A

Surname ____________________

Other Names ____________________

Centre Number ____________________

Candidate Number ____________________

Candidate Signature ____________________

I declare this is my own work.

A-level MATHEMATICS

Paper 1

7357/1

Time allowed: 1 hour 30 minutes

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

[Turn over]

J U N 2 2 7 3 5 7 1 0 1

- **You must have the AQA Formulae for A-level Mathematics booklet.**
- **You should have a graphical or scientific calculator that meets the requirements of the specification.**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).**
- **Do NOT write on blank pages.**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work that you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 A curve is defined by the parametric equations

$x = \cos\theta$ and $y = \sin\theta$ where $0 \leq \theta \leq 2\pi$

Which of the options shown below is a Cartesian equation for this curve?

Circle your answer. [1 mark]

$\frac{y}{x} = \tan\theta$

$x^2 + y^2 = 1$

$x^2 - y^2 = 1$

$x^2y^2 = 1$

2 **A periodic sequence is defined by**

$U_n = (-1)^n$

State the period of the sequence.

Circle your answer. [1 mark]

-1

0

1

2

[Turn over]

3 The curve

$$y = \log_4 x$$

is transformed by a stretch, scale factor 2, parallel to the y-axis.

State the equation of the curve after it has been transformed.

Circle your answer. [1 mark]

$$y = \frac{1}{2}\log_4 x$$

$$y = 2\log_4 x$$

$$y = \log_4 2x$$

$$y = \log_8 x$$

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[Turn over]

4 The graph of

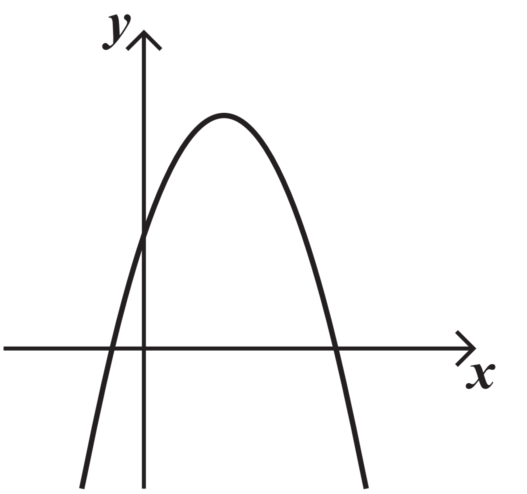
$y = \mathrm{f}(x)$

where

$\mathrm{f}(x) = ax^2 + bx + c$

is shown in FIGURE 1.

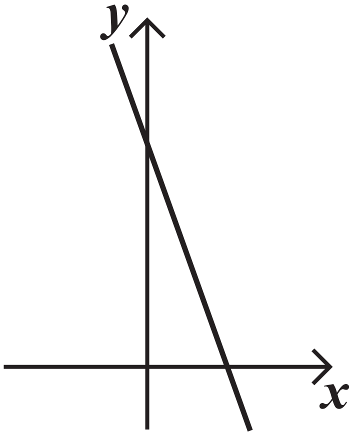
FIGURE 1



Which of the following shows the graph of $y = \mathrm{f}'(x)$?

Tick (✓) ONE box. [1 mark]

☐



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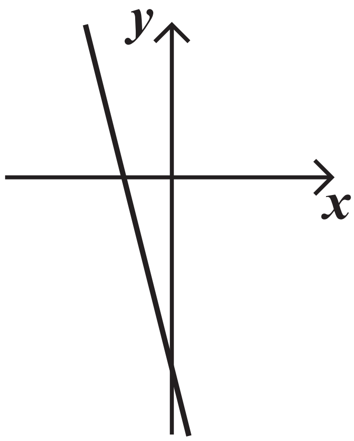
y
x

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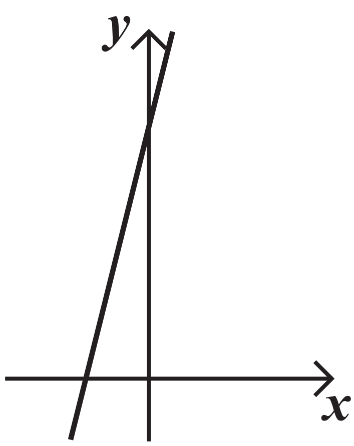
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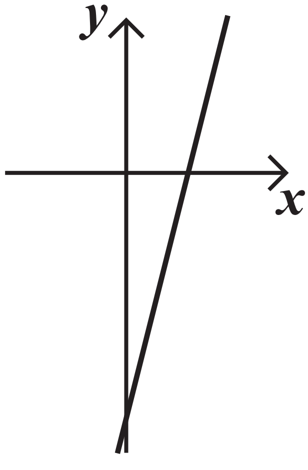
y
x

5 **Find an equation of the tangent to the curve**

$$y = (x - 2)^4$$

at the point where $x = 0$ [3 marks]

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[Turn over]

6 (a) **Find the first two terms, in ascending powers of x, of the binomial expansion of**

$$\left(1 - \frac{x}{2}\right)^{\frac{1}{2}}$$

[2 marks]

6 (b) **Hence, for small values of x, show that**

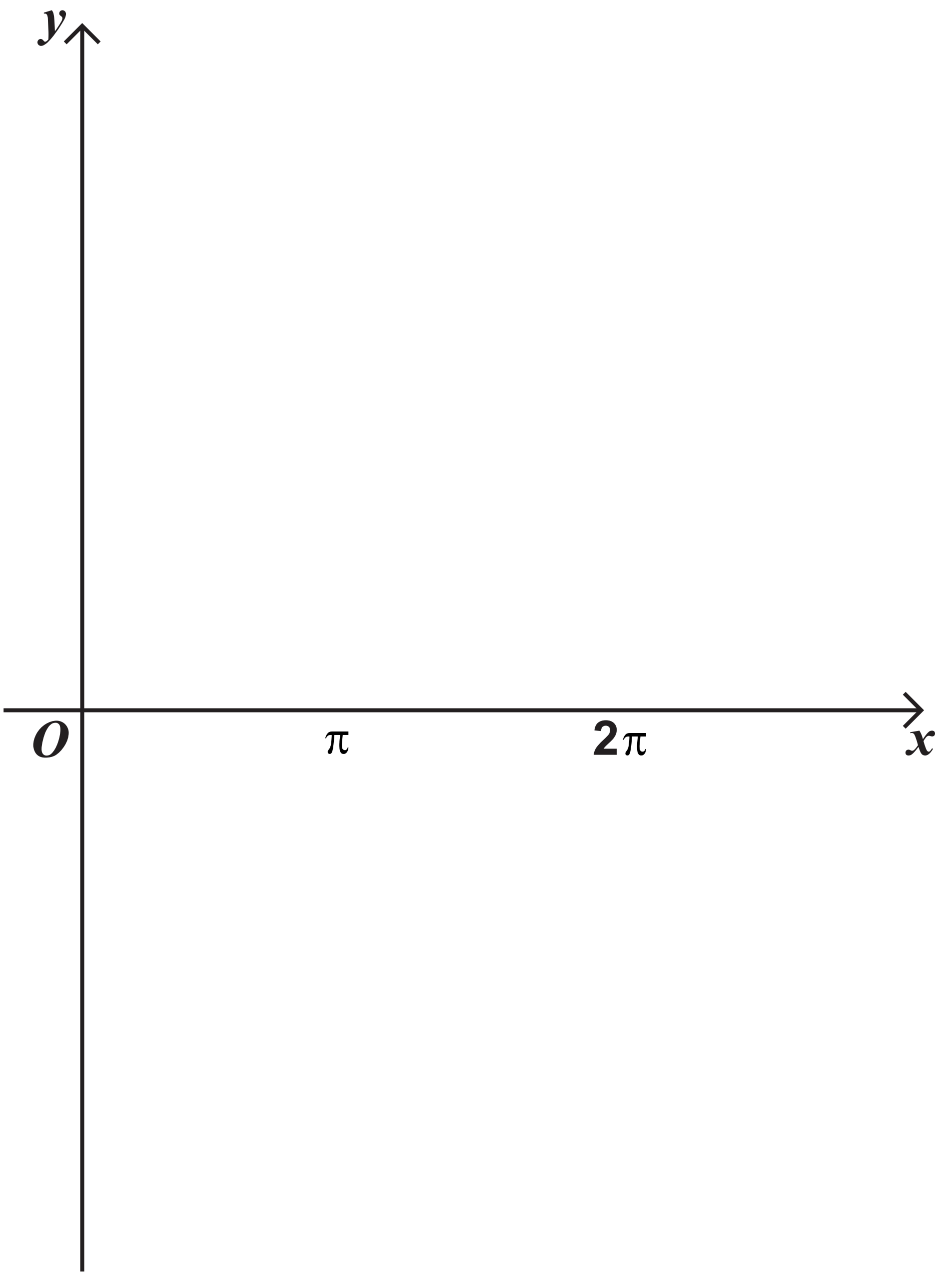
$$\sin 4x + \sqrt{\cos x} \approx A + Bx + Cx^2$$

where A, B and C are constants to be found.
[4 marks]

7 Sketch the graph of

$$y = \cot\left(x - \frac{\pi}{2}\right)$$

for $0 \leq x \leq 2\pi$ [3 marks]



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[Turn over]

8 The lines L_1 and L_2 are parallel.

L_1 has equation

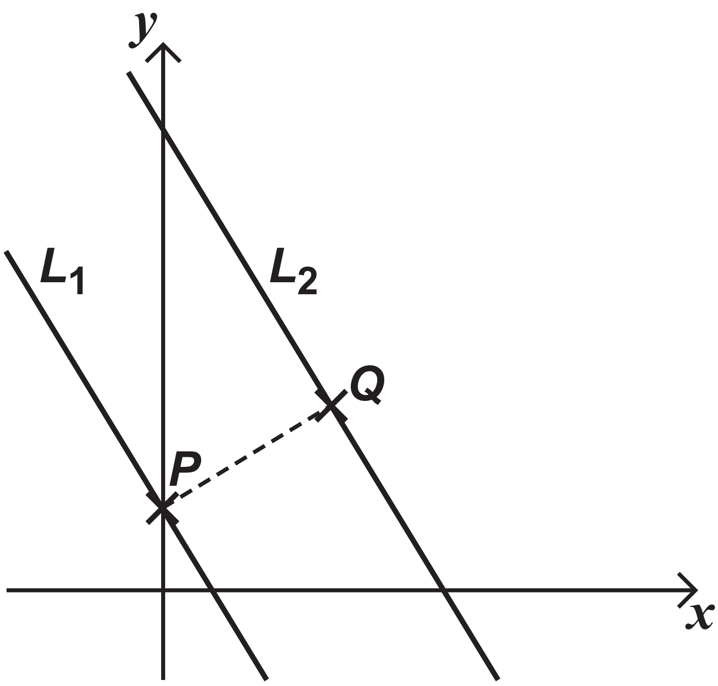
$5x + 3y = 15$

and L_2 has equation

$5x + 3y = 83$

L_1 intersects the y-axis at the point P.

The point Q is the point on L_2 closest to P, as shown in the diagram.



8 (a) (i) Find the coordinates of *Q*. [5 marks]

[Turn over]

8 (a) (ii) Hence show that $PQ = k\sqrt{34}$, where k is an integer to be found. [2 marks]

8 (b) A circle, C, has centre $(a, -17)$.

L_1 and L_2 are both tangents to C.

8 (b) (i) Find a. [2 marks]

8 (b) (ii) Find the equation of *C*. [2 marks]

[Turn over]

9 **The first three terms of an arithmetic sequence are given by**

$2x + 5$ $\quad\quad$ $5x + 1$ $\quad\quad$ $6x + 7$

9 (a) **Show that $x = 5$ is the only value which gives an arithmetic sequence. [3 marks]**

[Turn over]

9 (b) (i) Write down the value of the first term of the sequence. [1 mark]

9 (b) (ii) Find the value of the common difference of the sequence. [1 mark]

9 (c) The sum of the first N terms of the arithmetic sequence is S_N where

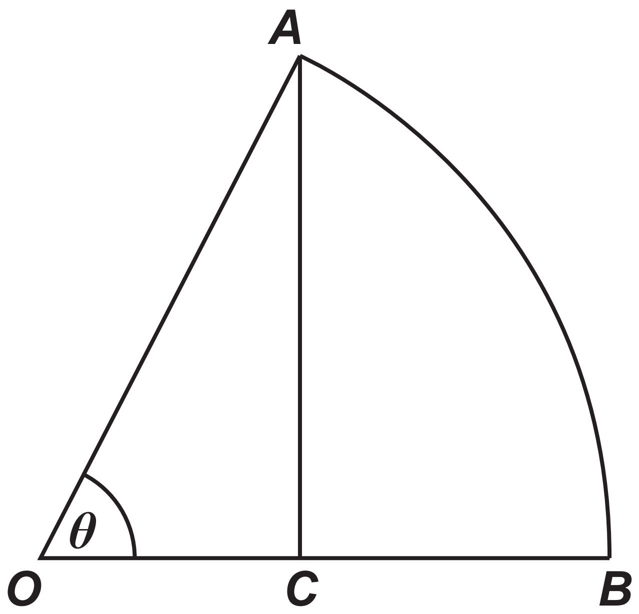
$$S_N < 100\,000$$

$$S_{N+1} > 100\,000$$

Find the value of N. [4 marks]

[Turn over]

10 **The diagram shows a sector of a circle *OAB*.**



The point *C* lies on *OB* such that *AC* is perpendicular to *OB*.

Angle *AOB* is θ radians.

10 (a) **Given the area of the triangle *OAC* is half the area of the sector *OAB*, show that**

$$\theta = \sin 2\theta$$

[4 marks]

[Turn over]

10 (b) **Use a suitable CHANGE OF SIGN to show that a solution to the equation**

$$\theta = \sin 2\theta$$

lies in the interval given by $\theta \in \left[\frac{\pi}{5}, \frac{2\pi}{5}\right]$

[2 marks]

[Turn over]

10 (c) **The Newton-Raphson method is used to find an approximate solution to the equation**

$$\theta = \sin 2\theta$$

10 (c) (i) **Using $\theta_1 = \frac{\pi}{5}$ as a first approximation for θ apply the Newton-Raphson method twice to find the value of θ_3**

Give your answer to three decimal places.
[3 marks]

10 (c) (ii) Explain how a more accurate approximation for θ can be found using the Newton-Raphson method. [1 mark]

[Turn over]

10 (c)(iii) **Explain why using $\theta_1 = \frac{\pi}{6}$ as a first approximation in the Newton-Raphson method does not lead to a solution for θ. [2 marks]**

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11 **The polynomial $p(x)$ is given by**

$$p(x) = x^3 + (b + 2)x^2 + 2(b + 2)x + 8$$

where b is a constant.

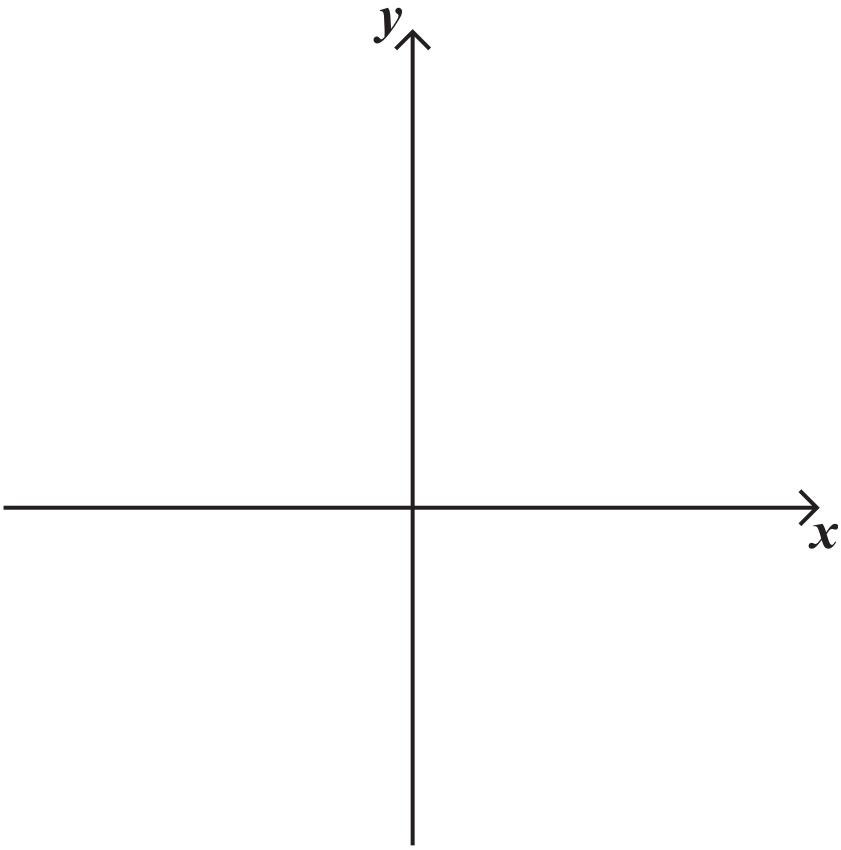
11 (a) **Use the factor theorem to prove that $(x + 2)$ is a factor of $p(x)$ for ALL values of b. [3 marks]**

QUESTION 11 CONTINUES ON THE NEXT PAGE

[Turn over]

11 (b) The graph of $y = \mathrm{p}(x)$ meets the x-axis at exactly two points.

11 (b) (i) Sketch a possible graph of $y = \mathrm{p}(x)$ [3 marks]



11 (b) (ii) Given $\mathrm{p}(x)$ can be written as

$$\mathrm{p}(x) = (x + 2)(x^2 + bx + 4)$$

find the value of b.

Fully justify your answer. [4 marks]

[Turn over]

12 (a) **A geometric sequence has first term 1 and common ratio $\frac{1}{2}$**

12 (a) (i) Find the sum to infinity of the sequence.
[2 marks]

QUESTION 12 CONTINUES ON THE NEXT PAGE

[Turn over]

12 (a) (ii) Hence, or otherwise, evaluate

$$\sum_{n=1}^{\infty} (\sin 30°)^n$$

[2 marks]

[Turn over]

12 (b) **Find the smallest positive exact value of θ, in RADIANS, which satisfies the equation**

$$\sum_{n=0}^{\infty} (\cos\theta)^n = 2 - \sqrt{2}$$

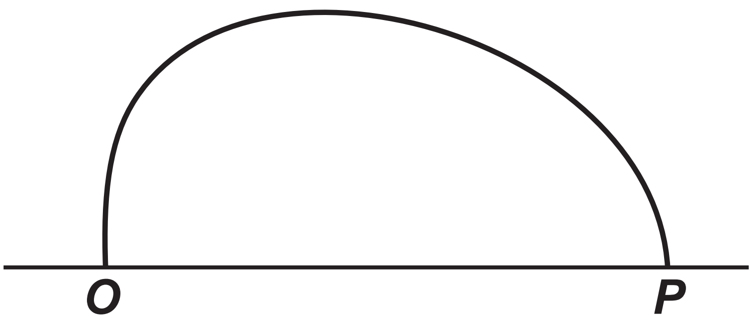
[4 marks]

[Turn over]

13 **FIGURE 2 shows the approximate shape of the vertical cross section of the entrance to a cave. The cave has a horizontal floor.**

The entrance to the cave joins the floor at the points *O* and *P*.

FIGURE 2



Garry models the shape of the cross section of the entrance to the cave using the equation

$$x^2 + y^2 = a\sqrt{x} - y$$

where a is a constant, and x and y are the horizontal and vertical distances respectively, in metres, measured from *O*.

13 (a) **The distance *OP* is 16 metres.**

Find the value of a that Garry should use in the model. [2 marks]

[Turn over]

13 (b) **Show that the maximum height of the cave above *OP* is approximately 10.5 metres.**
[6 marks]

13 (c) **Suggest one limitation of the model Garry has used. [1 mark]**

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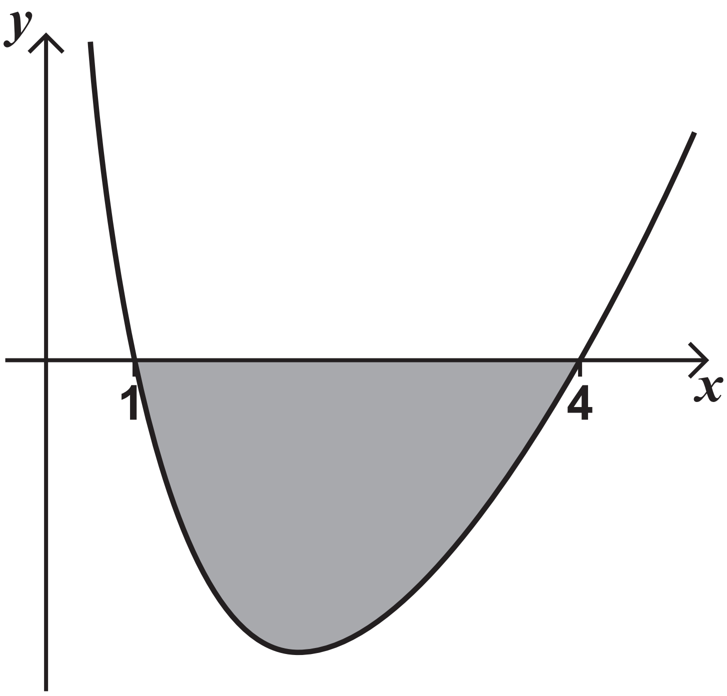
14 The region bounded by the curve

$$y = (2x - 8)\ln x$$

and the x-axis is shaded in the diagram below.



14 (a) Use the trapezium rule with 5 ORDINATES to find an estimate for the area of the shaded region.

Give your answer correct to three significant figures. [3 marks]

[Turn over]

14 (b) **Show that the exact area is given by**

$$32 \ln 2 - \frac{33}{2}$$

Fully justify your answer. [6 marks]

[Turn over]

15 (a) **Given that**

$y = \text{cosec}\,\theta$

15 (a) (i) **Express y in terms of $\sin\theta$.** **[1 mark]**

15 (a) (ii) **Hence, prove that**

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = -\text{cosec}\,\theta\cot\theta$$

[3 marks]

15 (a)(iii) **Show that**

$$\frac{\sqrt{y^2-1}}{y} = \cos\theta \qquad \text{for } 0 < \theta < \frac{\pi}{2}$$

[3 marks]

15 (b) (i) Use the substitution

$x = 2\operatorname{cosec} u$

to show that

$$\int \frac{1}{x^2\sqrt{x^2-4}}\,\mathrm{d}x \qquad \text{for } x > 2$$

can be written as

$$k\int \sin u \,\mathrm{d}u$$

where k is a constant to be found. [6 marks]

[Turn over]

15 (b) (ii) Hence, show

$$\int \frac{1}{x^2\sqrt{x^2-4}}\,\mathrm{d}x = \frac{\sqrt{x^2-4}}{4x} + c \qquad \text{for } x > 2$$

where c is a constant. [3 marks]

END OF QUESTIONS

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For Examiner's Use	
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