AQA

AS LEVEL
MATHEMATICS

7356/1
Report on the Examination

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General

This exam paper proved to be challenging to many students. There was a reduction in the overall mean score compared to the performance on this paper in 2019.

In some questions, students lacked the algebraic skills to be able to make any significant progress, such as in Q7, Q14 and Q16.

Good work was seen in Q3, Q4, Q5 and Q6 (a) and (b) in Section A and in Q13, Q15 and Q17 in Section B.

It is important that students know the relevant formulae to answer trigonometry and coordinate geometry questions. There was a significant minority of students who were unable to quote basic formulae such as the midpoint of two points correctly.

In some questions, a specific accuracy for the final answer is stated in the question. This requirement will be tested in at least one of these questions, but to be fair to students, it will not necessarily be tested on every occasion. It is therefore important that students do take note of any specific accuracy requirement stated in any question, and state their answers to this accuracy.

Students are advised that drawing a diagram to get a picture of what is going on in a question can often provide important clues to start and complete a solution. This was the case in Q6(c), where drawing the circle, labelling the points *D* and *E* where the circle cut the x-axis and labelling the centre of the circle *C*, gave an insight into how to find the area of triangle *DEC*. Drawing a diagram in Q15(a) also helped to identify the triangle needed to find the required angle between the two forces.

Question 1

This question on the laws of logarithms proved to be an excellent start for the vast majority of students, with around 90% identifying $\log_{10}\left(\frac{2}{x}\right)$ as the correct answer.

Question 2

Less than half of students identified the correct graph of $y = \cos\frac{1}{2}x$.

The graph could be identified from applying the correct stretch to $y = \cos x$ or by substituting two x values such as $x = 0^\circ$ and $x = 360^\circ$ into the given equation to identify two different points on the curve. The curve which matched these points could then be chosen.

Question 3

This question was generally well done. The majority of students were able to choose the required term from the expansion of $(3x + \frac{1}{2})^4$. Some students wasted time writing out the whole expansion, but this did not hinder their chances of obtaining full marks. A few students incorrectly evaluated $(3x)^3$ as $3x^3$.

The question asked for the **coefficient** of the x^3 term, so just quoting $54x^3$ as the final answer was not sufficient. It had to be clear that the coefficient was 54, either by stating this or by clearly identifying it.

Question 4

This question required the use of the substitution $\cos^2\theta = 1 - \sin^2\theta$ to obtain a quadratic equation in $\sin\theta$. This equation could then be solved using the calculator giving $\sin\theta = -5 \pm \sqrt{22}$
The two possible values of $\sin\theta$ could then be considered separately.

The solution $\sin\theta = -5 + \sqrt{22}$ generated two angles in the given range $\theta = 198^\circ$ and $\theta = 342^\circ$. For this question, it was necessary to give the final answers to the nearest degree to obtain full marks, as specified in the question.

The other value for $\sin\theta$ of $-5 - \sqrt{22}$ does not lie between –1 and 1 so does not generate any values for θ. There had to be some indication that this was the case. Any reasonable explanation was accepted, such as 'out of range', 'math error', 'no solutions' etc. but simply putting a '×' or just crossing $-5 - \sqrt{22}$ out was not accepted.

It is important that students know the few trig identities examined at AS level. There were a disappointing number of incorrect identities used such as $\sin\theta = 1 - \cos\theta$.

Question 5

This question was well done with over three-quarters of all students obtaining full marks. The majority of students used long division to achieve the correct answer; there were also successful attempts using comparing coefficients.

The question did ask for the final answer to be expressed in the form $(x - 2)(3x^2 + 11x - 5)$, but on this occasion full marks were given for either the term $3x^2 + 11x - 5$ or stating $a = 3$, $b = 11$ and $c = -5$.

Question 6

In part (a) there were a disappointing number of incorrect attempts at finding the midpoint, with (3 , −3) and (3 , 1) common wrong answers. However the great majority of students did get the correct answer of (4 , 1).

In part (b) there were two acceptable approaches.

The expected method was to work out the radius as the distance from point A or point B to the midpoint, or to find half of the diameter AB. Then using the coordinate of the midpoint (4 , 1) along with the radius of the circle $3\sqrt{2}$, the equation of the circle could be stated as

$(x - 4)^2 + (y - 1)^2 = (3\sqrt{2})^2$. This expression could then be multiplied out and simplified to obtain the given answer.

Some students started with the given answer and completed the square to obtain $(x - 4)^2 + (y - 1)^2 = 18$. However, this was insufficient to answer the question fully. It was still necessary to find the radius of the circle using the distance from point A or point B to the midpoint, or half of the diameter AB. Then a final conclusion comparing both the centre implied from the completed square form (4 , 1) to the midpoint of (a) and the radius squared of 18 from the completed square form to the square of the calculated radius was needed to obtain full marks.

Part (c) proved to be very challenging with over half of all students unable to make any progress. Students who drew a diagram of the scenario were often able to make more progress than those who did not. There were two main approaches that could be used to find the area of triangle DEC.

Firstly, the coordinates where the circle cuts the x-axis could be found by substituting $y = 0$ into the equation from part (b). This gave a quadratic in x which could be solved to obtain the exact values for x of $4 \pm \sqrt{17}$. The length DE could then be found as the difference between these two values of x. Finally, the area could be calculated using the y-coordinate of the midpoint as the vertical height of the triangle, along with the base of the triangle $DE = 2\sqrt{17}$. This gave a final exact area of $\sqrt{17}$.

An alternative approach was to use the fact that the triangle DEC is an isosceles triangle. If the mid-point of DE is M, then the length DM can be found in the right-angled triangle CDM with the hypotenuse $CD = \sqrt{18}$ and $CM = 1$. This gives $DM = \sqrt{17}$ and hence $DE = 2\sqrt{17}$. The area of triangle DEC could then be found using $\frac{1}{2} \times 2\sqrt{17} \times 1 = \sqrt{17}$.

Errors noted in this question included putting $x = 0$ rather than $y = 0$ into the equation and using an incorrect vertical height for the triangle.

Question 7

This was a challenging question. The inclusion of the constant 'a' in the equation of the curve proved to be a significant issue for most students.

The limits for the integral needed to be found by setting $y = 0$ and solving the equation $a^2 - x^2 = 0$. This gave limits of $x = \pm a$.

The equation $\int_{-a}^{a} \left(a^2 - x^2\right) dx = 36$ could then be set up using the given information.

The integration of a^2 with respect to x was the main stumbling block. Unfortunately, the vast majority of students integrated a^2 to $\frac{a^3}{3}$, rather than the correct a^2x. Those students who integrated correctly were then able to substitute the limits $x = a$ and $x = -a$ correctly to generate an equation in a^3. This equation could then be solved to obtain $a = 3$.

Question 8

There were cases seen in this question where students did work relevant to other parts of the question in the wrong part. For example, in part (a) rather than finding $\frac{dy}{dx}$, setting it to zero and showing the required equation, some students worked out the x-coordinates of the stationary points here. This approach still obtained credit, but students did not then show this same work which was required in part (b), instead looking for an alternative method which was invariably incorrect.

In part (a), the majority of students were able to differentiate at least one term correctly. The differentiation of the $\frac{9}{x}$ term did cause problems for some students because of mistakes made trying to write it as a term with a power of x.

$\frac{dy}{dx} = 3x^2 - 6 - \frac{9}{x^2}$ could then be put equal to 0, (condition for a stationary point), and the resulting equation rearranged to obtain the given answer.

In part (b) the deduction was meant to come from the equation generated in part (a). This required the equation to be factorised to $(x^2 - 3)(x^2 + 1) = 0$ which gave two possible real solutions for x of $x = \pm\sqrt{3}$, it could then be concluded that as $x^2 = -1$ has no real solutions, there are only 2 stationary points. Working from a calculator was accepted as long as the relevant points were still covered. Some students argued, from the quartic equation, that as $b^2 - 4ac > 0$, there are two real roots, so two stationary points. However, these roots were for x^2, so further work was need before a conclusion could be reached.

In part (c), it was necessary to differentiate $\frac{dy}{dx}$ again to obtain $\frac{d^2y}{dx^2}$ from which the nature of the turning points could be established. Unfortunately, a significant number of students used the equation generated in part (a) as $\frac{dy}{dx}$ rather than the correct $3x^2 - 6 - \frac{9}{x^2}$. This limited part (c) to 1 mark only. This was unfortunate, as students who used the correct $\frac{d^2y}{dx^2}$ were able to go on and successfully find the coordinates and nature of the two stationary points: $\left(\sqrt{3}, 0\right)$ is a minimum point on the curve and $\left(-\sqrt{3}, 0\right)$ is a maximum point on the curve.

In part (d) it was required to identify that the line $y = 0$ was a tangent to the curve. This answer was obtained by a larger proportion of students than expected, which was pleasing to see.

Question 9

There was evidence that students are engaging more with proof questions, with the vast majority of students making at least some attempt at this question. There were a number of issues setting up the proof which restricted the number of marks available in these cases:

- Some students defined two even numbers

- Some students defined two identical odd numbers e.g., $n = 2p + 1$ and $m = 2p + 1$
- Some students defined two consecutive odd numbers e.g., $n = 2n - 1$ and $m = 2n + 1$
- Some students defined two odd numbers using m and n in the definition, eg $n = 2n + 1$ and $m = 2m + 1$

Students needed to write down algebraic expressions for two odd numbers which were different, non-consecutive and not using 'n' or 'm' e.g., $n = 2p + 1$, $m = 2q + 1$

Then $m^2 + n^2$ could be found by expanding

$(2p + 1)^2 + (2q + 1)^2 = 4p^2 + 4p + 4q^2 + 4q + 2$

Factorising this gives $4p^2 + 4p + 4q^2 + 4q + 2 = 2(2p^2 + 2p + 2q^2 + 2q + 1)$

So, the factor of 2 shows that this is a multiple of 2, and the factor $2p^2 + 2p + 2q^2 + 2q + 1$ is odd, so $m^2 + n^2$ is a multiple of 2, but not a multiple of 4.

In all of these proof questions it is important that a final conclusion is stated to bring together all of the work done in the question.

Question 10

There were some good attempts at part (a) with a little over a quarter of all students obtaining full marks. The main issue for many students was the rewriting of $y = \frac{\sqrt{2}}{x^2}$ as $y = \sqrt{2}x^{-2}$ so that it could be differentiated. Some marks were still available for students who made an error doing this.

Differentiating $y = \sqrt{2}x^{-2}$ gives $\frac{dy}{dx} = -2\sqrt{2}x^{-3}$

Substituting $x = 2$ gives the gradient $\frac{dy}{dx} = -\frac{\sqrt{2}}{4}$

The equation of the tangent could then be stated as $y - \frac{\sqrt{2}}{4} = -\frac{\sqrt{2}}{4}(x-2)$

Part (b) proved to be an extremely challenging question with few students able to make any significant progress. There were two different methods which could be applied.

Firstly, the point where the tangent meets the curve again could be found by solving

$$\frac{\sqrt{2}}{x^2} = -\frac{\sqrt{2}}{4}x + \frac{3\sqrt{2}}{4}$$

This simplifies to the cubic equation $x^3 - 3x^2 + 4 = 0$

This equation could be solved by using the calculator or by factorisation.

So, the tangent intersects the curve again at x = −1

The gradient at x = −1 is $\dfrac{-2\sqrt{2}}{(-1)^3} = 2\sqrt{2}$

So

(Gradient of the curve at x = −1) × (Gradient of the curve at x = 2) = ($2\sqrt{2}$) × $\left(-\dfrac{\sqrt{2}}{4}\right)$ = -1

This shows that the tangent is perpendicular to the curve at x = –1, so it is a normal to the curve at another point on the curve.

An alternative approach was to work with the gradient of the curve and equate it to the gradient of the perpendicular to the curve.
So, solving $\dfrac{-2\sqrt{2}}{x^3} = 2\sqrt{2}$ gives x = −1 and substituting back gives $y = \sqrt{2}$.
To complete the argument it was then necessary to find the equation of the line with gradient $-\dfrac{\sqrt{2}}{4}$ passing through the point (−1 , $\sqrt{2}$) and confirm that this equation is identical to the answer found in part (a). There were some excellent solutions seen using this method.

Question 11

This multiple-choice question was very well answered with over three-quarters of all students correctly identifying the straight-line graph starting at the origin.

Question 12

This multiple-choice question caused some confusion because of the requirement to find the weight of the crate rather than its mass. Using $F = ma$ gives m = 15kg, so the weight is 15gN.

Question 13

The majority of students were able to make some progress on this question.

The first step was to find $\overrightarrow{AB}$ = 5**i** + 12**j**, then the magnitude of this vector could be found by Pythagoras $\left|\overrightarrow{AB}\right| = \sqrt{5^2 + 12^2}$ = 13

Then the correct force vector could be quoted as $\boldsymbol{F} = \dfrac{6.5}{13}$(5**i** + 12**j**) = 2.5**i** + 6**j**

Question 14

The question required careful setting up and execution to ensure that enough detail was included to convincingly obtain the given result, in particular justifying the inequality. One mark was available for a sensible set of variables defined, even if no further progress was made. Students should be aware that if g is part of the answer to be obtained, it is not acceptable to substitute in a numerical value for g. If students did this, they were allowed to recover this slip later in their solution.

$u = 0$ (from rest)
$a = g$ (falling under gravity)
$s = h$ (falls from a height of h metres)

Using $v^2 = u^2 + 2as$, gives $v^2 = 2gh$, then as $v \leq 10$, $v^2 \leq 100$ so $2gh \leq 100$ which can be rearranged to obtain the given result $h \leq \dfrac{50}{g}$

Some students used $v = 100$, but were then unable to justify the correct inequality, often stating ≤ without any justification.

Question 15

In part (a) a diagram would have helped students to identify the triangle needed to find the angle between the two vectors. As the force on *P* is horizontal in direction, the angle between the two forces is the angle that the force on *Q* makes with the horizontal.

Hence the required angle = $\tan^{-1}\left(\dfrac{15}{8}\right) = 62^{\circ}$ to the nearest degree (as requested in the question).

In part (b) $F = ma$ could be used on particle *P* to give $4 = 5m$, so $m = 0.8$

The magnitude of the acceleration of *Q* = $\sqrt{8^2 + 15^2} = 17$

Then using $F = ma$ for *Q* (as its mass is also equal to 0.8kg), $17 = 0.8a$ giving $a = 21.25\text{ms}^{-2}$

Question 16

Normally constant acceleration questions are a good source of marks for students. However, in this case, the algebraic nature of the question which required very careful use of variables proved beyond virtually all students. Hardly any students achieved full marks in part (a) of this question.

Firstly, looking at Jermaine's motion:

initial velocity = $(u - 0.2)$ (as moving 0.2 ms^{-1} slower than Meena)
acceleration = 2 ms^{-2}
displacement = s
time = t

So using $s = ut + \frac{1}{2}at^2$ gives $s = (u - 0.2)t + \frac{1}{2} \times 2 \times t^2$

Unfortunately, some students confused Jermaine's displacement over time t denoted by 's' with the distance he was behind Meena denoted by 'd'. A mark was available for finding at least one term of this equation correctly.

Further to this, students spotted that if you simply crossed out the 'u' in the above equation, the required result 'appeared'. So, disappointingly, that is what a large number of students did. This could be justified, but unless a clear explanation was giving for making this step, no credit was given.

Looking at Meena's displacement, as her speed is constant, in time t she will have a displacement of ut metres. So to catch Meena, Jermaine's displacement will have to be equal to Meena's displacement in the same time, ***plus*** the d metres he was behind her at the point where he started to run.

So $(u - 0.2)t + t^2 = ut + d$

$ut - 0.2t + t^2 = ut + d$

so $d = t^2 - 0.2t$

The main issue in part (b) was not adjusting the given value of u to obtain Jermaine's initial velocity. Many students used $u = 1.4$ and so obtained an incorrect value for t.

At the point where Jermaine reaches Meena, using $v = u + at$

$$7.8 = (1.4 - 0.2)t + 2t$$

which gives $t = 3.3$

Substituting $t = 3.3$ into the given result from part (a) gives $d = 10.23$

Question 17

In part (a) it was necessary to differentiate v to get an expression for the acceleration, and then substitute $t = 15$ to obtain the required value of 0.8 ms^{-2}. (It was also acceptable to show that when $a = 0.8$, $t = 15$ seconds).

A significant proportion of students just substituted $t = 15$ into the expression for v, and then tried, unsuccessfully, to obtain the given answer.

Part (b)(i) required the use of $F = ma$ on the caravan.

So $800 - R = 850 \times 0.8$ giving $R = 120\text{N}$

Some students made the mistake of working on the car alone or the car and caravan combined.

In part (b)(ii) it was also necessary to use $F = ma$, but this time it could be applied to the car alone or to the car and caravan combined.

For the car alone

D – 800 (resistance from towbar) – 100 (resistance force on car) = 1500 × 0.8

This gives D = 2100 N

For the car and caravan combined

D – 220 (resistance force on car + caravan) = 2350 (mass of car + caravan) × 0.8

This gives D = 2100 N

A mark was given if either side of an equation using either approach was correct. Errors noted included missing out forces, eg the 800N or the 100N force in the first method or the 120N force in the second method.

In part (c) any sensible valid assumption was accepted, and a little under two-thirds of all students obtained the mark for doing this.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.