AQA

A-level

FURTHER MATHEMATICS 7367/3S

Paper 3 Statistics

Mark scheme

June 2020

Version: 1.0 Final Mark Scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer.	1.1b	B1	$\frac{3}{5}$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer.	1.1b	B1	0.0500
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Obtains correct z value. AWRT 1.88 PI by a correct upper or lower limit of the confidence interval.	1.1b	B1	z = 1.88 $\bar{x} \pm z\sqrt{\frac{s^2}{n}}$ $= 1196 \pm 1.88 \frac{98}{\sqrt{600}}$ = (1188.5, 1203.5)
	Uses formula for upper or lower limit of a confidence interval using their z-value and sample mean and variance. Condone use of $\sqrt{98}$ May use t-value.	1.1a	M1	
	Obtains correct confidence interval AWRT 1dp.	1.1b	A1	
3(b)	States yes and explains that the sample size has changed and is part of the calculation or a t distribution would be used instead of a z distribution or the confidence interval will be wider.	2.4	E1	Yes as the sample size has changed and is part of the calculation.
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Uses formula for E(X)	1.1a	M1	E(X) = $\sum_{i=1}^{n} \frac{x}{n} = \frac{1+2+...+n}{n}$ $= \frac{\frac{n}{2}(1+n)}{n}$ $= \frac{n+1}{2}$ E(X^2) = $\sum_{i=1}^{n} \frac{x^2}{n} = \frac{1^2+2^2+...+n^2}{n}$ $= \frac{\frac{1}{6}n(n+1)(2n+1)}{n}$ $= \frac{(n+1)(2n+1)}{6}$ Var (X) = $\frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$ $= \frac{2n^2+3n+1}{6} - \frac{n^2+2n+1}{4}$ $= \frac{4n^2+6n+2-3n^2-6n-3}{12}$ $= \frac{n^2-1}{12}$ Var (Y) = Var (2X) = 2^2 Var (X) $= 4 \times \frac{n^2-1}{12}$ $= \frac{n^2-1}{3}$
	Obtains an expression for E(X) by using the formula for $\sum n$	1.1b	A1	
	Uses formula for E(X^2)	1.1a	M1	
	Obtains an expression for E(X^2) by using the formula for $\sum n^2$	1.1b	A1	
	Uses the formula for Var (X) or uses Var (Y) = 2^2 Var (X)	1.1a	M1	
	Obtains correct expression for Var (X)	1.1b	A1	
	Completes a rigorous algebraic proof by using Var (Y) = 2^2 Var (X) to show that Var (Y) = $\frac{n^2-1}{3}$	2.1	R1	
4(b)	Obtains probability that next value spinner lands on is greater than 2 0.75 OE	1.1b	B1	$3 \times \frac{1}{4} = 0.75$
4(c)	Explains that the spinner toy is unbiased or the probability of obtaining each score is equal.	3.5b	E1	The spinner toy is unbiased.
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	States both hypotheses using correct language.	2.5	B1	H_0: $\lambda = 8$ H_1: $\lambda \neq 8$
5(b)	Evaluates the Poisson model by comparing the sample with the critical region.	3.5a	R1	3 is not in the critical region. Accept H_0 There is no significant evidence to suggest that the average number of runners per minute passing the shop is not 8
	Infers H_0 not rejected. FT 'their' comparison.	2.2b	E1F	
	Concludes in context. (Conclusion should not be definite) FT their incorrect rejection of H_0 if stated or 'their' comparison if not.	3.2a	E1F	
5(c)	Uses Poisson model with $\lambda = 7$ to calculate a cumulative probability.	1.1a	M1	$P(X \leq 2) = 0.0296$ $P(X \geq 14) = 0.0128$ Power = 0.0296 + 0.0128 = 0.0424
	Uses Poisson model with $\lambda = 7$ to calculate both $P(X \leq 2)$ = AWRT 0.0296 and $P(X \geq 14)$ = AWRT 0.0128 or $P(3 \leq X \leq 13)$ = AWRT 0.958	1.1b	A1	
	Obtains power of the test AWRT 0.0424	1.1b	A1F	
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Uses the mean equalling 25 to find the value of λ	3.3	M1	$\frac{1}{\lambda} = 25$ $\lambda = 0.04$ $P(X \le 2) = 1 - e^{-0.04 \times 2}$ $= 0.077 > 0.05$ The probability of a 'Red' rating has increased.
	Evaluates an exponential model to find $P(X \le 2)$ or uses it to find x	3.4	M1	
	Find $P(X \le 2)$ = AWRT 0.077 or x = AWRT 1.3 metres.	1.1b	A1	
	Concludes probability higher or probability will be increased because 2 metres is higher than 1.3 metres. FT 'their' probability or x value.	3.2a	E1F	
6(b)	States the correct probability density function for $x \ge 0$ for their value of λ	1.1b	B1F	$f(x) = \begin{cases} 0.04e^{-0.04x} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$
	States the complete correct probability density function including $x < 0$ (or otherwise).	1.2	B1F	
6(c)	Obtains the standard deviation of the exponential model or the value of λ (0.2) corresponding to a standard deviation of 5 metres.	1.1b	B1	s.d. = $\sqrt{\frac{1}{0.04^2}} = 25$ metres The model is not appropriate as the actual standard deviation of 5 metres is very different from the standard deviation of the model.
	Explains that the model is not appropriate as the actual standard deviation is not close to the standard deviation of the model or λ values are different.	3.5b	E1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	States both hypotheses using correct language.	2.5	B1	$H_0: \mu = 1.8$ $H_1: \mu > 1.8$
	Obtains t test statistic with 'their' sample mean and variance. Condone z =	1.1a	M1	$t = \dfrac{2-1.8}{\dfrac{0.4}{\sqrt{12}}}$
	Obtains t test statistic correctly or find p = AWRT 0.0556 Condone z =	1.1b	A1	$t = 1.73$
	Evaluates t model by comparing 'their' test statistic and correct critical value or by comparing p value with 0.05	3.5a	M1	t_{11} at 95% = 1.796 1.73 < 1.796
	Infers H_0 not rejected. FT 'their' comparison using the t model.	2.2b	E1F	Accept H_0
	Concludes in context. (Conclusion must not be definite) FT their incorrect rejection of H_0 or 'their' test statistic (or p-value) and 'their' critical value (or 0.05) if not.	3.2a	E1F	No significant evidence to suggest that the rainfall in February in the town has increased.
7(b)	States to accept that the mean rainfall per day in February has not increased when it has.	3.2a	E1	To conclude that the rainfall per day in February has not increased when it has.
7(c)	Recognises that the assumption is not appropriate because the distribution is not symmetrical or is skewed.	3.5b	E1	The assumption of a normal distribution is not appropriate because the distribution is not symmetrical.
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Deduces that two columns will need to be merged as there is an expected frequency less than 5	2.2a	E1	There is an expected frequency less than 5 so two columns will need to be merged. This means that the test will only have 1 degree of freedom. This means that Yates' correction will need to be used and so the test statistic will be $\sum\frac{(\lvert O-E\rvert-0.5)^2}{E}$ rather than $\sum\frac{(O-E)^2}{E}$
	Deduces that there is only 1 degree of freedom or that this will form a 2×2 table. Allow if only seen in 8(b).	2.2a	E1	
	Explains that Yates' correction will need to be used.	3.5c	E1	
8(b)	Evaluates χ^2-test statistic by comparing the correct critical value with the test statistic.	3.5a	R1	χ^2 cv for 1 df = 6.635 8.74 > 6.635 Reject H_0 Significant evidence to suggest that there is an association between time of day and number of snacks eaten.
	Infers H_0 rejected. FT 'their' critical value.	2.2b	E1F	
	Concludes in context. (Conclusion must not be definite) FI their incorrect acceptance of H_0 if stated or 'their' comparison if not.	3.2a	E1F	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
9	Differentiates the cumulative distribution function of X to obtain f(x) in the form $ax^2 + bx + c$	1.1a	M1	f(x) = $\frac{1}{124}(24x^2 + 24x + 6)$ $= \frac{3}{62}(4x^2 + 4x + 1)$
	Uses correct integral of the form $\int x^3 \text{ f}(x)\, dx$ with any limits to find E(X^3)	1.1a	M1	E(X^3) = $\frac{3}{62}\int_0^2 x^3(4x^2 + 4x + 1)\, dx$
	Obtains E(X^3) = $\frac{542}{155}$ or AWRT 3.5	1.1b	A1	= $\frac{542}{155}$
	Obtains E(Y) by evaluating 2 × 0.5 + 7 × 0.1 + 13 × 0.1 + 19 × 0.3	1.1a	M1	E(Y) = 2 × 0.5 + 7 × 0.1 + 13 × 0.1 + 19 × 0.3 = $\frac{87}{10}$
	Uses formula E(X^3 + Y) = E(X^3) + E(Y)	1.1a	M1	E(X^3 + Y) = E(X^3) + E(Y) = $\frac{542}{155}$ + $\frac{87}{10}$
	Completes a rigorous method to correctly show that E(X^3 + Y) = $\frac{3781}{310}$	2.1	R1	= $\frac{3781}{310}$
	Total		**6**	