A

Surname ______________________________

Other Names ______________________________

Centre Number ______________________________

Candidate Number ______________________________

Candidate Signature ______________________________
I declare this is my own work.

A-level
MATHEMATICS

Paper 1

7357/1

Time allowed: 2 hours

For this paper:

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

[Turn over]

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INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do NOT write on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the space provided.

1 State the set of values of x which satisfies the inequality

$(x - 3)(2x + 7) > 0$

Tick (✓) ONE box. [1 mark]

☐ $\left\{x : -\frac{7}{2} < x < 3\right\}$

☐ $\left\{x : x < -3 \text{ or } x > \frac{7}{2}\right\}$

☐ $\left\{x : x < -\frac{7}{2} \text{ or } x > 3\right\}$

☐ $\left\{x : -3 < x < \frac{7}{2}\right\}$

2 Given that $y = \ln(5x)$

find $\dfrac{dy}{dx}$

Circle your answer. [1 mark]

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{5x}$$

$$\frac{dy}{dx} = \frac{5}{x}$$

$$\frac{dy}{dx} = \ln 5$$

[Turn over]

3 A geometric sequence has a sum to infinity of -3

A second sequence is formed by multiplying each term of the original sequence by -2

What is the sum to infinity of the new sequence?

Circle your answer. [1 mark]

The sum to infinity does not exist

-6

-3

6

4 Millie is attempting to use proof by contradiction to show that the result of multiplying an irrational number by a non-zero rational number is always an irrational number.

Select the assumption she should make to start her proof.

Tick (✓) ONE box. [1 mark]

☐ Every irrational multiplied by a non-zero rational is irrational.

☐ Every irrational multiplied by a non-zero rational is rational.

☐ There exists a non-zero rational and an irrational whose product is irrational.

☐ There exists a non-zero rational and an irrational whose product is rational.

[Turn over]

5 **The line *L* has equation**

$$3y - 4x = 21$$

The point *P* has coordinates (15, 2)

5 (a) **Find the equation of the line perpendicular to *L* which passes through *P*. [2 marks]**

5 (b) **Hence, find the shortest distance from *P* to *L*. [4 marks]**

[Turn over]

6 (a) The ninth term of an arithmetic series is 3

The sum of the first n terms of the series is S_n and $S_{21} = 42$

Find the first term and common difference of the series. [4 marks]

[Turn over]

6 (b) **A second arithmetic series has first term -18 and common difference $\frac{3}{4}$**

The sum of the first n terms of this series is T_n

Find the value of n such that $T_n = S_n$
[3 marks]

[Turn over]

7 **The equation $x^2 = x^3 + x - 3$ has a single solution, $x = \alpha$**

7 (a) **By considering a suitable change of sign, show that α lies between 1.5 and 1.6 [2 marks]**

7 (b) **Show that the equation $x^2 = x^3 + x - 3$ can be rearranged into the form**

$$x^2 = x - 1 + \frac{3}{x}$$

[2 marks]

[Turn over]

7 (c) **Use the iterative formula**

$$x_{n+1} = \sqrt{x_n - 1 + \frac{3}{x_n}}$$

with $x_1 = 1.5$, to find x_2, x_3 and x_4, giving your answers to four decimal places. [2 marks]

7 (d) **Hence, deduce an interval of width 0.001 in which α lies. [1 mark]**

[Turn over]

8 (a) Given that

$$9\sin^2\theta + \sin 2\theta = 8$$

show that

$$8\cot^2\theta - 2\cot\theta - 1 = 0$$

[4 marks]

[Turn over]

8 (b) **Hence, solve**

$$9\sin^2\theta + \sin 2\theta = 8$$

in the interval $0 < \theta < 2\pi$

Give your answers to two decimal places.
[3 marks]

8 (c) **Solve**

$$9\sin^2\left(2x-\frac{\pi}{4}\right)+\sin\left(4x-\frac{\pi}{2}\right)=8$$

in the interval $0<x<\frac{\pi}{2}$

Give your answers to one decimal place.
[2 marks]

9 The table below shows the annual global production of plastics, P, measured in millions of tonnes per year, for six selected years.

YEAR	1980	1985	1990	1995	2000	2005
P	75	94	120	156	206	260

It is thought that P can be modelled by

$$P = A \times 10^{kt}$$

where t is the number of years after 1980 and A and k are constants.

9 (a) Show algebraically that the graph of $\log_{10} P$ against t should be linear. [3 marks]

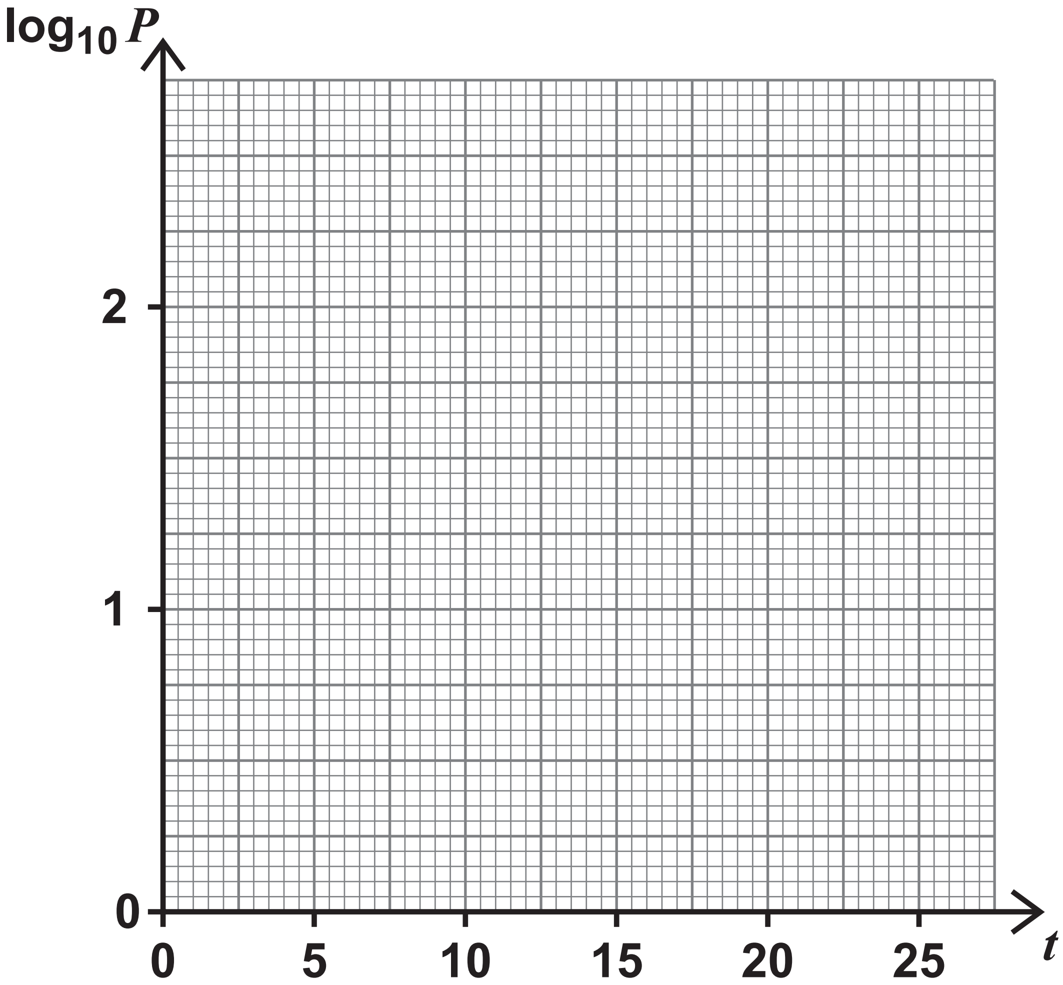
9 (b) (i) Complete the table below.

t	0	5	10	15	20	25
$\log_{10}P$	1.88	1.97	2.08		2.31	

[1 mark]

[Turn over]

9 (b) (ii) Plot $\log_{10} P$ against t, and draw a line of best fit for the data. [2 marks]



9 (c) (i) Hence, show that k is approximately 0.02
[2 marks]

9 (c) (ii) Find the value of A. [1 mark]

[Turn over]

9 (d) **Using the model with $k = 0.02$ predict the number of tonnes of annual global production of plastics in 2030. [2 marks]**

9 (e) **Using the model with $k = 0.02$ predict the year in which P first exceeds 8000 [3 marks]**

9 (f) Give a reason why it may be inappropriate to use the model to make predictions about future annual global production of plastics. [1 mark]

10 (a) Given that

$y = \tan x$

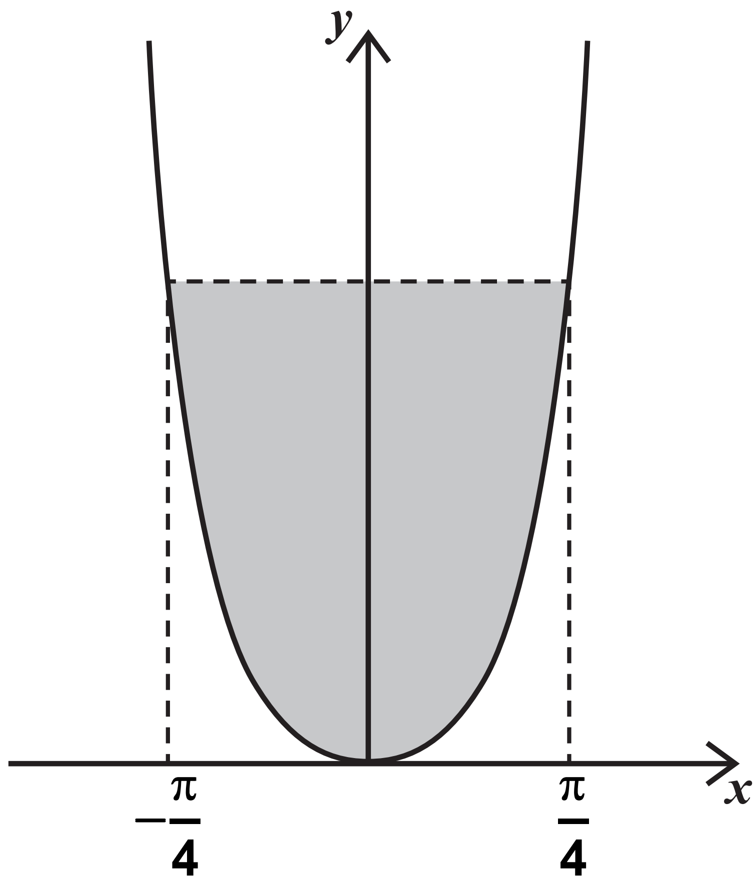
use the quotient rule to show that

$$\frac{dy}{dx} = \sec^2 x$$

[3 marks]

[Turn over]

10 (b) The region enclosed by the curve $y = \tan^2 x$ and the horizontal line, which intersects the curve at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, is shaded in the diagram below.



Show that the area of the shaded region is

$\pi - 2$

Fully justify your answer. [5 marks]

[Turn over]

11 A curve, C, passes through the point with coordinates (1, 6)

The gradient of C is given by

$$\frac{dy}{dx} = \frac{1}{6}(xy)^2$$

Show that C intersects the coordinate axes at exactly one point and state the coordinates of this point.

Fully justify your answer. [8 marks]

[Turn over]

12 **The equation of a curve is**

$$(x + y)^2 = 4y + 2x + 8$$

The curve intersects the positive x-axis at the point P.

12 (a) **Show that the gradient of the curve at P is $-\frac{3}{2}$**

[6 marks]

[Turn over]

12 (b) **Find the equation of the normal to the curve at *P*, giving your answer in the form $ax + by = c$, where a, b and c are integers. [2 marks]**

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[Turn over]

13 (a) **Given that**

$$\mathrm{P}(x) = 125x^3 + 150x^2 + 55x + 6$$

use the factor theorem to prove that $(5x + 1)$ is a factor of $\mathrm{P}(x)$. [2 marks]

13 (b) Factorise $P(x)$ completely. [3 marks]

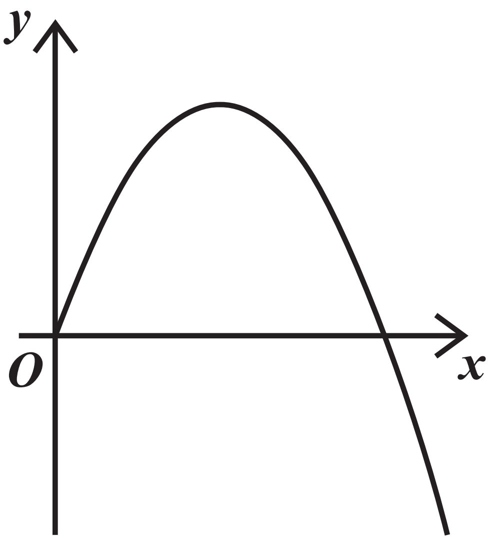
13 (c) **Hence, prove that $250n^3 + 300n^2 + 110n + 12$ is a multiple of 12 when n is a positive whole number. [3 marks]**

[Turn over]

14 **The curve *C* is defined for $t \geq 0$ by the parametric equations**

$$x = t^2 + t \quad \text{and} \quad y = 4t^2 - t^3$$

***C* is shown in the diagram below.**



14 (a) **Find the gradient of *C* at the point where it intersects the positive x-axis.** **[5 marks]**

14 (b) (i) The area A enclosed between C and the x-axis is given by

$$A = \int_0^b y\,\mathrm{d}x$$

Find the value of b. [1 mark]

14 (b) (ii) Use the substitution $y = 4t^2 - t^3$ to show that

$$A = \int_0^4 (4t^2 + 7t^3 - 2t^4)\,\mathrm{d}t$$

[3 marks]

14 (b) (iii) **Find the value of A. [1 mark]**

[Turn over]

15 (a) **Show that**

$$\sin x - \sin x \cos 2x \approx 2x^3$$

for small values of x. [3 marks]

15 (b) **Hence, show that the area between the graph with equation**

$$y = \sqrt{8(\sin x - \sin x \cos 2x)}$$

the positive x-axis and the line $x = 0.25$ can be approximated by

$$\text{Area} \approx 2^m \times 5^n$$

where m and n are integers to be found.
[4 marks]

15 (c) (i) Explain why

$$\int_{6.3}^{6.4} 2x^3 \, \mathrm{d}x$$

is NOT a suitable approximation for

$$\int_{6.3}^{6.4} (\sin x - \sin x \cos 2x) \, \mathrm{d}x$$

[1 mark]

[Turn over]

15 (c) (ii) Explain how

$$\int_{6.3}^{6.4} (\sin x - \sin x \cos 2x)\, \mathrm{d}x$$

may be approximated by

$$\int_a^b 2x^3\, \mathrm{d}x$$

for suitable values of a and b. [2 marks]

END OF QUESTIONS

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