

For use in exams from the June 2017 Series onwards

DATA - FUNDAMENTAL CONSTANTS AND VALUES

| Quantity | Symbol | Value | Units |
|--|-----------------|---------------------------|-----------------------------------|
| speed of light in vacuo | c | 3.00×10^8 | m s^{-1} |
| permeability of free space | μ_0 | $4\pi \times 10^{-7}$ | H m^{-1} |
| permittivity of free space | ϵ_0 | 8.85×10^{-12} | F m^{-1} |
| magnitude of the charge of electron | e | 1.60×10^{-19} | C |
| the Planck constant | h | 6.63×10^{-34} | J s |
| gravitational constant | G | 6.67×10^{-11} | $\text{N m}^2 \text{kg}^{-2}$ |
| the Avogadro constant | N_A | 6.02×10^{23} | mol^{-1} |
| molar gas constant | R | 8.31 | $\text{J K}^{-1} \text{mol}^{-1}$ |
| the Boltzmann constant | k | 1.38×10^{-23} | J K^{-1} |
| the Stefan constant | σ | 5.67×10^{-8} | $\text{W m}^{-2} \text{K}^{-4}$ |
| the Wien constant | α | 2.90×10^{-3} | m K |
| electron rest mass (equivalent to 5.5×10^{-4} u) | m_e | 9.11×10^{-31} | kg |
| magnitude of electron charge/mass ratio | $\frac{e}{m_e}$ | 1.76×10^{11} | C kg^{-1} |
| proton rest mass (equivalent to 1.00728 u) | m_p | $1.67(3) \times 10^{-27}$ | kg |
| proton charge/mass ratio | $\frac{e}{m_p}$ | 9.58×10^7 | C kg^{-1} |
| neutron rest mass (equivalent to 1.00867 u) | m_n | $1.67(5) \times 10^{-27}$ | kg |
| gravitational field strength | g | 9.81 | N kg^{-1} |
| acceleration due to gravity | g | 9.81 | m s^{-2} |
| atomic mass unit (1u is equivalent to 931.5 MeV) | u | 1.661×10^{-27} | kg |

ALGEBRAIC EQUATION

quadratic equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

ASTRONOMICAL DATA

| Body | Mass/kg | Mean radius/m |
|-------|-----------------------|--------------------|
| Sun | 1.99×10^{30} | 6.96×10^8 |
| Earth | 5.97×10^{24} | 6.37×10^6 |

GEOMETRICAL EQUATIONS

arc length = $r\theta$

circumference of circle = $2\pi r$

area of circle = πr^2

curved surface area of cylinder = $2\pi r h$

surface area of sphere = $4\pi r^2$

volume of sphere = $\frac{4}{3}\pi r^3$

Particle Physics

| Class | Name | Symbol | Rest energy/MeV |
|---------|-------------|-----------|-----------------|
| photon | photon | γ | 0 |
| lepton | neutrino | ν_e | 0 |
| | | ν_μ | 0 |
| | electron | e^\pm | 0.510999 |
| | muon | μ^\pm | 105.659 |
| mesons | π meson | π^\pm | 139.576 |
| | | π^0 | 134.972 |
| | K meson | K^\pm | 493.821 |
| | | K^0 | 497.762 |
| baryons | proton | p | 938.257 |
| | neutron | n | 939.551 |

Properties of quarks

antiquarks have opposite signs

| Type | Charge | Baryon number | Strangeness |
|----------|-----------------|----------------|-------------|
| u | $+\frac{2}{3}e$ | $+\frac{1}{3}$ | 0 |
| d | $-\frac{1}{3}e$ | $+\frac{1}{3}$ | 0 |
| s | $-\frac{1}{3}e$ | $+\frac{1}{3}$ | -1 |

Properties of Leptons

| | | Lepton number |
|----------------|--|---------------|
| Particles: | $e^-, \nu_e; \mu^-, \nu_\mu$ | +1 |
| Antiparticles: | $e^+, \bar{\nu}_e, \mu^+, \bar{\nu}_\mu$ | -1 |

Photons and energy levels

photon energy $E = hf = \frac{hc}{\lambda}$

photoelectricity $hf = \phi + E_{k(\max)}$

energy levels $hf = E_1 - E_2$

de Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$

Waves

wave speed $c = f\lambda$ period $f = \frac{1}{T}$

first harmonic $f = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$

fringe spacing $w = \frac{\lambda D}{s}$ diffraction grating $d \sin \theta = n\lambda$

refractive index of a substance s, $n = \frac{c}{c_s}$

for two different substances of refractive indices n_1 and n_2 ,
law of refraction $n_1 \sin \theta_1 = n_2 \sin \theta_2$

critical angle $\sin \theta_c = \frac{n_2}{n_1}$ for $n_1 > n_2$

Mechanics

moments moment = Fd

velocity and acceleration $v = \frac{\Delta s}{\Delta t}$ $a = \frac{\Delta v}{\Delta t}$

equations of motion $v = u + at$ $s = \left(\frac{u+v}{2}\right)t$

$v^2 = u^2 + 2as$ $s = ut + \frac{at^2}{2}$

force $F = ma$

force $F = \frac{\Delta(mv)}{\Delta t}$

impulse $F \Delta t = \Delta(mv)$

work, energy and power $W = F s \cos \theta$

$E_k = \frac{1}{2} m v^2$ $\Delta E_p = mg\Delta h$

$P = \frac{\Delta W}{\Delta t}, P = Fv$

efficiency = $\frac{\text{useful output power}}{\text{input power}}$

Materials

density $\rho = \frac{m}{V}$ Hooke's law $F = k \Delta L$

Young modulus = $\frac{\text{tensile stress}}{\text{tensile strain}}$ tensile stress = $\frac{F}{A}$

tensile strain = $\frac{\Delta L}{L}$

energy stored $E = \frac{1}{2} F \Delta L$

Electricity

current and pd $I = \frac{\Delta Q}{\Delta t}$ $V = \frac{W}{Q}$ $R = \frac{V}{I}$

resistivity $\rho = \frac{RA}{L}$

resistors in series $R_T = R_1 + R_2 + R_3 + \dots$

resistors in parallel $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

power $P = VI = I^2R = \frac{V^2}{R}$

emf $\varepsilon = \frac{E}{Q}$ $\varepsilon = I(R + r)$

Circular motion

magnitude of angular speed $\omega = \frac{v}{r}$

$\omega = 2\pi f$

centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r$

centripetal force $F = \frac{mv^2}{r} = m\omega^2 r$

Simple harmonic motion

acceleration $a = -\omega^2 x$

displacement $x = A \cos(\omega t)$

speed $v = \pm \omega \sqrt{(A^2 - x^2)}$

maximum speed $v_{\max} = \omega A$

maximum acceleration $a_{\max} = \omega^2 A$

for a mass-spring system $T = 2\pi \sqrt{\frac{m}{k}}$

for a simple pendulum $T = 2\pi \sqrt{\frac{l}{g}}$

Thermal physics

energy to change temperature $Q = mc\Delta\theta$

energy to change state $Q = ml$

gas law $pV = nRT$
 $pV = NkT$

kinetic theory model $pV = \frac{1}{3}Nm(c_{\text{rms}})^2$

kinetic energy of gas molecule $\frac{1}{2}m(c_{\text{rms}})^2 = \frac{3}{2}kT = \frac{3RT}{2N_A}$

Gravitational fields

force between two masses $F = \frac{Gm_1m_2}{r^2}$

gravitational field strength $g = \frac{F}{m}$

magnitude of gravitational field strength in a radial field $g = \frac{GM}{r^2}$

work done $\Delta W = m\Delta V$

gravitational potential $V = -\frac{GM}{r}$

$g = -\frac{\Delta V}{\Delta r}$

Electric fields and capacitors

force between two point charges $F = \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{r^2}$

force on a charge $F = EQ$

field strength for a uniform field $E = \frac{V}{d}$

work done $\Delta W = Q\Delta V$

field strength for a radial field $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

electric potential $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

field strength $E = \frac{\Delta V}{\Delta r}$

capacitance $C = \frac{Q}{V}$

$C = \frac{A\epsilon_0\epsilon_r}{d}$

capacitor energy stored $E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2} \frac{Q^2}{C}$

capacitor charging $Q = Q_0(1 - e^{-\frac{t}{RC}})$

decay of charge $Q = Q_0e^{-\frac{t}{RC}}$

time constant RC

Magnetic fields

| | |
|---------------------------------------|---|
| <i>force on a current</i> | $F = BIl$ |
| <i>force on a moving charge</i> | $F = BQv$ |
| <i>magnetic flux</i> | $\Phi = BA$ |
| <i>magnetic flux linkage</i> | $N\Phi = BAN \cos \theta$ |
| <i>magnitude of induced emf</i> | $\varepsilon = N \frac{\Delta\Phi}{\Delta t}$ |
| | $N\Phi = BAN \cos \theta$ |
| <i>emf induced in a rotating coil</i> | $\varepsilon = BAN\omega \sin \omega t$ |
| <i>alternating current</i> | $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \quad V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ |
| <i>transformer equations</i> | $\frac{N_s}{N_p} = \frac{V_s}{V_p}$ |
| | $\text{efficiency} = \frac{I_s V_s}{I_p V_p}$ |

Nuclear physics

| | |
|---|--|
| <i>inverse square law for γ radiation</i> | $I = \frac{k}{x^2}$ |
| <i>radioactive decay</i> | $\frac{\Delta N}{\Delta t} = -\lambda N, N = N_0 e^{-\lambda t}$ |
| <i>activity</i> | $A = \lambda N$ |
| <i>half-life</i> | $T_{1/2} = \frac{\ln 2}{\lambda}$ |
| <i>nuclear radius</i> | $R = R_0 A^{1/3}$ |
| <i>energy-mass equation</i> | $E = mc^2$ |

OPTIONS

Astrophysics

| | |
|--|--|
| 1 astronomical unit | $= 1.50 \times 10^{11} \text{ m}$ |
| 1 light year | $= 9.46 \times 10^{15} \text{ m}$ |
| 1 parsec | $= 2.06 \times 10^5 \text{ AU} = 3.08 \times 10^{16} \text{ m}$ $= 3.26 \text{ ly}$ |
| Hubble constant, H | $= 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ |
| $M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}}$ | |
| <i>telescope in normal adjustment</i> | $M = \frac{f_o}{f_e}$ |
| <i>Rayleigh criterion</i> | $\theta \approx \frac{\lambda}{D}$ |
| <i>magnitude equation</i> | $m - M = 5 \log \frac{d}{10}$ |
| <i>Wien's law</i> | $\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K}$ |
| <i>Stefan's law</i> | $P = \sigma AT^4$ |
| <i>Schwarzschild radius</i> | $R_s \approx \frac{2GM}{c^2}$ |
| <i>Doppler shift for $v \ll c$</i> | $\frac{\Delta f}{f} = -\frac{\Delta \lambda}{\lambda} = \frac{v}{c}$ |
| <i>red shift</i> | $z = -\frac{v}{c}$ |
| <i>Hubble's law</i> | $v = Hd$ |

Medical physics

| | |
|-----------------------------|---|
| <i>lens equations</i> | $P = \frac{1}{f}$ $m = \frac{v}{u}$ $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ |
| <i>threshold of hearing</i> | $I_0 = 1.0 \times 10^{-12} \text{ W m}^{-2}$ |
| <i>intensity level</i> | $\text{intensity level} = 10 \log \frac{I}{I_0}$ |
| <i>absorption</i> | $I = I_0 e^{-\mu x}$ $\mu_m = \frac{\mu}{\rho}$ |
| <i>ultrasound imaging</i> | $Z = p c$ $\frac{I_r}{I_i} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$ |
| <i>half-lives</i> | $\frac{1}{T_E} = \frac{1}{T_B} + \frac{1}{T_P}$ |

Engineering physics

| | |
|-----------------------------|--|
| moment of inertia | $I = \Sigma mr^2$ |
| angular kinetic energy | $E_k = \frac{1}{2} I \omega^2$ |
| equations of angular motion | $\omega_2 = \omega_1 + \alpha t$ $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ $\theta = \omega_1 t + \frac{\alpha t^2}{2}$ $\theta = \frac{(\omega_1 + \omega_2) t}{2}$ |
| torque | $T = I \alpha$ $T = F r$ |
| angular momentum | angular momentum = $I\omega$ |
| angular impulse | $T\Delta t = \Delta(I\omega)$ |
| work done | $W = T\theta$ |
| power | $P = T\omega$ |
| thermodynamics | $Q = \Delta U + W$ $W = p\Delta V$ |
| adiabatic change | $pV^\gamma = \text{constant}$ |
| isothermal change | $pV = \text{constant}$ |
| heat engines | efficiency = $\frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$ maximum theoretical efficiency = $\frac{T_H - T_C}{T_H}$ work done per cycle = area of loop input power = calorific value \times fuel flow rate indicated power = (area of $p - V$ loop) \times (number of cycles per second) \times (number of cylinders) output or brake power $P = T\omega$ friction power = indicated power - brake power heat pumps and refrigerators refrigerator: $COP_{\text{ref}} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$ heat pump: $COP_{\text{hp}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$ |

Turning points in physics

| | |
|-----------------------|---|
| electrons in fields | $F = \frac{eV}{d}$ $F = Bev$ $r = \frac{mv}{Be}$ $\frac{1}{2} mv^2 = eV$ |
| Millikan's experiment | $\frac{QV}{d} = mg$ $F = 6\pi\eta r v$ |
| Maxwell's formula | $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$ |
| special relativity | $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$ $E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$ |

Electronics

| | |
|-----------------------------------|--|
| resonant frequency | $f_0 = \frac{1}{2\pi \sqrt{LC}}$ |
| Q-factor | $Q = \frac{f_0}{f_B}$ |
| operational amplifiers: open loop | $V_{\text{out}} = A_{OL}(V_+ - V_-)$ |
| inverting amplifier | $\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$ |
| non-inverting amplifier | $\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_f}{R_1}$ |
| summing amplifier | $V_{\text{out}} = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots \right)$ |
| difference amplifier | $V_{\text{out}} = (V_+ - V_-) \frac{R_f}{R_1}$ |
| Bandwidth requirement: | |
| for AM | bandwidth = $2f_M$ |
| for FM | bandwidth = $2(\Delta f + f_M)$ |

