A-LEVEL
MATHEMATICS

7357/3 Paper 3
Report on the Examination

7357
Autumn 2021

Version: 1.0

General Introduction to the Autumn Series

This has been another unusual exam series in many ways. Entry patterns have been very different from those normally seen in the summer, and students had a very different experience in preparation for these exams. It is therefore more difficult to make meaningful comparisons between the range of student responses seen in this series and those seen in a normal summer series. The smaller entry also means that there is less evidence available for examiners to comment on.

In this report, senior examiners will summarise the performance of students in this series in a way that is as helpful as possible to teachers preparing future cohorts while taking into account the unusual circumstances and limited evidence available.

Overview of Entry

This paper provided the opportunity for all students to demonstrate their knowledge and skills in pure mathematics and statistics, with the full range of available marks being scored.

Topics that were done well included:

- binomial distribution
- data presentation and interpretation
- differentiation
- Normal distribution.

Topics which students found challenging included:

- algebraic manipulation
- arc length and area of a sector
- geometric series
- integration
- systematic sampling

Comments on Individual Questions

Section A

Question 1
Just over half of the students answered this question correctly. The most common incorrect answer was $(-\pi, 1)$

Question 2
This was the least well answered of all the multiple-choice questions on the paper with less than half of students obtaining the correct answer.

Question 3
This question was answered well by the majority of students who demonstrated good knowledge of differentiation from first principles.

Question 4
There was clear evidence of good understanding of the binomial expansion in part (a) with many students scoring at least two marks. Very few students attempted to show how the first term, $1024x^{10}$, was derived, hence losing the final mark. It is worth noting that those who extracted a factor of either 2 or -3 and proceeded to expand $\left(x-\frac{3}{2}\right)^{10}$ or $\left(1-\frac{2x}{3}\right)^{10}$ were usually unsuccessful in their approach.

Many incorrect attempts were seen in part (b) with less than 20% obtaining full marks. A few students scored only 1 mark for deducing that the constant term comes from $(\ldots x)^5\left(\ldots\frac{3}{x}\right)^5$. However, they then struggled with the subsequent working and could not evaluate this term.

Question 5
This question was not well answered with very few students able to gain full marks in each part of the question. Many couldn't recall the formulae for the area of the sector ($\frac{1}{2}r^2\theta$) and arc length ($r\theta$) correctly, often swapping them round or omitting the ½. There were a few attempts to convert the angle from radians to degrees and then find the length. The final mark in (a)(ii) was a strict mark for appreciating that it was sensible in this context to express the answer to the nearest penny. Some therefore lost the mark by leaving their answer as £24.3 instead of £24.30

In part (b)(i) some managed to re-arrange their equation successfully to show the required result. However, a few who managed to form two correct expressions for area and perimeter couldn't eliminate θ to obtain an expression for the perimeter in terms of r. Part (b)(ii) was poorly attempted with little more than 10% of students being able to complete a fully reasoned argument to show the minimum cost. It was pleasing to see that most could recognise the need to use differentiation. However, they often couldn't explain that a minimum occurs when the derivative is 0. Many didn't "fully justify [their] answer" because there was no use of a gradient test or the second derivative to confirm that the stationary point was a minimum.

Question 6
Algebraic manipulation appears to be an area of significant weakness. It was commendable to see many students begin to solve the problem using factorisation or multiplying the given fraction by $\frac{5+\sqrt{x}}{5+\sqrt{x}}$. Those who used the latter method had more success and scored at least three marks but often lost the final mark as they could not factorise the quadratic expression in the numerator. Some factors were expressed without the use of brackets and as a result students couldn't complete the manipulation by cancelling common factors to obtain the final answer.

Question 7
Students continue to struggle with geometric sequences and series. In part (a), most students obtained the value of the second term. Many stated the value of A correctly in part (b) but very few were able to state that the model is a geometric sequence, explaining that the 2% reduction leads to a common ratio of 0.98

Few could use the formula to find the sum of a geometric series in part (c). Many students confused the 15th term and the sum of the first 15 terms.. Several candidates found S_{15} by calculating each of the first 15 terms of the geometric series and adding them, which although correct, is very time-consuming. In part (d), about 10% of students scored two marks for calculating the sum to infinity correctly, but they forgot to add 4 litres to their final answer. Nearly one in five students did not attempt this part of the question. There was clear evidence of a better understanding of the context as many students scored one mark for identifying an environmental factor that affects the volume of water. However, very few students recognised that the model used assumes the drips continue indefinitely.

Question 8
This question was not well attempted overall with only about one-quarter of students achieving full marks. At the start of the question, students often recognised that integration by parts was required and started with $u = x$ and $v' = \cos x$. However, some could not apply the integration by parts formula correctly by substituting their *u, u', v* and *v*'. Common errors were to state that the integral of $\cos x$ was $-\sin x$ or to differentiate $\cos x$. Fortunately, despite not achieving the correct integrated expression, many correctly substituted the integral limits and used an exact value for $\sin\theta$ or $\cos\theta$ to obtain further marks. A few students lost the final mark for incorrect further "simplification" of the surd expressions.

Question 9
A high proportion of students made good progress in this question involving differentiation. In part (a)(i), many differentiated f(x) and found at least one correct term in the second derivative. Determining the nature of the stationary points in part (a)(ii) was found to be challenging, although many were competent in showing that the minimum occurred at $x = -\frac{15}{4}$, by showing that $\mathrm{f}''\left(-\frac{15}{4}\right) > 0$. However, very few students knew the full approach to take in determining the nature of the stationary point at $x = 0$. A common misconception was to provide an incomplete justification by stating that a point of inflection occurred at $x = 0$ because $\mathrm{f}''(0) = 0$. Those who substituted two values either side of $x = 0$ went on to score full marks.

In part (b), just under one-third of students scored the mark and those who could not respond to this question lost a further mark in part (c)(ii). The somewhat awkward consideration of whether the stationary values should be included was ignored,with students able to score the mark irrespective of whether or not either of these were included in the answer.

In (c)(i) about 40% of students deduced that the transformation is a reflection in the line x = 0. An incorrect response such as reflection in the line $y = x$ or translation was often seen.

Section B

Question 10
A little over 60% of the students answered correctly here.

Question 11
A significant minority of students could not use the formulae for the mean and variance of a binomial distribution, which are stated in the formulae booklet.

Question 12
Very few students scored full marks. Most students scored one mark for obtaining the number 80. However, only some could set up an enumerated population using a valid numbering system. Unfortunately, they were not able to make good progress from this point onward as they could not explain the need to randomly select the first person from the first 80 people and thereafter every subsequent 80th person. The most common approach was to choose the 80th, 160th, 240th person and so on.

Question 13
There was a clear lack of use of a calculator which affected students' performance in part (a)(i). Some tried to work out the mean and standard deviation using formulae rather than with a calculator, wasting time and often obtaining the wrong answers. In part (a)(ii), the majority of students achieved one mark for calculating the lower or upper bound correctly. However, they could not score further as they did not make a clear comparison with 192, and some only worked out the upper outlier bound, ignoring the lower bound.

Students lacked familiarity with the Large Data Set and did not score well in part (b). Those who got one mark explained that the 0 value is an error because every car has a mass or the mass of an average driver is included in the mass of each car in the LDS. However, very few students could explain that the blank cell may not be an error as the LDS only has particulate emissions recorded for some cars.

Question 14
The use of a Venn diagram was common in part (a). However, some lost marks when they set up an incorrect algebraic expression for P(A) and P(B). Students who used the formula for $\mathrm{P}(A \cup B)$ were more successful in obtaining full marks than those who used a Venn diagram. In part (b), just over 50% of the students could use the conditional probability formula with 0.1 and their P(A) substituted correctly. Common errors involved writing P(B) as the denominator for P$(B|A)$. Although students recalled the equation to be used to show the independence of two events, very few scored the mark in part (c) because they had not obtained the correct P(A) previously.

Question 15
Hypothesis testing continues to be an area requiring further improvement. Most students were able to gain at least two marks by stating the hypotheses and calculating the sample mean correctly. However, not many more than 10% of students scored full marks. The most successful students found the test statistic and compared it with the critical value or compared the probability with 0.01, but those who tried to define the acceptance region did less well. Not many could conclude correctly in context using appropriate wording such as 'sufficient evidence'.

Question 16
Just over 40% of students scored full marks in each of parts (a) and (b). Many struggled with substituting the given x values into the probability function to obtain at least three correct expressions in terms of c. Those who attempted the question by creating a table of values scored very well.

Question 17
In part (a) many could not correctly state relevant binomial assumptions in context. Students scored well in calculations involving the binomial distribution in parts (b) and (c). However, a fully correct solution for the hypothesis test in part (d) was very rare. Most could state the null and alternative hypotheses using the correct notation. However, many students were confused about the value of p and incorrectly used $\frac{12}{15}$. Very few successfully calculated $P(X \geq 12)$.

Question 18
Students scored reasonably well in this question involving a normal distribution. A little over 25% of students scored the mark in part (a)(i) where the majority could not recall that in a Normal distribution P$(X = x)$ = 0. However, the majority scored both marks in part (a)(ii). In part (b)(i) many could not explain that the 1.96 is obtained from the inverse normal distribution function. However,

they managed to form an equation with unknown μ, σ and 1.96 correctly, then successfully rearranged the equation to show the required result. Students were often caught out by needing to use the inverse Normal distribution function again in part (b)(ii). Common errors were the use of 0.14 as their z-value and the omission of the negative sign in the z-value. Of those who used the correct simultaneous equations, many went on to successfully obtain the correct values of μ and σ.

Concluding Remarks

There was a good balance of knowledge between the areas of pure mathematics and statistics with a similar mean score in both sections. However, there is clear evidence that students did not take full advantage of the calculator and often spent time doing a long manual calculation which often led to an incorrect answer. Recalling and using formulae correctly continued to be a problem on many occasions.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.