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Other Names \_\_\_\_\_

Centre Number \_\_\_\_\_

Candidate Number \_\_\_\_\_

Candidate Signature \_\_\_\_\_

I declare this is my own work.

# A-level MATHEMATICS

Paper 2

**7357/2**

Time allowed: 2 hours

## MATERIALS

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

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J U N 2 1 7 3 5 7 2 0 1

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## INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do NOT write on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

## INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

## ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

**DO NOT TURN OVER UNTIL TOLD TO DO SO**



## SECTION A

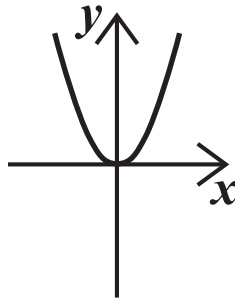
Answer ALL questions in the spaces provided.

- 1 Four possible sketches of  $y = ax^2 + bx + c$  are shown below and opposite.

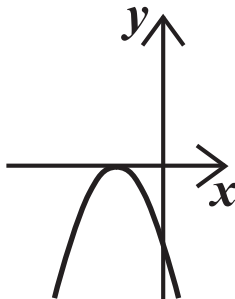
Given  $b^2 - 4ac = 0$  and  $a$ ,  $b$  and  $c$  are non-zero constants, which sketch is the only one that could possibly be correct?

Tick (✓) ONE box. [1 mark]

A

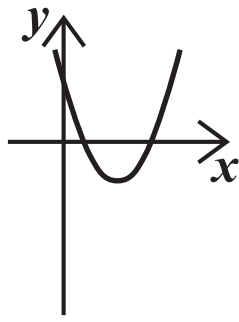


B

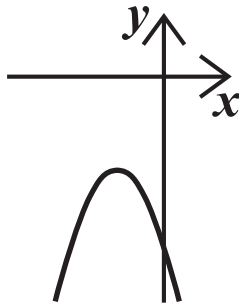




C



D



[Turn over]

2 A curve has equation  $y = f(x)$

The curve has a point of inflection at  $x = 7$

It is given that  $f'(7) = a$  and  $f''(7) = b$ , where  $a$  and  $b$  are real numbers.

Identify which one of the statements below must be true.

Circle your answer. [1 mark]

$$f'(7) \neq 0$$

$$f'(7) = 0$$

$$f''(7) \neq 0$$

$$f''(7) = 0$$



3 A sequence is defined by

$$u_1 = a \text{ and } u_{n+1} = -1 \times u_n$$

Find  $\sum_{n=1}^{95} u_n$

Circle your answer. [1 mark]

$-a$

0

$a$

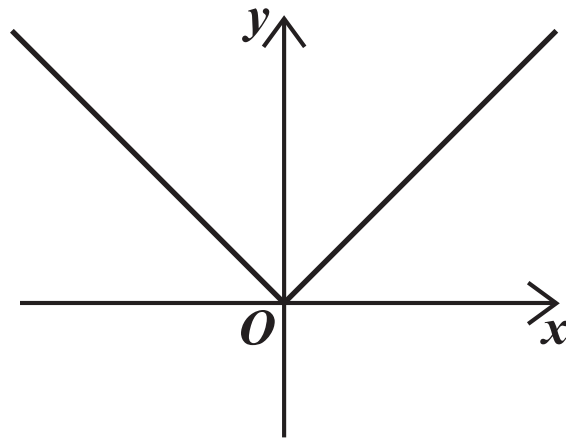
$95a$

[Turn over]



4 FIGURE 1 shows the graph of  $y = |2x|$

FIGURE 1



4 (a) On FIGURE 1 add a sketch of the graph of

$$y = |3x - 6|$$

[2 marks]

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**[Turn over]**



7 A circle has equation

$$x^2 + y^2 - 6x - 8y = p$$

7 (a) (i) State the coordinates of the centre of the circle.  
[1 mark]

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- 8 Kai is proving that  $n^3 - n$  is a multiple of 3 for all positive integer values of  $n$ .

Kai begins a proof by exhaustion.

Step 1

$$n^3 - n = n(n^2 - 1)$$

Step 2

When  $n = 3m$ ,  
where  $m$  is a  
non-negative  
integer

$$n^3 - n = 3m(9m^2 - 1)$$

which is a multiple of 3

Step 3

$$\text{When } n = 3m + 1, \quad n^3 - n = (3m + 1)((3m + 1)^2 - 1)$$

Step 4

$$= (3m + 1)(9m^2)$$

$$= 3(3m + 1)(3m^2)$$

which is a multiple of 3

Step 5

Therefore  $n^3 - n$  is a multiple of 3 for all positive integer values of  $n$



8 (a) Explain the two mistakes that Kai has made after Step 3. [2 marks]

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[Turn over]





9 A robotic arm which is attached to a flat surface at the origin  $O$ , is used to draw a graphic design.

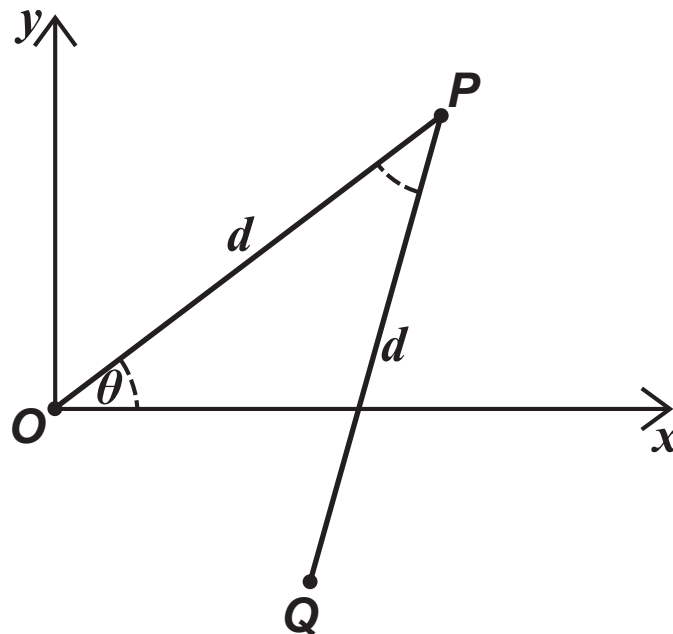
The arm is made from two rods  $OP$  and  $PQ$ , each of length  $d$ , which are joined at  $P$ .

A pen is attached to the arm at  $Q$ .

The coordinates of the pen are controlled by adjusting the angle  $OPQ$  and the angle  $\theta$  between  $OP$  and the  $x$ -axis.

For this particular design the pen is made to move so that the two angles are always equal to each other with  $0 \leq \theta \leq \frac{\pi}{2}$  as shown in FIGURE 2.

FIGURE 2





9 (b) Hence, show that

$$x = d(1 + \cos \theta - 2 \cos^2 \theta)$$

[2 marks]

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9 (c) It can be shown that

$$x = \frac{9d}{8} - d \left( \cos \theta - \frac{1}{4} \right)^2$$

State the greatest possible value of  $x$  and the corresponding value of  $\cos \theta$  [2 marks]

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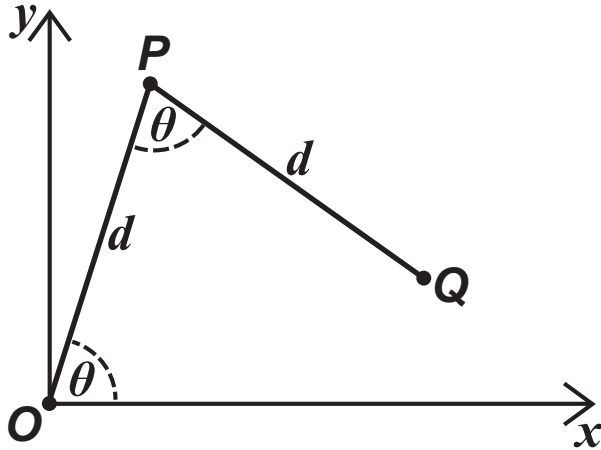
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- 9 (d) FIGURE 3 below shows the arm when the  $x$ -coordinate is at its greatest possible value.

FIGURE 3



Find, in terms of  $d$ , the exact distance  $OQ$ .  
[3 marks]

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**[Turn over]**



10 The function  $h$  is defined by

$$h(x) = \frac{\sqrt{x}}{x-3}$$

where  $h$  has its maximum possible domain.

10 (a) Find the domain of  $h$ .

Give your answer using set notation. [3 marks]

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10 (b) Alice correctly calculates

$$h(1) = -0.5 \quad \text{and} \quad h(4) = 2$$

She then argues that since there is a change of sign there must be a value of  $x$  in the interval  $1 < x < 4$  that gives  $h(x) = 0$

Explain the error in Alice's argument. [2 marks]

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[Turn over]







## SECTION B

Answer ALL questions in the spaces provided.

- 11 A particle's displacement,  $r$  metres, with respect to time,  $t$  seconds, is defined by the equation

$$r = 3e^{0.5t}$$

Find an expression for the velocity,  $v \text{ m s}^{-1}$ , of the particle at time  $t$  seconds.

Circle your answer. [1 mark]

$$v = 1.5e^{0.5t}$$

$$v = 6e^{0.5t}$$

$$v = 1.5te^{0.5t}$$

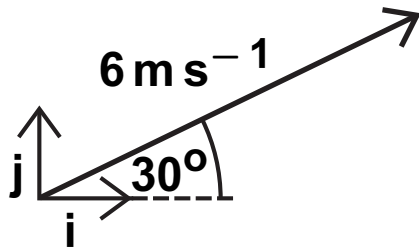
$$v = 6te^{0.5t}$$





12

A particle has a speed of  $6 \text{ m s}^{-1}$  in a direction relative to unit vectors  $i$  and  $j$  as shown in the diagram below.



The velocity of this particle can be expressed as a vector  $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \text{ m s}^{-1}$

Find the correct expression for  $v_2$

Circle your answer. [1 mark]

$$v_2 = 6 \cos 30^\circ$$

$$v_2 = 6 \sin 30^\circ$$

$$v_2 = -6 \sin 30^\circ$$

$$v_2 = -6 \cos 30^\circ$$

[Turn over]





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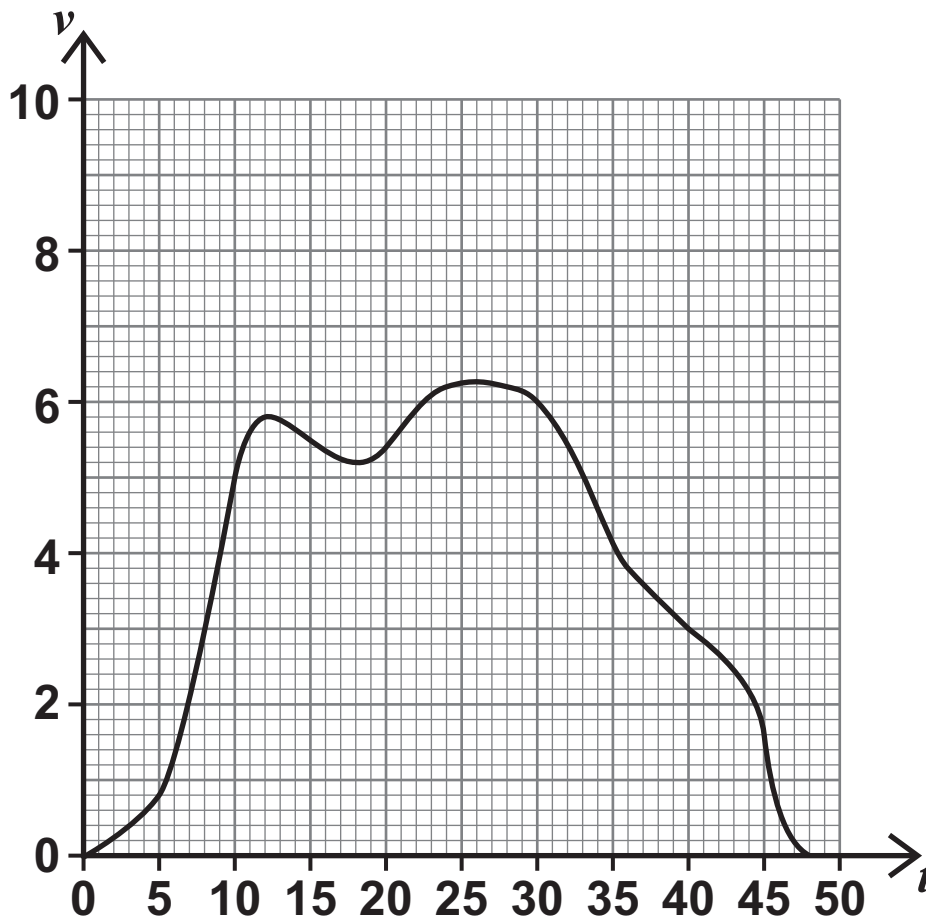
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14

A motorised scooter is travelling along a straight path with velocity  $v \text{ m s}^{-1}$  over time  $t$  seconds as shown by the following graph.



Noosha says that, in the period  $12 \leq t \leq 36$ , the scooter travels approximately 130 metres.

Determine if Noosha is correct, showing clearly any calculations you have used. [4 marks]

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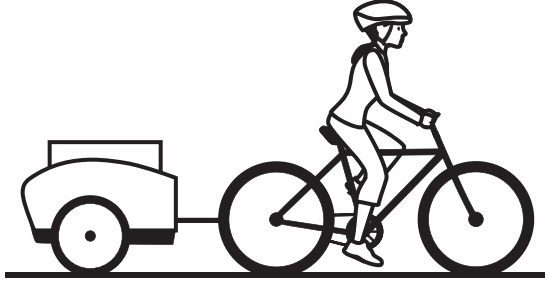


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- 15 **A cyclist is towing a trailer behind her bicycle.**
- She is riding along a straight, horizontal path at a constant speed.**



**A tension of  $T$  newtons acts on the connecting rod between the bicycle and the trailer.**

**The cyclist is causing a constant driving force of 40 N to be applied whilst pedalling forwards on her bicycle.**

**The constant resistance force acting on the trailer is 12 N**



- 15 (a) State the value of  $T$  giving a clear reason for your answer. [2 marks]

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**15 (b) State one assumption you have made in reaching your answer to part (a). [1 mark]**

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15 (c) Find the external resistance force acting on the cyclist and her bicycle. [2 marks]

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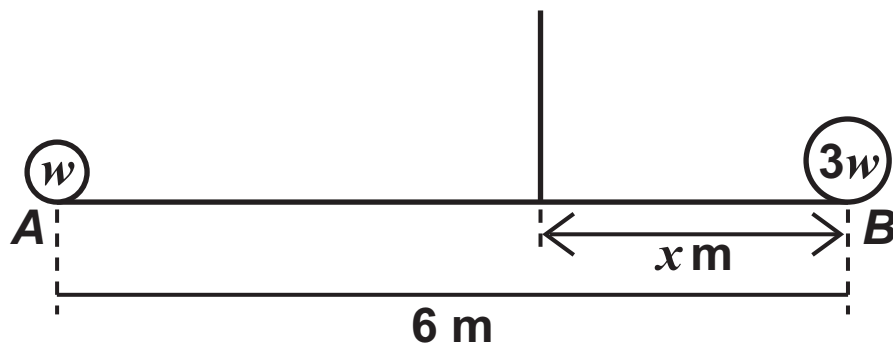
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[Turn over]

- 16 A straight uniform rod,  $AB$ , has length 6 m and mass 0.2 kg
- A particle of weight  $w$  newtons is fixed at  $A$ .
- A second particle of weight  $3w$  newtons is fixed at  $B$ .
- The rod is suspended by a string from a point  $x$  metres from  $B$ .
- The rod rests in equilibrium with  $AB$  horizontal and the string hanging vertically as shown in the diagram below.



Show that

$$x = \frac{3w + 0.3g}{2w + 0.1g}$$

[4 marks]

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**17 (c) Comment on the value of  $\nu$  for the two models as  $t$  becomes large. [2 marks]**

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**[Turn over]**



18 Two particles,  $P$  and  $Q$ , are projected at the same time from a fixed point  $X$ , on the ground, so that they travel in the same vertical plane.

$P$  is projected at an acute angle  $\theta^\circ$  to the horizontal, with speed  $u \text{ m s}^{-1}$

$Q$  is projected at an acute angle  $2\theta^\circ$  to the horizontal, with speed  $2u \text{ m s}^{-1}$

Both particles land back on the ground at exactly the same point,  $Y$ .

Resistance forces to motion may be ignored.

18 (a) Show that

$$\cos 2\theta = \frac{1}{8}$$

[6 marks]

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**18 (c) State one modelling assumption you have chosen to make in this question. [1 mark]**

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19 Two skaters, Jo and Amba, are separately skating across a smooth, horizontal surface of ice.

Both are moving in the same direction, so that their paths are straight and are parallel to each other.

Jo is moving with constant velocity  $(2.8\mathbf{i} + 9.6\mathbf{j}) \text{ m s}^{-1}$

At time  $t = 0$  seconds Amba is at position  $(2\mathbf{i} - 7\mathbf{j})$  metres and is moving with a constant speed of  $8 \text{ m s}^{-1}$

19 (a) (i) Explain why Amba's velocity must be in the form  $k(2.8\mathbf{i} + 9.6\mathbf{j}) \text{ m s}^{-1}$ , where  $k$  is a constant. [1 mark]

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19 (a) (ii) Verify that  $k = 0.8$  [1 mark]

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