AQA

Please write clearly in block capitals.

Centre number	
Candidate number	
Surname	
Forename(s)	
Candidate signature	I declare this is my own work.

A-level
FURTHER MATHEMATICS
Paper 2

Thursday 4 June 2020 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

Answer **all** questions in the spaces provided.

1 Three of the four expressions below are equivalent to each other.

Which of the four expressions is **not** equivalent to any of the others?

Circle your answer.

[1 mark]

$\mathbf{a} \times (\mathbf{a} + \mathbf{b})$ $\quad$ $(\mathbf{a} + \mathbf{b}) \times \mathbf{b}$ $\quad$ $(\mathbf{a} - \mathbf{b}) \times \mathbf{b}$ $\quad$ $\mathbf{a} \times (\mathbf{a} - \mathbf{b})$

2 Given that $\arg(a + b\mathrm{i}) = \varphi$, where a and b are positive real numbers and $0 < \varphi < \frac{\pi}{2}$, three of the following four statements are correct.

Which statement is **not** correct?

Tick (✓) **one** box.

[1 mark]

$\arg(-a - b\mathrm{i}) = \pi - \varphi$ ☐

$\arg(a - b\mathrm{i}) = -\varphi$ ☐

$\arg(b + a\mathrm{i}) = \frac{\pi}{2} - \varphi$ ☐

$\arg(b - a\mathrm{i}) = \varphi - \frac{\pi}{2}$ ☐

3 Find the gradient of the tangent to the curve

$$y = \sin^{-1} x$$

at the point where $x = \dfrac{1}{5}$

Circle your answer.

[1 mark]

$\dfrac{5\sqrt{6}}{12}$ $\qquad$ $\dfrac{2\sqrt{6}}{5}$ $\qquad$ $\dfrac{4\sqrt{3}}{25}$ $\qquad$ $\dfrac{25}{24}$

Turn over for the next question

Do not write outside the box

4 The matrices **A** and **B** are defined as follows:

$$\mathbf{A} = \begin{bmatrix} x+1 & 2 \\ x+2 & -3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} x-4 & x-2 \\ 0 & -2 \end{bmatrix}$$

Show that there is a value of x for which $\mathbf{AB} = k\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix and k is an integer to be found.

[3 marks]

5 Solve the inequality

$$\frac{2x+3}{x-1} \leq x+5$$

[5 marks]

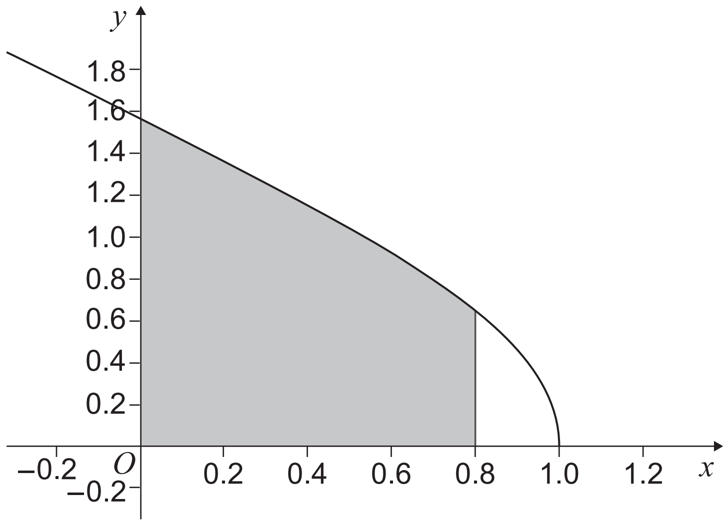
Do not write outside the box

6 Find the sum of all the integers from 1 to 999 inclusive that are not square or cube numbers.

[5 marks]

Do not write outside the box

7 The diagram shows part of the graph of $y = \cos^{-1} x$



The finite region enclosed by the graph of $y = \cos^{-1} x$, the y-axis, the x-axis and the line $x = 0.8$ is rotated by 2π radians about the x-axis.

Use Simpson's rule with five ordinates to estimate the volume of the solid formed.
Give your answer to four decimal places.

[5 marks]

Do not write outside the box

8 (a) Factorise $\begin{vmatrix} 2a+b+x & x+b & x^2+b^2 \\ 0 & a & -a^2 \\ a+b & b & b^2 \end{vmatrix}$ as fully as possible.

[6 marks]

Do not write outside the box

8 (b) The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} 13+x & x+3 & x^2+9 \\ 0 & 5 & -25 \\ 8 & 3 & 9 \end{bmatrix}$$

Under the transformation represented by **M**, a solid of volume $0.625\,\text{m}^3$ becomes a solid of volume $300\,\text{m}^3$

Use your answer to part **(a)** to find the possible values of x.

[3 marks]

Do not write outside the box

9 The matrix $\mathbf{C} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, where a and b are positive real numbers,

and $\mathbf{C}^2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Use $\mathbf{C}$ to show that $\cos\frac{\pi}{12}$ can be written in the form $\frac{\sqrt{\sqrt{m}+n}}{2}$, where m and n are integers.

[7 marks]

Turn over for the next question

10 The sequence u_1, u_2, u_3, … is defined by

$$u_1 = 0 \qquad u_{n+1} = \frac{5}{6 - u_n}$$

Prove by induction that, for all integers $n \geq 1$,

$$u_n = \frac{5^n - 5}{5^n - 1}$$

[6 marks]

Turn over for the next question

Do not write outside the box

11 (a) Starting from the series given in the formulae booklet, show that the general term of the Maclaurin series for

$$\frac{\sin x}{x} - \cos x$$

is

$$(-1)^{r+1}\frac{2r}{(2r+1)!}x^{2r}$$

[4 marks]

Do not write outside the box

11 (b) Show that

$$\lim_{x \to 0}\left[\frac{\dfrac{\sin x}{x} - \cos x}{1 - \cos x}\right] = \frac{2}{3}$$

[4 marks]

Turn over ▶

Do not write outside the box

12 (a) Given that $I = \int_a^b e^{2t} \sin t \, dt$, show that

$$I = \left[q e^{2t} \sin t + r e^{2t} \cos t \right]_a^b$$

where q and r are rational numbers to be found.

[6 marks]

Do not write outside the box

12 (b) A small object is initially at rest. The subsequent motion of the object is modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} + v = 5\mathrm{e}^{t}\sin t$$

where v is the velocity at time t.

Find the speed of the object when $t = 2\pi$, giving your answer in exact form.

[6 marks]

Turn over ▶

13 Charlotte is trying to solve this mathematical problem:

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=10e^{-2x}$$

Charlotte's solution starts as follows:

Particular integral: $y=\lambda e^{-2x}$

so

$$\frac{dy}{dx}=-2\lambda e^{-2x}$$

and

$$\frac{d^2y}{dx^2}=4\lambda e^{-2x}$$

13 (a) Show that Charlotte's method will fail to find a particular integral for the differential equation.

[2 marks]

Do not write outside the box

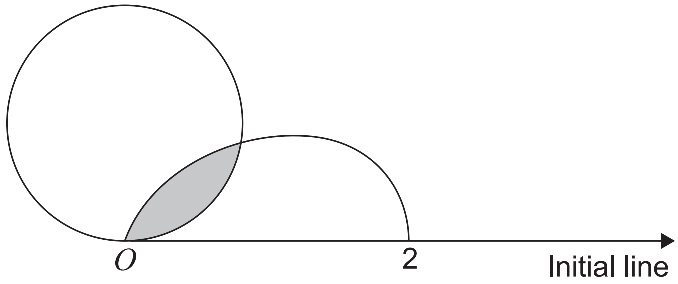
13 (b) Explain how Charlotte should have started her solution differently and find the general solution of the differential equation.

[8 marks]

Turn over ▶

14 The diagram shows the polar curve C_1 with equation $r = 2\sin\theta$

The diagram also shows part of the polar curve C_2 with equation $r = 1 + \cos 2\theta$



14 (a) On the diagram above, complete the sketch of C_2

[2 marks]

14 (b) Show that the area of the region shaded in the diagram is equal to

$$k\pi + m\alpha - \sin 2\alpha + q\sin 4\alpha$$

where $\alpha = \sin^{-1}\left(\dfrac{\sqrt{5}-1}{2}\right)$, and k, m and q are rational numbers.

[9 marks]

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15 The points $A(7, 2, 8)$, $B(7, -4, 0)$ and $C(3, 3.2, 9.6)$ all lie in the plane Π.

15 (a) Find a Cartesian equation of the plane Π.

[3 marks]

15 (b) The line L_1 has equation $\mathbf{r} = \begin{bmatrix} 5 \\ -0.4 \\ 4.8 \end{bmatrix} + \mu \begin{bmatrix} 15 \\ 3 \\ 4 \end{bmatrix}$

15 (b) (i) Show that L_1 lies in the plane Π.

[2 marks]

15 (b) (ii) Show that every point on L_1 is equidistant from B and C.

[4 marks]

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15 (c) The line L_2 lies in the plane Π, and every point on L_2 is equidistant from A and B.

Find an equation of the line L_2

[4 marks]

15 (d) The points A, B and C all lie on a circle G.
The point D is the centre of circle G.

Find the coordinates of D.

[3 marks]

END OF QUESTIONS

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