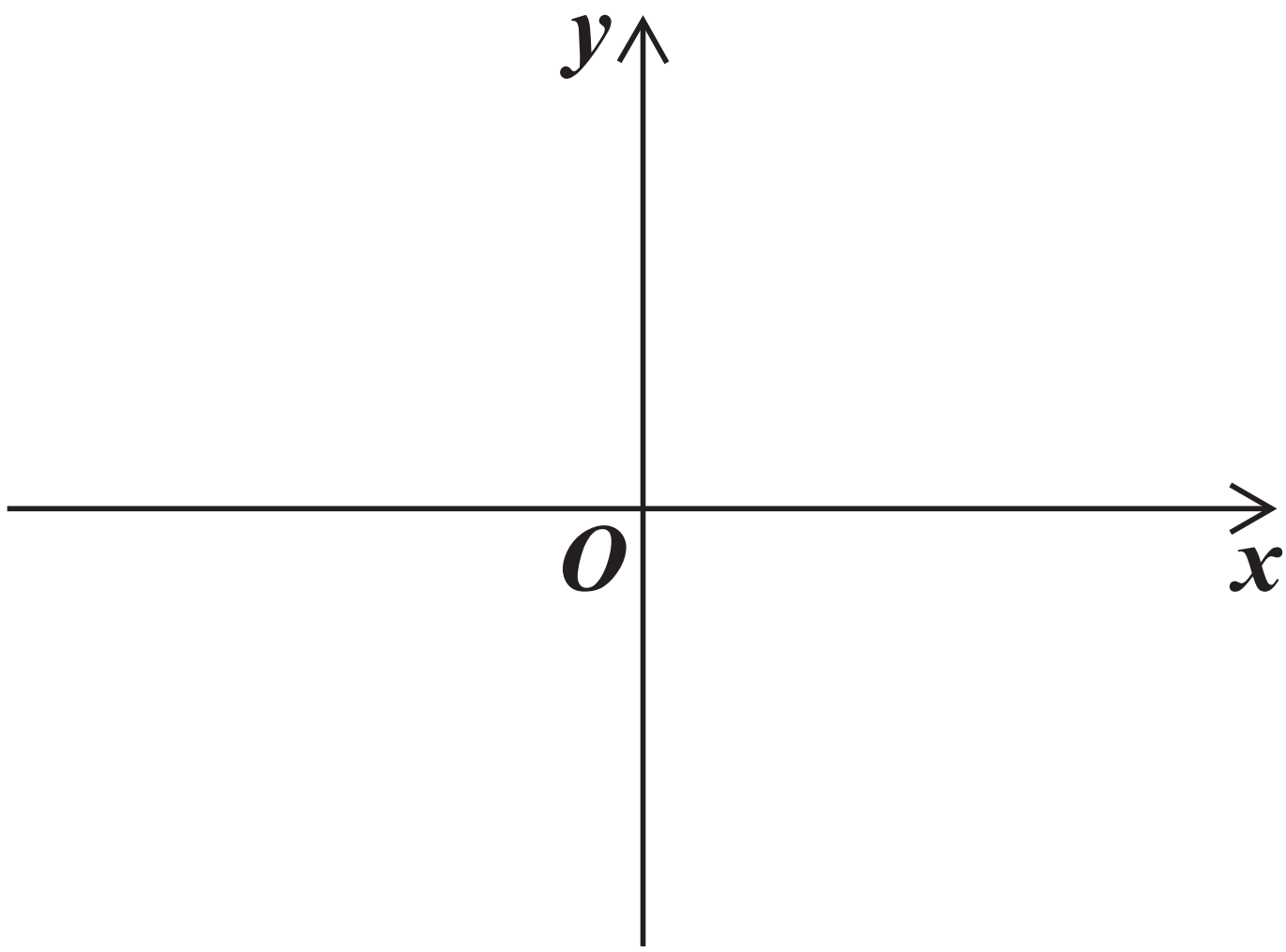
6 The graph of $y = \mathrm{f}(x)$ is shown below.



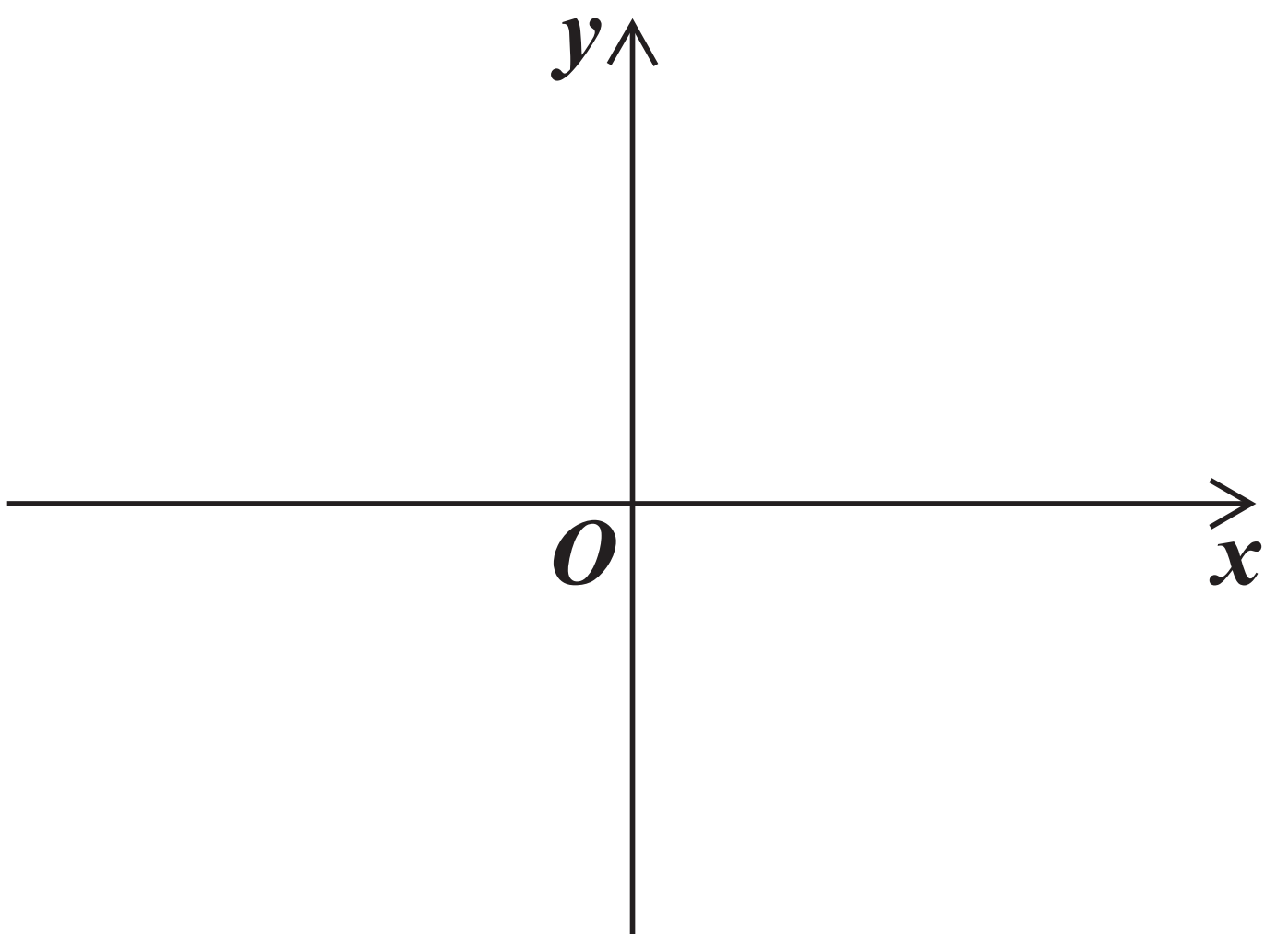

6 (a) Sketch the graph of $y = \mathrm{f}(-x)$ [2 marks]



6(b) Sketch the graph of $y = 2f(x) - 4$
[2 marks]




6(c) Sketch the graph of $y = f'(x)$
[3 marks]






[Turn over]

7 (a) Using $^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$ show that

$$^{n}C_{2} = \frac{n(n-1)}{2}$$

[2 marks]

7(b) (i) Show that the equation

$$2 \times {}^{n}C_{4} = 51 \times {}^{n}C_{2}$$

simplifies to

$$n^2 - 5n - 300 = 0$$

[3 marks]

[Turn over]

7(b) (ii) Hence, solve the equation

$$2 \times {}^{n}C_{4} = 51 \times {}^{n}C_{2}$$

[2 marks]

[Turn over]

8 The sum to infinity of a geometric series is 96

The first term of the series is less than 30

The second term of the series is 18

8 (a) Find the first term and common ratio of the series. [5 marks]

[Turn over]

8(b) (i) Show that the $n$th term of the series, $u_n$, can be written as

$$u_n = \frac{3^n}{2^{2n-5}}$$

[4 marks]

[Turn over]

8(b) (ii) Hence show that

$$\log_3 u_n = n(1 - 2\log_3 2) + 5\log_3 2$$

[3 marks]

[Turn over]

9 (a) For $\cos\theta \neq 0$, prove that

$$\text{cosec}\,2\theta + \cot 2\theta = \cot\theta$$

[4 marks]

[Turn over]

9(b) Explain why

$\cot\theta \neq \operatorname{cosec} 2\theta + \cot 2\theta$

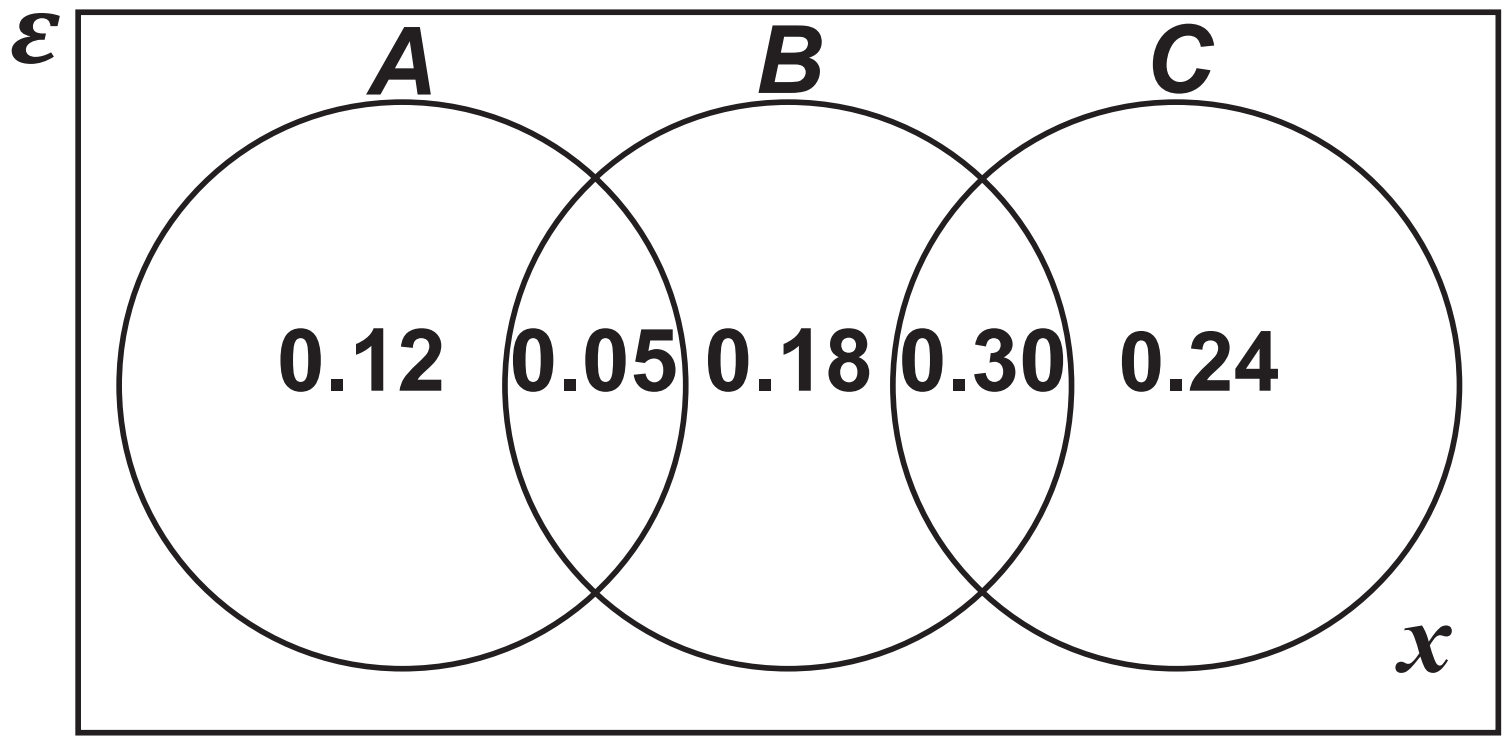
when $\cos\theta = 0$ [1 mark]

[Turn over]

# SECTION B

**Answer ALL questions in the spaces provided.**

**10** **The probabilities of events *A*, *B* and *C* are related, as shown in the Venn diagram below.**




**Find the value of $x$.**

**Circle your answer. [1 mark]**

**0.11** **0.46** **0.54** **0.89**

**11** The table below shows the temperature on Mount Everest on the first day of each month.

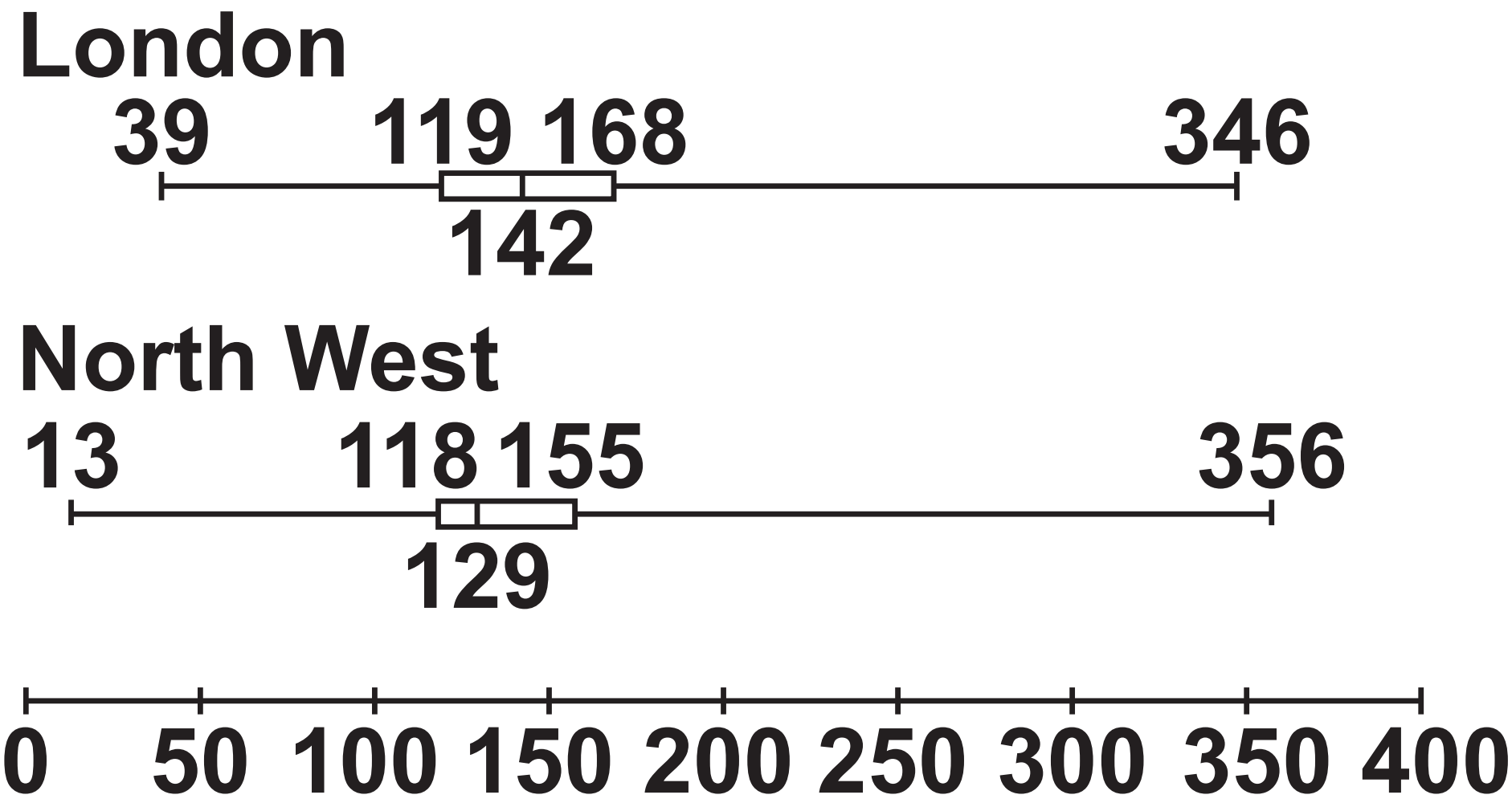
| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temperature (°C) | −17 | −16 | −14 | −9 | −2 | 2 | 6 | 5 | −3 | −4 | −11 | −18 |

**Calculate the standard deviation of these temperatures.**

**Circle your answer.** **[1 mark]**

−6.75 5.82 8.24 67.85

12 The box plot below summarises the $CO_2$ emissions, in g/km, for cars in the Large Data Set from the London and North West regions.




12 (a) Using the box plot, give ONE comparison of central tendency and ONE comparison of spread for the two regions. [2 marks]

Comparison of central tendency

Comparison of spread

[Turn over]

12(b) **Jaspal, an environmental researcher, used all of the data in the Large Data Set to produce a statistical comparison of the $CO_2$ and CO emissions in regions of England.**

**Using your knowledge of the Large Data Set, give TWO reasons why his conclusions may be invalid. [2 marks]**

[Turn over]

13 Diedre is a head teacher in a school which provides primary, secondary and sixth-form education.

There are 200 teachers in her school.

The number of teachers in each level of education along with their gender is shown in the table below.

| | Primary | Secondary | Sixth-form |
|---|---|---|---|
| Male | 9 | 24 | 23 |
| Female | 35 | 85 | 24 |

13 (a) A teacher is selected at random. Find the probability that:

13 (a) (i) the teacher is female [1 mark]

13 (a) (ii) the teacher is NOT a sixth-form teacher. [1 mark]

13(b) Given that a randomly chosen teacher is male, find the probability that this teacher is NOT a primary teacher.
[2 marks]

13 (c) Diedre wants to select three different teachers at random to be part of a school project.

Calculate the probability that all three chosen are secondary teachers. [2 marks]

14 It is known that a hospital has a mean waiting time of 4 hours for its Accident and Emergency (A&E) patients.

After some new initiatives were introduced, a random sample of 12 patients from the hospital's A&E Department had the following waiting times, in hours.

4.25 3.90 4.15 3.95 4.20 4.15

5.00 3.85 4.25 4.05 3.80 3.95

Carry out a hypothesis test at the 10% significance level to investigate whether the mean waiting time at this hospital's A&E department has changed.

**You may assume that the waiting times are normally distributed with standard deviation 0.8 hours. [7 marks]**

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**[Turn over]**

15 A political party is holding an election to choose a new leader.

A statistician within the party decides to sample 70 party members to find their opinions of the leadership candidates.

There are 4735 members under 30 years old and 8565 members 30 years old and over.

The statistician wants to use a sample of 70 party members in the survey.

He decides to use a random stratified sample.

15 (a) Calculate how many of each age group should be included in his sample. [2 marks]

[Turn over]

15(b) **Explain how he could collect the random sample of members under 30 years old. [3 marks]**

[Turn over]

16 An educational expert found that the correlation coefficient between the hours of revision and the scores achieved by 25 students in their A-level exams was 0.379

Her data came from a bivariate normal distribution.

Carry out a hypothesis test at the 1% significance level to determine if there is a positive correlation between the hours of revision and the scores achieved by students in their A-level exams.

The critical value of the correlation coefficient is 0.4622
[4 marks]

[Turn over]

17 The lifetime of Zaple smartphone batteries, $X$ hours, is normally distributed with mean 8 hours and standard deviation 1.5 hours.

17 (a) (i) Find $P(X \neq 8)$ [1 mark]

17 (a) (ii) Find $P(6 < X < 10)$ [1 mark]

17(b) Determine the lifetime exceeded by 90% of Zaple smartphone batteries. [2 marks]

17 (c) A different smartphone, Kaphone, has its battery's lifetime, $Y$ hours, modelled by a normal distribution with mean 7 hours and standard deviation $\sigma$.

25% of randomly selected Kaphone batteries last less than 5 hours.

Find the value of $\sigma$, correct to three significant figures.
[4 marks]

[Turn over]

18 Tiana is a quality controller in a clothes factory. She checks for four possible types of defects in shirts.

Of the shirts with defects, the proportion of each type of defect is as shown in the table below.

| Type of defect | Colour | Fabric | Sewing | Sizing |
|---|---|---|---|---|
| Probability | 0.25 | 0.30 | 0.40 | 0.05 |

Shirts with defects are packed in boxes of 30 at random.

18 (a) Find the probability that:

18 (a) (i) a box contains exactly 5 shirts with a colour defect [2 marks]

18 (a) (ii) a box contains fewer than 15 shirts with a sewing defect [2 marks]

18 (a) (iii) a box contains at least 20 shirts which do NOT have a fabric defect. [3 marks]

[Turn over]

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18 (b) Tiana wants to investigate the proportion, $p$, of defective shirts with a fabric defect.

She wishes to test the hypotheses

$$\mathrm{H}_0 : p = 0.3$$

$$\mathrm{H}_1 : p < 0.3$$

She takes a random sample of 60 shirts with a defect and finds that $x$ of them have a fabric defect.

18 (b) (i) Using a 5% level of significance, find the critical region for $x$.
[5 marks]

[Turn over]

18 (b) (ii) In her sample she finds 13 shirts with a fabric defect.

Complete the test stating her conclusion in context. [2 marks]

END OF QUESTIONS

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| **TOTAL** | |

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