AS
# FURTHER MATHEMATICS
# 7366/1
Paper 1

**Mark scheme**

June 2021

Version: 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

# Mark scheme instructions to examiners

## General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

**Key to mark types**

| | |
|---|---|
| M | mark is for method |
| R | mark is for reasoning |
| A | mark is dependent on M marks and is for accuracy |
| B | mark is independent of M marks and is for method and accuracy |
| E | mark is for explanation |
| F | follow through from previous incorrect result |

**Key to mark scheme abbreviations**

| | |
|---|---|
| CAO | correct answer only |
| CSO | correct solution only |
| ft | follow through from previous incorrect result |
| 'their' | indicates that credit can be given from previous incorrect result |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| NMS | no method shown |
| PI | possibly implied |
| sf | significant figure(s) |
| dp | decimal place(s) |

Examiners should consistently apply the following general marking principles:

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**Diagrams**

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

**Work erased or crossed out**

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

**Choice**

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

## AS/A-level Maths/Further Maths assessment objectives

| AO | | Description |
|---|---|---|
| **AO1** | AO1.1a | Select routine procedures |
| | AO1.1b | Correctly carry out routine procedures |
| | AO1.2 | Accurately recall facts, terminology and definitions |
| **AO2** | AO2.1 | Construct rigorous mathematical arguments (including proofs) |
| | AO2.2a | Make deductions |
| | AO2.2b | Make inferences |
| | AO2.3 | Assess the validity of mathematical arguments |
| | AO2.4 | Explain their reasoning |
| | AO2.5 | Use mathematical language and notation correctly |
| **AO3** | AO3.1a | Translate problems in mathematical contexts into mathematical processes |
| | AO3.1b | Translate problems in non-mathematical contexts into mathematical processes |
| | AO3.2a | Interpret solutions to problems in their original context |
| | AO3.2b | Where appropriate, evaluate the accuracy and limitations of solutions to problems |
| | AO3.3 | Translate situations in context into mathematical models |
| | AO3.4 | Use mathematical models |
| | AO3.5a | Evaluate the outcomes of modelling in context |
| | AO3.5b | Recognise the limitations of models |
| | AO3.5c | Where appropriate, explain how to refine models |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 1 | Ticks correct box | 1.1b | B1 | $\cos(2) + i\sin(2)$ |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 2 | Circles correct answer | 1.1b | B1 | 17 |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 3 | Ticks correct box | 1.1b | B1 | $a = -\frac{1}{2} \quad b = -\frac{\sqrt{3}}{2}$ |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 4 | Circles correct answer | 1.1b | B1 | $\begin{bmatrix} 4 & 6 \\ 2 & 5 \end{bmatrix}$ |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 5 | Calculates the scalar product of the two vectors.<br>Accept one miscopy of an element, or one incorrect product if the elements are not shown. | 1.1a | M1 | $\begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix} . \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix} = 1 \times 7 + (-3) \times 4 + 5 \times 1$<br>$= 7 - 12 + 5 = 0$<br>$\therefore$ the two vectors are perpendicular |
| | Completes a fully correct proof to reach the required result.<br>Must include at least one calculation line and =0 linked with perpendicular conclusion.<br>Ignore incorrect vector magnitudes. | 2.1 | R1 | |
| | **Total** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 6 | Recalls exponential definitions of hyperbolic functions and substitutes $\frac{e^x+e^{-x}}{2}$ **and** $\frac{e^x-e^{-x}}{2}$<br>Condone cosh $x$ replaced with $\frac{e^x-e^{-x}}{2}$ and sinh $x$ replaced with $\frac{e^x+e^{-x}}{2}$ | 1.2 | M1 | $\cosh^2 x - \sinh^2 x =$<br>$\left(\frac{e^x+e^{-x}}{2}\right)^2 - \left(\frac{e^x-e^{-x}}{2}\right)^2$<br>$= \frac{e^{2x}+2+e^{-2x}}{4} - \frac{e^{2x}-2+e^{-2x}}{4}$<br>$= \frac{4}{4}$<br>$= 1$ |
| | Completes a fully correct proof to reach the required result. | 2.1 | R1 | |
| | **Total** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 7 | Writes the given expression in the form $m + \ln(1 + \mathrm{f}(x))$ where $m$ is a non-zero constant (allow unsimplified).<br>Or obtains the first three derivatives of $\ln(\mathrm{e} + 2\mathrm{e}x)$ in the form $p(q + rx)^n$ | 3.1a | M1 | $\ln(\mathrm{e} + 2\mathrm{e}x) = \ln(e(1 + 2x))$<br>$= \ln \mathrm{e} + \ln(1 + 2x)$<br>$= 1 + 2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \ldots$<br>$= 1 + 2x - 2x^2 + \frac{8}{3}x^3 - \ldots$ |
| | From an expression of the form $\ln(1 + \mathrm{f}(x))$, substitutes $\mathrm{f}(x)$ into the Maclaurin series.<br>Condone sign errors and missing brackets.<br>Or substitutes $x = 0$ into their f(0), f'(0), f''(0) and f'''(0) of the form $p(q + rx)^n$ | 1.1a | M1 | |
| | Completes a fully correct argument to reach the required result and correctly calculates $a$.<br>Do not condone missing brackets in their working. | 2.1 | R1 | |
| | **Total** | | **3** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **8(a)** | Gives a correct explanation.<br>Condone an explanation which suggests that conjugate pairs only occur if the coefficients are real. | 2.3 | B1 | Can only assume that complex roots of a polynomial equation are in conjugate pairs if the coefficients are all real. |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **8(b)** | Writes an equation for the 3rd root using the sum of roots.<br>Condone $-2\mathrm{i}$ or $2$ or $-2$ for the sum of roots.<br>Or writes an equation in $p$ and $q$ (may be unsimplified) from substitution of one known root into the equation. | 3.1a | M1 | $(1+\mathrm{i})+(-1)+\alpha=2\mathrm{i}$<br>$\alpha=\mathrm{i}$<br>$p=(1+\mathrm{i})(-1)+(-1)\mathrm{i}+\mathrm{i}(1+\mathrm{i})$<br>$p=-1-\mathrm{i}-\mathrm{i}+\mathrm{i}+\mathrm{i}^2$<br>$p=-2-\mathrm{i}$<br>$q=-(1+\mathrm{i})(-1)\mathrm{i}$<br>$q=\mathrm{i}+\mathrm{i}^2$<br>$q=\mathrm{i}-1$ |
| | Obtains correct 3rd root.<br>PI by a factor of $(z-\mathrm{i})$<br>Or writes two correct simultaneous equations in $p$ and $q$ from substitution of both known roots into the equation (any $i^2$ must be replaced with –1). | 1.1b | A1 | |
| | Writes an expression for $p$ using the sum of pairwise products of roots with their 3rd root and the two given roots.<br>Or eliminates $q$ from their simultaneous equations.<br>Or multiplies $\big(z-(1+\mathrm{i})\big)(z+1)(z-\alpha)$ to obtain a cubic expression where $\alpha$ is their 3rd root. May be unsimplified.<br>Allow sign errors. | 3.1a | M1 | |
| | Writes an expression for $q$ using the product of roots with their 3rd root and the two given roots.<br>Or eliminates $p$ from their simultaneous equations.<br>Or compares the coefficients of the given polynomial and their cubic expansion.<br>Allow sign errors. | 3.1a | M1 | |
| | Obtains $p=-2-\mathrm{i}$ **and** $q=\mathrm{i}-1$ | 1.1b | A1 | |
| | **Total** | | **5** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **6** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **9(a)** | Writes the sum in the form $\alpha\sum r^2 + \beta\sum r$ where $\alpha$ and $\beta$ are numbers.<br>PI by use of correct standard formulae. | 1.1a | M1 | $\sum_{r=1}^{n} r(r+3) = \sum_{r=1}^{n} r^2 + 3\sum_{r=1}^{n} r$ |
| | Recalls and uses standard formulae to obtain a correct expression for the sum in terms of $n$<br>May be unsimplified. | 1.2 | A1 | $= \frac{1}{6}n(n+1)(2n+1) + 3 \times \frac{1}{2}n(n+1)$ |
| | Identifies $n$ and $(n+1)$ as common factors.<br>Must be at least one correct term inside the remaining bracket. | 1.1a | M1 | $= \frac{1}{6}n(n+1)\big((2n+1)+9\big)$<br>$= \frac{1}{6}n(n+1)(2n+10)$ |
| | Completes a fully correct proof to reach the required result, including a clear statement that the original sum can be written in the form $\alpha\sum r^2 + \beta\sum r$<br>Must have $a = \frac{1}{3}$ and $b = 5$ | 2.1 | R1 | $= \frac{1}{3}n(n+1)(n+5)$ |
| | **Total** | | **4** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **9(b)** | Substitutes $5n$ into their expression of the form $an(n+1)(n+b)$ **and** subtracts their part **(a)**. | 1.1a | M1 | $\sum_{r=n+1}^{5n} r(r+3)$<br>$= \sum_{r=1}^{5n} r(r+3) - \sum_{r=1}^{n} r(r+3)$ |
| | Obtains a correct expression.<br>May be unsimplified. | 1.1b | A1 | $= \frac{1}{3}5n(5n+1)(5n+5) - \frac{1}{3}n(n+1)(n+5)$ |
| | Obtains a correct fully factorised expression. | 1.1b | A1 | $= \frac{1}{3}n(n+1)\big(25(5n+1)-(n+5)\big)$<br>$= \frac{1}{3}n(n+1)(124n+20)$<br>$= \frac{4}{3}n(n+1)(31n+5)$ |
| | **Total** | | **3** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **7** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **10(a)** | Writes a correct unsimplified expression for det **A** | 1.1a | M1 | $3 \times 2 - \mathrm{i}(\mathrm{i}-1) = 6 - \mathrm{i}^2 + \mathrm{i}$<br>$= 7 + \mathrm{i}$ |
| | Completes a fully correct proof to reach the required result.<br>Must have $a = 7$ | 2.1 | R1 | |
| | **Total** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 10(b) | Recognises that **B** is equal to a multiple of $\mathbf{A}^{-1}$<br>Or multiplies **A** and **B** to find at least one correct unsimplified element of **AB** | 3.1a | M1 | $\mathbf{B} = p\mathbf{A}^{-1}$<br>$\begin{bmatrix} 14-2\text{i} & b \\ c & d \end{bmatrix} = \dfrac{p}{7+\text{i}}\begin{bmatrix} 2 & 1-\text{i} \\ -\text{i} & 3 \end{bmatrix}$<br>$14 - 2\text{i} = \dfrac{2p}{7+\text{i}}$<br>$(7-\text{i})(7+\text{i}) = p$<br>$p = 50$<br>$b = \dfrac{50(1-\text{i})}{7+\text{i}} = 6 - 8\text{i}$<br>$c = \dfrac{-50\text{i}}{7+\text{i}} = -1 - 7\text{i}$<br>$d = \dfrac{150}{7+\text{i}} = 21 - 3\text{i}$ |
| | Sets up at least one correct non-matrix equation in one or two unknowns by equating a pair of corresponding elements. | 1.1a | M1 | |
| | Sets up another correct non-matrix equation in one unknown only. | 1.1a | M1 | |
| | Obtains at least one correct value of $p$, $b$, $c$ or $d$.<br>Could be seen as an element of a matrix,<br>e.g. $\frac{1}{50}\begin{bmatrix} 14-2\text{i} & * \\ * & * \end{bmatrix}$ or $k\begin{bmatrix} 14-2\text{i} & 6-8\text{i} \\ * & * \end{bmatrix}$<br>or $k\begin{bmatrix} 14-2\text{i} & * \\ -1-7\text{i} & * \end{bmatrix}$ or $k\begin{bmatrix} 14-2\text{i} & * \\ * & 21-3\text{i} \end{bmatrix}$ | 1.1b | A1 | |
| | Obtains at least **two** correct values of $p$, $b$, $c$ or $d$.<br>Could be seen as an element of a matrix,<br>e.g. $\frac{1}{50}\begin{bmatrix} 14-2\text{i} & * \\ * & * \end{bmatrix}$ or $k\begin{bmatrix} 14-2\text{i} & 6-8i \\ * & * \end{bmatrix}$<br>or $k\begin{bmatrix} 14-2\text{i} & * \\ -1-7\text{i} & * \end{bmatrix}$ or $k\begin{bmatrix} 14-2\text{i} & * \\ * & 21-3\text{i} \end{bmatrix}$ | 1.1b | A1 | |
| | Obtains all four correct values of $p$, $b$, $c$ and $d$.<br>Accept $p = 50$ and<br>$\mathbf{B} = \begin{bmatrix} 14-2\text{i} & 6-8\text{i} \\ -1-7\text{i} & 21-3\text{i} \end{bmatrix}$ | 1.1b | A1 | |
| | **Total** | | **6** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **8** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 11(a) | Obtains the correct result including at least one intermediate step.<br>All lines must be correct.<br>Condone original expression omitted. | 2.1 | B1 | $\frac{1}{(r-1)!}-\frac{1}{r!}=\frac{r}{r!}-\frac{1}{r!}=\frac{r-1}{r!}$ |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 11(b) | Writes the first two pairs (or last two pairs) of corresponding terms of $\frac{1}{(r-1)!}$ and $\frac{1}{r!}$ | 1.1a | M1 | $\sum_{r=1}^{n}\frac{r-1}{r!}=\sum_{r=1}^{n}\left(\frac{1}{(r-1)!}-\frac{1}{r!}\right)$<br>$=\frac{1}{0!}-\cancel{\frac{1}{1!}}$<br>$+\cancel{\frac{1}{1!}}-\cancel{\frac{1}{2!}}$<br>$+\cancel{\ldots\ldots}$<br>$+\cancel{\frac{1}{(n-2)!}}-\cancel{\frac{1}{(n-1)!}}$<br>$+\cancel{\frac{1}{(n-1)!}}-\frac{1}{n!}$<br>$=\frac{1}{0!}-\frac{1}{n!}$<br>$=1-\frac{1}{n!}$ |
| | Writes at least the first pair and the last pair of corresponding terms of $\frac{1}{(r-1)!}$ and $\frac{1}{r!}$ and shows the pattern of cancelling. | 1.1a | M1 | |
| | Completes a fully correct proof to reach the required result with $a=1$ and $b=-1$<br>Must include the 1st and $n$th terms and at least one pair of cancelling terms when completing the method of differences process. | 2.1 | R1 | |
| | **Total** | | **3** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **4** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **12(a)** | Obtains the other two roots. | 1.1b | B1 | $x^3 - 2x^2 - x + 2 = (x-2)(x^2-1)$<br>$= (x-2)(x-1)(x+1)$<br><br>$x = 1$ and $x = -1$ |
| | **Total** | | **1** | |

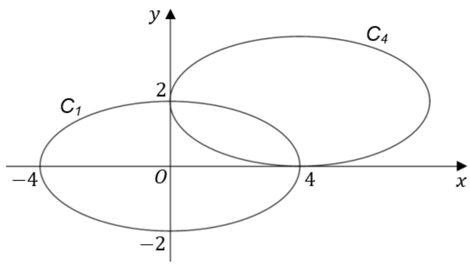
| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **12(b)** | Obtains at least one correct value for $\cosh\theta$<br>Or uses their answer to part **(a)** to write down a value for $\cosh\theta$ | 1.1a | M1 | $\cosh\theta = 2$ ,<br>$\cosh\theta = 1$ ,<br>$\cosh\theta = -1$<br><br>but $\cosh\theta \geq 1$ $\therefore$ $\cosh\theta \neq -1$<br><br>$\theta = \pm\ln\left(2+\sqrt{3}\right)$ , $\theta = 0$ |
| | Rejects $\cosh\theta = -1$<br>Follow through their part **(a)** $< 1$<br>PI by only considering values for which $\cosh\theta \geq 1$ | 1.1b | B1ft | |
| | Correctly finds at least one non-zero root of the equation.<br>Follow through any of their answers to part **(a)** if greater than $1$. | 1.1b | A1ft | |
| | Obtains the three correct values of $\theta$ with no incorrect answers.<br>Accept $\theta = \pm\cosh^{-1}(2)$ | 1.1b | A1 | |
| | **Total** | | **4** | |

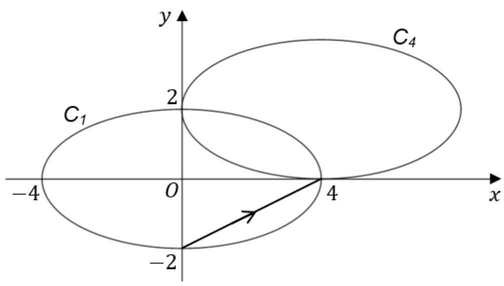
| | | | | |
|---|---|---|---|---|
| | **Question total** | | **5** | |

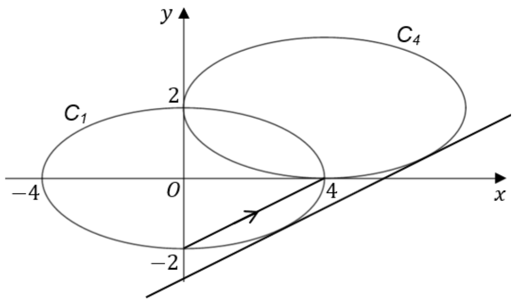
| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **13** | Demonstrates the rule is correct for $n = 1$ | 2.2a | B1 | Try $n = 1$:<br>LHS = $2^{-1} = \frac{1}{2}$<br>and<br>RHS = $1 - 2^{-1} = \frac{1}{2}$<br>LHS = RHS $\therefore$ the rule is true for $n = 1$ |
| | Assumes the rule is true for $n = k$ **and** adds $2^{-(k+1)}$ to $1 - 2^{-k}$<br>Condone incorrect or missing brackets for this mark only. | 2.4 | M1 | Assume the rule is true for $n = k$:<br>$\sum_{r=1}^{k} 2^{-r} = 1 - 2^{-k}$<br>$\sum_{r=1}^{k} 2^{-r} + 2^{-(k+1)} = 1 - 2^{-k} + 2^{-(k+1)}$<br>$\sum_{r=1}^{k+1} 2^{-r} = 1 + 2^{-(k+1)}(-2^{1} + 1)$<br>$\sum_{r=1}^{k+1} 2^{-r} = 1 - 2^{-(k+1)}$<br>$\therefore$ the rule is also true for $n = k + 1$ |
| | Correctly obtains $1 - 2^{-(k+1)}$ from $1 - 2^{-k} + 2^{-(k+1)}$ | 2.2a | A1 | |
| | Concludes a reasoned argument by stating that<br><br>The rule is true for $n = 1$;<br><br>that if the rule is true for $n = k$ then it is also true for $n = k + 1$<br><br>and hence, by induction, the rule is true for all integers $n \geq 1$. | 2.1 | R1 | As the rule is true for $n = 1$, and if true for $n = k$ then it is also true for $n = k + 1$, then by induction the rule is true for all integers $n \geq 1$ |
| | **Total** | | **4** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 14(a) | Writes down a correct equation. | 1.1b | B1 | $\frac{y^2}{16}+\frac{x^2}{4}=1$ |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 14(b) | Indicates a stretch.<br>Condone a stretch in any direction. | 3.1a | M1 | $y \longrightarrow \frac{y}{2}$ |
| | Identifies the correct transformation. | 1.1b | A1 | stretch, parallel to the $y$-axis, scale factor 2 |
| | **Total** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 14(c)(i) | Draws one loop centred on the origin and a second loop, approximately the same shape as the first, in the 1st quadrant with the positive $x$ and $y$ axes as tangents.<br>Or draws one correct graph with one correct $x$-intercept and one correct $y$-intercept. | 1.1a | M1 |  |
| | Draws two correct graphs with all four intercepts correctly indicated. | 1.1b | A1 | |
| | **Total** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 14(c)(ii) | States a translation vector which contains either $2$ or $-2$ **and** $4$ or $-4$<br>Follow through their intercepts. | 3.1a | M1 |  |
| | Obtains the correct translation vector. | 1.1b | A1 | $\begin{bmatrix}4\\2\end{bmatrix}$ |
| | **Total** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 14(c)(iii) | Calculates $\frac{b}{a}$ or $\frac{a}{b}$ for their translation vector $\begin{bmatrix} a \\ b \end{bmatrix}$ | 3.1a | M1 |   $m = \frac{2}{4} = \frac{1}{2}$ |
| | Obtains the correct gradient.<br>Follow through their part (**cii**) | 2.2a | A1F | |
| | **Total** | | **2** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **9** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 15(a)(i) | Forms one equation by substituting **two** of $x = 2 + 5\lambda$, $y = -1 + 3\lambda$, $z = 4 - 2\lambda$ into the equation of the second line.<br><br>Or introduces a second parameter (eg $\mu$) to form at least **two** equations in $\lambda$ and $\mu$,<br>e.g. $2 + 5\lambda = 4\mu + 5$, $-1 + 3\lambda = 2\mu$, $4 - 2\lambda = 4 - \mu$ | 1.1a | M1 | $\dfrac{2+5\lambda-5}{4} = \dfrac{-1+3\lambda}{2}$<br><br>$2(5\lambda - 3) = 4(3\lambda - 1)$<br>$10\lambda - 12\lambda = -4 + 6$<br>$-2\lambda = 2$<br>$\lambda = -1$ |
| | Solves their equation(s) to obtain the correct value of $\lambda$ (or $\mu$). | 1.1b | A1 | $\dfrac{2+5\lambda-5}{4} = 4 - (4 - 2\lambda)$<br><br>$5\lambda - 3 = 4 \times 2\lambda$<br>$-3 = 8\lambda - 5\lambda$<br>$-3 = 3\lambda$<br>$\lambda = -1$ |
| | Completes a correct argument to conclude that the lines intersect.<br><br>eg shows that $\lambda$ = -1 satisfies another equation formed by substituting a different combination of $x$, $y$ and $z$<br><br>Or shows that $\lambda$ = -1 and $\mu$ = -2 satisfy a third equation.<br><br>Or shows that (-3, -4, 6) lies on both lines. | 2.1 | R1 | same value of $\lambda$ $\therefore$ the lines intersect |
| | **Total** | | **3** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 15(a)(ii) | Obtains $\begin{bmatrix} -3 \\ -4 \\ 6 \end{bmatrix}$<br><br>Accept $-3\boldsymbol{i} - 4\boldsymbol{j} + 6\boldsymbol{k}$<br><br>Condone $(-3, -4, 6)$ | 1.1b | B1 | $\boldsymbol{r} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} - 1\begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$<br><br>$= \begin{bmatrix} -3 \\ -4 \\ 6 \end{bmatrix}$ |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 15(b) | States a correct explanation. | 3.2a | B1 | The submarines might not be at the intersection point at the same time. |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **15(c)** | Calculates the scalar product of the two direction vectors.<br>Allow one error in each direction vector. | 1.1a | M1 | $\begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} . \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$<br>$= 5 \times 4 + 3 \times 2 + (-2) \times (-1) = 28$ |
| | Calculates the product of the direction vector magnitudes. | 1.1a | M1 | $\sqrt{5^2 + 3^2 + (-2)^2} \times \sqrt{4^2 + 2^2 + (-1)^2}$<br>$= \sqrt{38 \times 21} = \sqrt{798}$ |
| | Obtains correct angle.<br>Condone 0.13 (radians).<br>Accept 7.61° or better. | 1.1b | A1 | $\cos\theta = \dfrac{28}{\sqrt{798}}$<br>$\theta = 7.6°$ (1dp) |
| | **Total** | | **3** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **8** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 16(a) | Obtains $a = 3$<br><br>Accept $y = \frac{3x}{x+b}$ with any non-zero value of *b* | 2.2a | B1 | $y = \dfrac{3x}{x+2}$<br><br>$a = 3$ and $b = 2$ |
| | Obtains $b = 2$<br><br>Accept $y = \frac{ax}{x+2}$ with any non-zero value of *a* | 2.2a | B1 | |
| | **Total** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 16(b) | Explains that $y = \frac{3x}{2}$ is a tangent to the curve.<br>Condone 'tangent' only. | 2.4 | B1 | $y = \frac{3x}{2}$ is a tangent to the hyperbola.<br><br>Any tangent to a hyperbola only intersects once.<br><br>So there is exactly one root of the equation. |
| | Explains that a tangent to a hyperbola meets the curve exactly once.<br>Accept 'conic' instead of 'hyperbola'.<br>Condone no conclusion given. | 2.4 | B1 | |
| | **Total** | | **2** | |

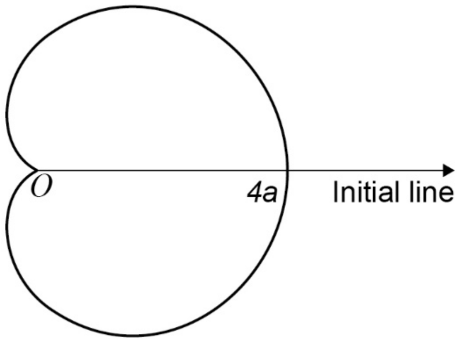
| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **16(c)** | Forms a simplified quadratic equation from $ax - (1-x)(x+b) = 0$<br><br>Accept any inequality sign instead of $=$<br><br>PI by $x^2 + 4x - 2$ or $-x^2 - 4x + 2$<br><br>PI by $-2+\sqrt{6}$ or $-2-\sqrt{6}$<br><br>Or forms a simplified cubic equation from $ax(x+2) - (1-x)(x+b)(x+2) = 0$ | 1.1a | M1 | $\frac{3x}{x+2} = 1 - x$<br><br>$3x = (1-x)(x+2)$<br><br>$3x = -x^2 - x + 2$<br><br>$x^2 + 4x - 2 = 0$<br><br>$x = \frac{-4 \pm \sqrt{16 + 4 \times 2}}{2}$<br><br>$x = -2 \pm \sqrt{6}$ |
| | Identifies $-2+\sqrt{6}$ or $-2-\sqrt{6}$ as a critical value.<br>Follow through their $a$ and $b$ | 1.1b | A1F | |
| | Identifies $-2$ as a critical value. | 1.1b | B1 | $x \leq -2-\sqrt{6}$ ,<br><br>$-2 < x \leq -2+\sqrt{6}$ |
| | Obtains the correct set of values of $x$ | 3.2a | A1 | |
| | **Total** | | **4** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **8** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **17(a)** | States correct polar coordinates.<br><br>Condone $(0, 2a)$ after seeing $r = 2a$ and $\theta = 0$ | 1.1b | B1 | $r = 2a(1 + \sin 0) = 2a$<br><br>$M = (2a, 0)$ |
| | **Total** | | **1** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **17(b)** | Substitutes $\sin\theta = 1$<br>PI by $r = 4a$ or $\theta = \frac{\pi}{2}$ | 3.1a | M1 | $r = 2a(1 + 1) = 4a$<br><br>$N = \left(4a, \frac{\pi}{2}\right)$ |
| | Obtains the correct polar coordinates.<br><br>Condone $\left(\frac{\pi}{2}, 4a\right)$ after $r = 4a$ and $\theta = \frac{\pi}{2}$<br><br>Condone $\left(\frac{\pi}{2}, 4a\right)$ if $(0, 2a)$ has been penalised in **(a)**. | 1.1b | A1 | |
| | **Total** | | **2** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| 17(c) | Forms an equation by equating the two polar equations. | 3.1a | M1 | $3a = 2a(1+\sin\theta)$<br>$\frac{3}{2} = 1 + \sin\theta$<br>$\sin\theta = \frac{1}{2}$<br>$\theta = \frac{\pi}{6}, \quad \theta = \frac{5\pi}{6}$ |
| | Obtains at least one correct $\theta$ value for *P* or Q<br>Implied by a correct area. | 1.1b | A1 | |
| | Calculates the $x$-coordinate of $P$ or $Q$,<br>ie $x = 3a\cos\theta$<br>Implied by $6a\cos\theta$ for the length $PQ$.<br>Or calculates the $y$-coordinate of $P$ or $Q$,<br>ie $y = 3a\sin\theta$ | 1.1a | M1 | $x = 3a\cos\frac{\pi}{6}$<br>$x = \frac{3\sqrt{3}a}{2}$ |
| | Calculates the required area (or half of the required area).<br>ie $\frac{1}{2} \times (y_N - y_P) \times 6a\cos\theta$ | 2.2a | M1 | $y = 3a\sin\frac{\pi}{6}$<br>$y = \frac{3a}{2}$ |
| | Obtains the correct area. | 3.2a | A1 | $\text{area} = \frac{1}{2} \times \frac{6\sqrt{3}a}{2} \times \left(4a - \frac{3a}{2}\right)$<br>$= \frac{3\sqrt{3}a}{2} \times \frac{5a}{2}$<br>$= \frac{15}{4}\sqrt{3}a^2$ |
| | **Total** | | **5** | |

| Q | Marking instructions | AO | Marks | Typical solution |
|---|---|---|---|---|
| **17(d)** | Draws correct shape in each of the four quadrants with a cusp at the pole. | 1.1b | B1 |  |
| | Marks the point $(4a, 0)$ on the diagram.<br>Or marks $4a$ as the intercept on the initial line.<br>Condone omission of (0,0). | 1.1b | B1 | |
| | **Total** | | **2** | |

| | | | | |
|---|---|---|---|---|
| | **Question total** | | **10** | |

| | | | | |
|---|---|---|---|---|
| | **Paper total** | | **80** | |