AQA

A-level

FURTHER MATHEMATICS 7367/3M

Paper 3 Mechanics

Mark scheme

June 2021

Version: 1.0 Final Mark Scheme

216A7367/3M/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	1.1b	B1	1600
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	1.1b	B1	1.4 N m
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Uses vectors or diagram to show the change in momentum	3.3	M1	$\mathbf{u} = 6\mathbf{i}$ $\mathbf{v} = 8\mathbf{j}$ $\mathbf{I} = 0.4 \times 8\mathbf{j} - 0.4 \times 6\mathbf{i}$ $= -2.4\mathbf{i} + 3.2\mathbf{j}$ $I = \sqrt{2.4^2 + 3.2^2} = 4 \text{ Ns}$
	Applies the principle that impulse is change in momentum working in two dimensions.	1.1a	M1	
	Obtains the correct magnitude, from a valid argument	2.1	R1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Determines the correct dimensions of stiffness, CAO.	1.2	B1	MT^{-2}
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(b)	Formulates the problem using dimensional analysis with at least two dimensions correct.	3.3	M1	$T = M^a\left(LT^{-2}\right)^b\left(MT^{-2}\right)^c$ $a+c=0$ $b=0$ $-2b-2c=1$ $c=-\dfrac{1}{2}$ $a=\dfrac{1}{2}$
	Compares dimensions to deduce three equations in terms of a, b and c (PI)	2.2a	M1	
	Obtains the correct values of a, b and c	1.1b	A1	
	Total		**3**	

	Question total		**4**	

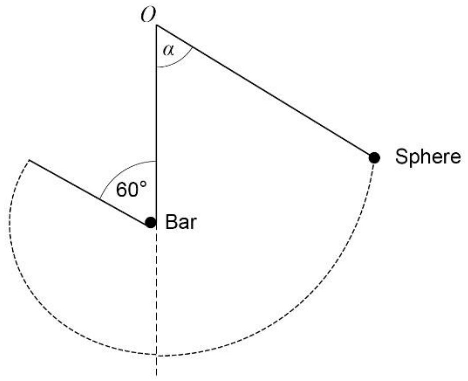
Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Obtains correct mass or area, PI	1.1b	B1	$\text{Mass} = \rho\int_0^4 \left(x^2+1\right)dx$ $= \dfrac{76}{3}\rho$ $\bar{x}\times\dfrac{76}{3}\rho = \rho\int_0^4\left(x^3+x\right)dx$ $\dfrac{76}{3}\bar{x} = 72$ $\bar{x} = \dfrac{54}{19}$ $\bar{y}\times\dfrac{76}{3}\rho = \dfrac{1}{2}\rho\int_0^4 (x^2+1)^2 dx$ $\dfrac{76}{3}\bar{y} = \dfrac{3772}{30}$ $\bar{y} = \dfrac{943}{190}$ $\left(\dfrac{54}{19}, \dfrac{943}{190}\right)$
	Sets up equation to find $\bar{x}$ using appropriate integral Condone missing density	1.1a	M1	
	Sets up equation to find $\bar{y}$ using appropriate integral Condone missing density	1.1a	M1	
	Obtains correct exact value for $\bar{x}$ and for $\bar{y}$, written as fractions.	1.1b	A1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(b)	Drawing a diagram showing the CM and the point of suspension and the required angle, PI.	3.4	M1	$\tan\alpha = \dfrac{4-\dfrac{54}{19}}{17-\dfrac{943}{190}}$ $\alpha = 5^{\circ}$
	Uses their model to set up a suitable trigonometric equation using their coordinates to find the angle	1.1a	M1	
	Obtains the correct angle (FT from part a)	1.1b	A1F	
	Total		**3**	

	Question total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
6	Finds impact speed or speed^2 or KE in terms of h	3.3	B1	Let U = Impact speed $U^2 = 0^2 + 2gh$ $U = \sqrt{2gh}$ Let V = Rebound speed $V = e\sqrt{2gh}$ KE Lost $= \frac{1}{2}mU^2 - \frac{1}{2}mV^2$ $= mgh - mghe^2$ $= mgh\left(1 - e^2\right)$
	Uses the law of restitution to find rebound speed in terms of h or impact speed based on their impact speed	1.1a	M1	
	Forms expression for the KE lost using their impact and rebound speeds	1.1a	M1	
	Completes a rigorous argument to show the required result	2.1	R1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	States that the 60° angle is too small	2.3	E1	The 60° angle is too small The sphere must start at a higher level than where the string becomes slack
	States that the initial position must be higher than the point at which the string becomes slack	2.3	E1	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(b)	Draws a diagram that shows the initial position more accurately, where the sphere starts at higher level than the final level shown.	2.3	B1	
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(c)	Introduces a variable for the mass and uses it in an equation	1.1b	B1	Let mass = m and speed when the string becomes slack = v $mg\cos 60° = \frac{mv^2}{0.6}$ $v^2 = 0.6g\cos 60°$ Take GPE as zero at the bar $mgh = \frac{1}{2}m \times 0.6g\cos 60° + mg \times 0.6\cos 60°$ $h = 0.45$ $\cos\alpha = \frac{0.45}{1.5} = 0.3$ $\alpha = 73°$
	Resolves radially at the point where the string becomes slack and uses Newton's second law.	3.3	M1	
	Obtains correct expression for v or v^2	1.1b	A1	
	Forms a three term energy equation and substitutes their value for v	3.4	M1	
	Obtains a correct energy equation	1.1b	A1	
	Deduces the correct angle	2.2a	A1	
	Total		**6**	

	Question total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Recognises that WD is the integral of force with respect to x Condone missing dx	3.3	M1	$\text{WD} = \int_0^{12} k(x+1)(12-x) + 2450dx$ $= k\int_0^{12}(x+1)(12-x)dx + \int_0^{12} 2450dx$ $= 360k + 29400$
	Obtains correct integral with correct limits	1.1b	A1	
	Obtains correct expression for the work done from a rigorous argument	2.1	R1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(b)	Forms an energy equation to find k	3.3	M1	$29400 + 360k = 250\times 9.8\times 12 + \frac{1}{2}\times 250\times 3^2$ $360k = 1125$ $k = \frac{25}{8}$
	Obtains the correct value for k	1.1b	A1	
	Finds the work done when the crate has risen from 0 to15 metres or from 12 to 15 metres, FT their k	1.1a	M1	$\text{Work Done} = 36750 + \frac{25}{8}\times\frac{585}{2}$
	Forms an energy equation to find the speed at 15 metres	3.4	M1	$36750 + \frac{25}{8}\times\frac{585}{2} = 250\times 9.8\times 15 + \frac{1}{2}\times 250\times v^2$ $v = \sqrt{\frac{2\times 25\times 585}{16\times 250}}$ $= 2.704 = 2.7 \text{ m s}^{-1}$ to 2sf
	Finds the correct speed and gives the answer to 2 sig fig, with correct units	3.2a	A1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(c)	Forms an equation to find x (PI)	3.4	M1	$12kx + \frac{11kx^2}{2} - \frac{kx^3}{3} = 0$ $kx\left(12 + \frac{11x}{2} - \frac{x^2}{3}\right) = 0$ $x = 0,\ 18.45,\ -1.95$ $x = 18$ metres to 2 sf
	Solves the equation and deduces correct value of x, condone missing units. AWRT 18	2.2a	A1	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(d)	Gives valid explanation with reference to low speeds	3.5a	E1	Reasonable because the speed of the crate is very low
	Total		**1**	

	Question total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Forms an equation that relates the length of the string to the radius of the circle	3.3	M1	(1.6+e)sinα =0.5 $e = \dfrac{0.5}{\sin\alpha} - 1.6$
	Forms an expression for the tension using Hooke's Law	1.1a	M1	$T = \dfrac{200\left(\dfrac{0.5}{\sin\alpha} - 1.6\right)}{1.6}$ $= \dfrac{62.5}{\sin\alpha} - 200$
	Resolves tension vertically to form an equation	3.4	M1	$T\cos\alpha = 0.2g = 1.962$
	Combines the three equations to eliminate tension and extension	1.1a	M1	$T = \dfrac{1.962}{\cos\alpha}$
	Derives the required result using a rigorous argument, must see justification for $\tan\alpha$	2.1	R1	$\dfrac{1.962}{\cos\alpha} = \dfrac{62.5}{\sin\alpha} - 200$ $\dfrac{1.962}{\cos\alpha}\sin\alpha = \dfrac{62.5}{\sin\alpha}\sin\alpha - 200\sin\alpha$ $1.962\tan\alpha = 62.5 - 200\sin\alpha$
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(b)	Obtains correct solution from calculator or numerical method. AWRT 18	1.1b	B1	18.0
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(c)	Uses their angle to find an expression for tension	1.1a	B1	$T = \dfrac{1.962}{\cos(18.02°)} = 2.0632$
	Resolves horizontally to form an equation in v	3.4	M1	$2.0632\sin\alpha = 0.2\dfrac{v^2}{0.5}$
	Obtains correct equation	1.1b	A1	$v = \sqrt{\dfrac{2.0632\sin(18.02°)}{0.4}}$
	Solves their equation to find the correct speed	3.4	A1	$= 1.263$ $= 1.26$ m s^{-1} to 3 sf
	Total		**4**	

	Question total		**10**	

	Paper total		**50**	