A

AQA

Surname ____________________

Other Names ____________________

Centre Number ____________________

Candidate Number ____________________

Candidate Signature ____________________
I declare this is my own work.

AS
FURTHER MATHEMATICS
Paper 1

7366/1

Monday 11 May 2020 Afternoon

Time allowed: 1 hour 30 minutes

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

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JUN2073661O1

For this paper you must have:

- **the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.**
- **You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)**

INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question. Do NOT write on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 Express the complex number $1 - \mathrm{i}\sqrt{3}$ in modulus-argument form.

Tick (✓) ONE box. [1 mark]

☐ $2\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$

☐ $2\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right)$

☐ $2\left(\cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$

☐ $2\left(\cos\left(-\frac{2\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{2\pi}{3}\right)\right)$

2 **Given that 1 – i is a root of the equation $z^3 - 3z^2 + 4z - 2 = 0$, find the other two roots.**

Tick (✓) ONE box. [1 mark]

- [ ] **–1 + i and –1**
- [ ] **1 + i and 1**
- [ ] **–1 + i and 1**
- [ ] **1 + i and –1**

[Turn over]

3 Given $(x - 1)(x - 2)(x - a) < 0$ and $a > 2$

Find the set of possible values of x.

Tick (✓) ONE box. [1 mark]

☐ $\{x : x < 1\} \cup \{x : 2 < x < a\}$

☐ $\{x : 1 < x < 2\} \cup \{x : x > a\}$

☐ $\{x : x < -a\} \cup \{x : -2 < x < -1\}$

☐ $\{x : -a < x < -2\} \cup \{x : x > -1\}$

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[Turn over]

4 **The matrices A and B are such that**

$$\mathbf{A} = \begin{bmatrix} 2 & a & 3 \\ 0 & -2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -3 \\ -2 & 4a \\ 0 & 5 \end{bmatrix}$$

4 (a) **Find the product AB in terms of a. [2 marks]**

4 (b) Find the determinant of AB in terms of a.
[1 mark]

4 (c) **Show that AB is singular when $a = -1$**
[2 marks]

[Turn over]

5 (a) **Show that**

$$r^2(r + 1)^2 - (r - 1)^2r^2 = pr^3$$

where p is an integer to be found. [1 mark]

5 (b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n + 1)^2$$

[3 marks]

[Turn over]

6 Anna has been asked to describe the transformation given by the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

She writes her answer as follows:

The transformation is a rotation about the x-axis through an angle of θ, where

$$\sin\theta = \frac{1}{2} \quad \text{and} \quad -\sin\theta = -\frac{1}{2}$$

$$\theta = 30°$$

Identify and correct the error in Anna's work.
[2 marks]

[Turn over]

7 **Prove by induction that, for all integers $n \geq 1$, the expression $7^n - 3^n$ is divisible by 4**
[4 marks]

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[Turn over]

8 (a) **Prove that**

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

[5 marks]

[Turn over]

8 (b) **Prove that the graphs of**

y **= sinh** x **and** y **= cosh** x

do NOT intersect. [3 marks]

[Turn over]

9 The quadratic equation $2x^2 + px + 3 = 0$ has two roots, α and β, where $\alpha > \beta$.

9 (a) (i) Write down the value of $\alpha\beta$. [1 mark]

9 (a) (ii) Express $\alpha + \beta$ in terms of p. [1 mark]

9 (b) Hence find $(\alpha - \beta)^2$ in terms of p. [2 marks]

9 (c) **HENCE find, in terms of p, a quadratic equation with roots $\alpha - 1$ and $\beta + 1$ [4 marks]**

[Turn over]

10 (a) **Show that the equation**

$$y = \frac{3x - 5}{2x + 4}$$

can be written in the form

$$(x + a)(y + b) = c$$

where a, b and c are integers to be found.
[3 marks]

10 (b) Write down the equations of the asymptotes of the graph of

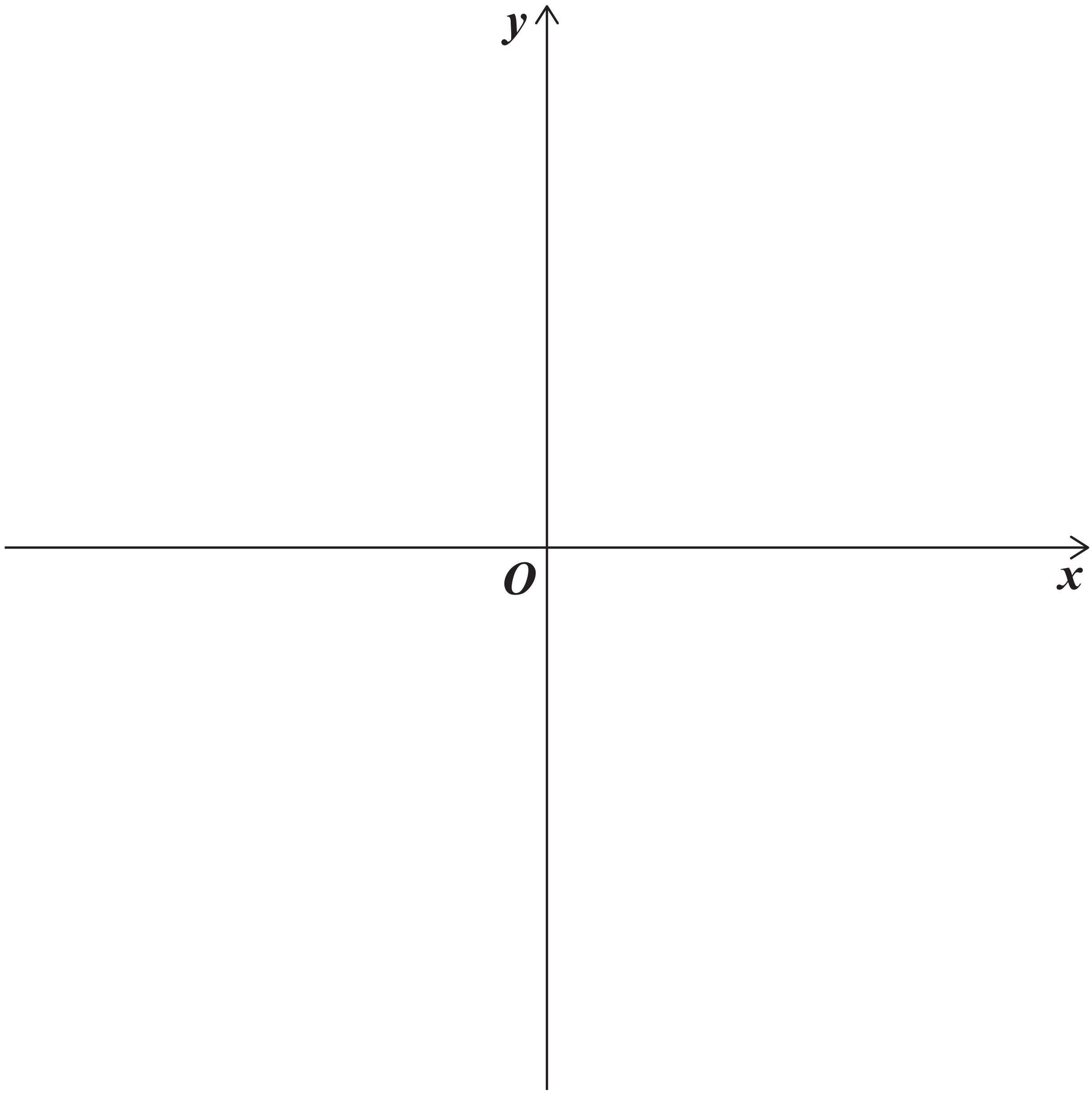
$$y = \frac{3x - 5}{2x + 4}$$

[2 marks]

10 (c) **Sketch, on the axes provided, the graph of**

$$y = \frac{3x - 5}{2x + 4}$$

[3 marks]



[Turn over]

11 **Sketch the polar graph of**

$$r = \sinh\theta + \cosh\theta$$

for $0 \le \theta \le 2\pi$ [3 marks]

[Turn over]

12 **The mean value of the function f over the interval $1 \le x \le 5$ is m.**

The graph of $y = \mathrm{g}(x)$ is a reflection in the x-axis of $y = \mathrm{f}(x)$.

The graph of $y = \mathrm{h}(x)$ is a translation of $y = \mathrm{g}(x)$ by $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Determine, in terms of m, the mean value of the function h over the interval $4 \le x \le 8$
[2 marks]

[Turn over]

13 Line l_1 has equation

$$\frac{x-2}{3} = \frac{1-2y}{4} = -z$$

and line l_2 has equation

$$\mathbf{r} = \begin{bmatrix} -7 \\ 4 \\ -2 \end{bmatrix} + \mu \begin{bmatrix} 12 \\ a+3 \\ 2b \end{bmatrix}$$

13 (a) In the case when l_1 and l_2 are parallel, show that $a = -11$ and find the value of b. [4 marks]

13 (b) **In a DIFFERENT case, the lines l_1 and l_2 intersect at exactly one point, and the value of b is 3**

Find the value of a. [5 marks]

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[Turn over]

14 (a) Given

$$\frac{x+7}{x+1} \leq x+1$$

show that

$$\frac{(x+a)(x+b)}{x+c} \geq 0$$

where a, b, and c are integers to be found.
[4 marks]

14 (b) **Briefly explain why this statement is incorrect.**

$$\frac{(x + p)(x + q)}{x + r} \geq 0 \Leftrightarrow (x + p)(x + q)(x + r) \geq 0$$

[1 mark]

14 (c) **Solve**

$$\frac{x+7}{x+1} \leq x+1$$

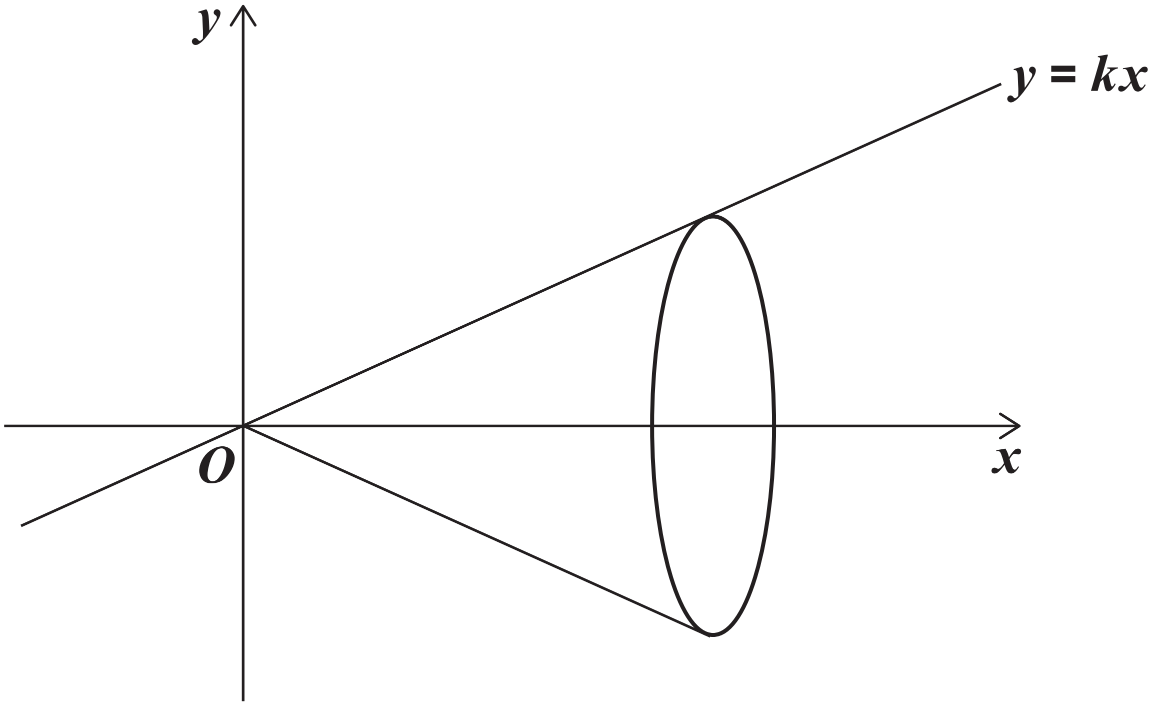
[2 marks]

[Turn over]

15 A segment of the line $y = kx$ is rotated about the x-axis to generate a cone with vertex O.

The distance of O from the centre of the base of the cone is h.

The radius of the base of the cone is r.



15 (a) Find k in terms of r and h. [1 mark]

15 (b) **Use calculus to prove that the volume of the cone is**

$$\frac{1}{3}\pi r^2 h$$

[3 marks]

[Turn over]

16 **A and B are non-singular square matrices.**

16 (a) **Write down the product AA^{-1} as a single matrix. [1 mark]**

16 (b) **M is a matrix such that M = AB.**

Prove that $M^{-1} = B^{-1}A^{-1}$ [3 marks]

17 The polar equation of the circle C is

$$r = a(\cos\theta + \sin\theta)$$

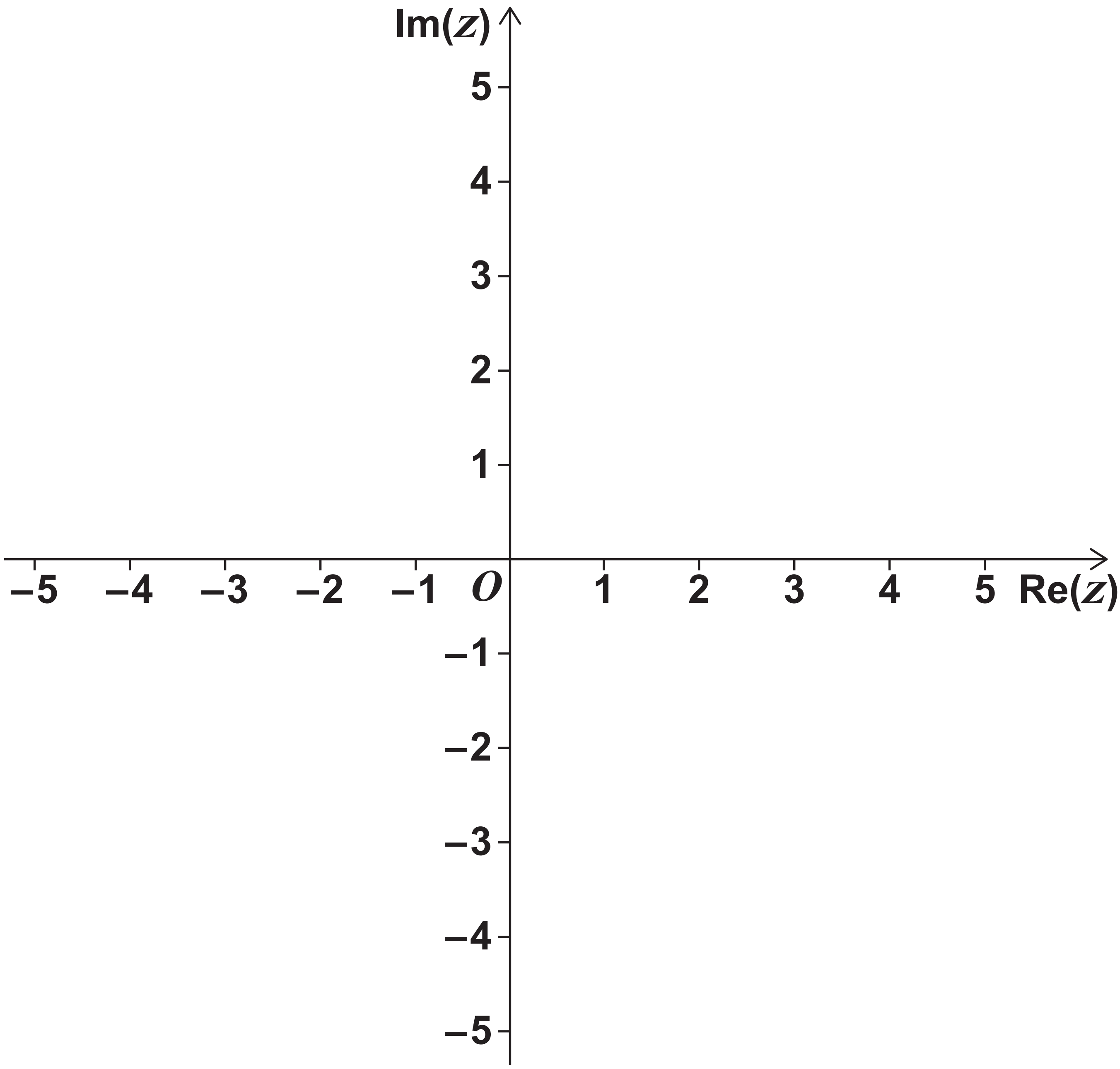
Find, in terms of a, the radius of C.

Fully justify your answer. [4 marks]

18 The locus of points L_1 satisfies the equation $|z| = 2$

The locus of points L_2 satisfies the equation $\arg(z + 4) = \frac{\pi}{4}$

18 (a) Sketch L_1 on the Argand diagram below.
[1 mark]



18 (b) Sketch L_2 on the Argand diagram on page 60. **[1 mark]**

18 (c) The complex number $a + \mathrm{i}b$, where a and b are real, lies on L_1

The complex number $c + \mathrm{i}d$, where c and d are real, lies on L_2

Calculate the least possible value of the expression

$$(c - a)^2 + (d - b)^2$$

[3 marks]

END OF QUESTIONS

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For Examiner's Use	
Question	Mark
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