



**Surname** \_\_\_\_\_

**Other Names** \_\_\_\_\_

**Centre Number** \_\_\_\_\_

**Candidate Number** \_\_\_\_\_

**Candidate Signature** \_\_\_\_\_

**I declare this is my own work.**

**A-level  
MATHEMATICS**

**Paper 1**

**7357/1**

**Time allowed: 2 hours**

**At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.**

**[Turn over]**



- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

## INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do NOT write on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.



## **INFORMATION**

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

## **ADVICE**

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

**DO NOT TURN OVER UNTIL TOLD TO DO SO**



**Answer ALL questions in the spaces provided.**

**1 A curve is defined by the parametric equations**

$$x = \cos \theta \quad \text{and} \quad y = \sin \theta$$

where  $0 \leq \theta \leq 2\pi$

**Which of the options shown below is a Cartesian equation for this curve?**

**Circle your answer. [1 mark]**

$$\frac{y}{x} = \tan \theta$$

$$x^2 + y^2 = 1$$

$$x^2 - y^2 = 1$$

$$x^2 y^2 = 1$$



5

2 A periodic sequence is defined by

$$U_n = (-1)^n$$

State the period of the sequence.

Circle your answer. [1 mark]

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[Turn over]



**3 The curve**

$$y = \log_4 x$$

**is transformed by a stretch, scale factor 2, parallel to the  $y$ -axis.**

**State the equation of the curve after it has been transformed.**

**Circle your answer. [1 mark]**

$$y = \frac{1}{2} \log_4 x$$

$$y = 2 \log_4 x$$

$$y = \log_4 2x$$

$$y = \log_8 x$$



4 The graph of

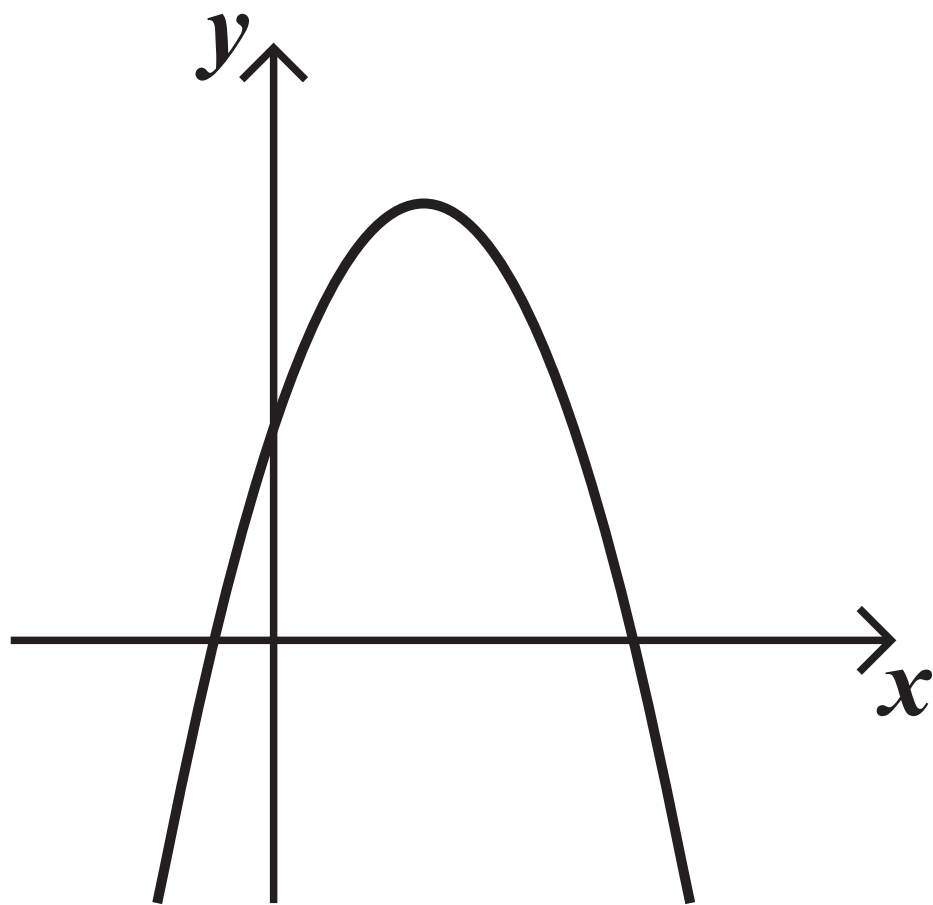
$$y = f(x)$$

where

$$f(x) = ax^2 + bx + c$$

is shown in FIGURE 1.

**FIGURE 1**



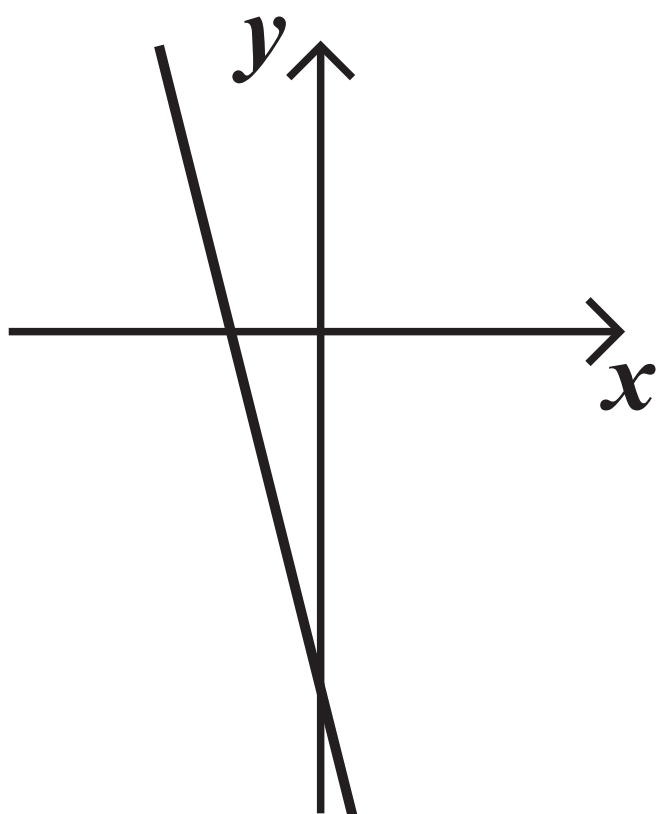
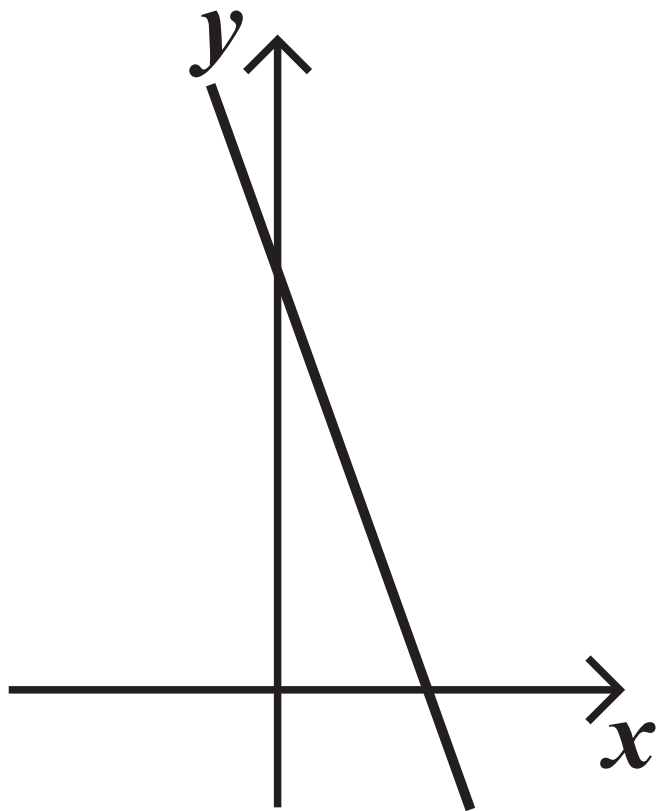
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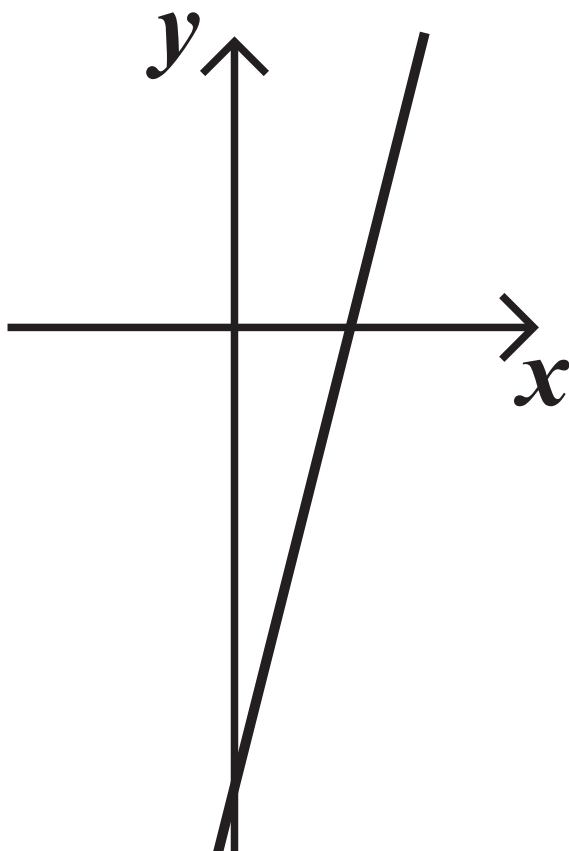
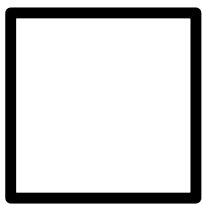
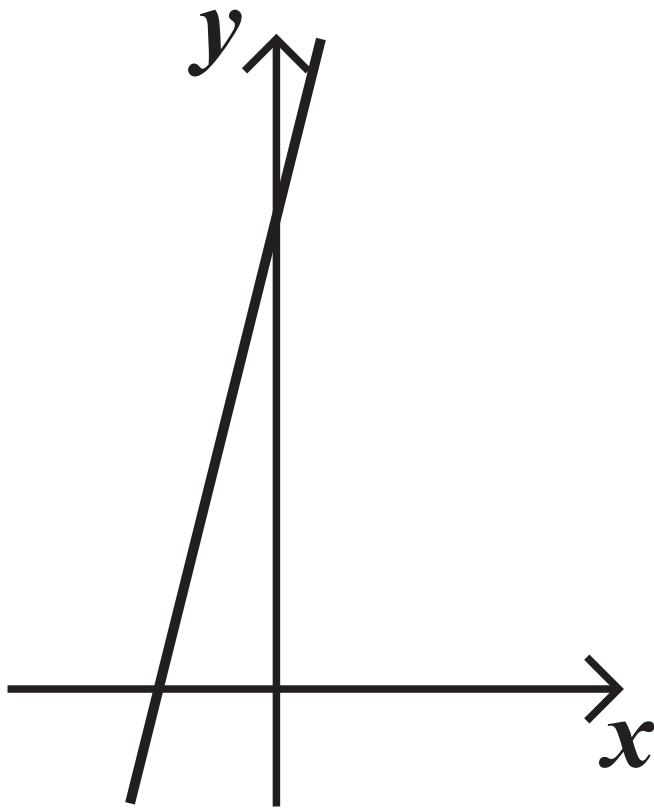
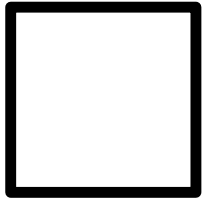
Which of the following shows the graph of  $y = f'(x)$  ?

Tick (✓) ONE box. [1 mark]





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**[Turn over]**



0 9



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**[Turn over]**



**6 (a)** Find the first two terms, in ascending powers of  $x$ , of the binomial expansion of

$$\left(1 - \frac{x}{2}\right)^{\frac{1}{2}}$$

**[2 marks]**

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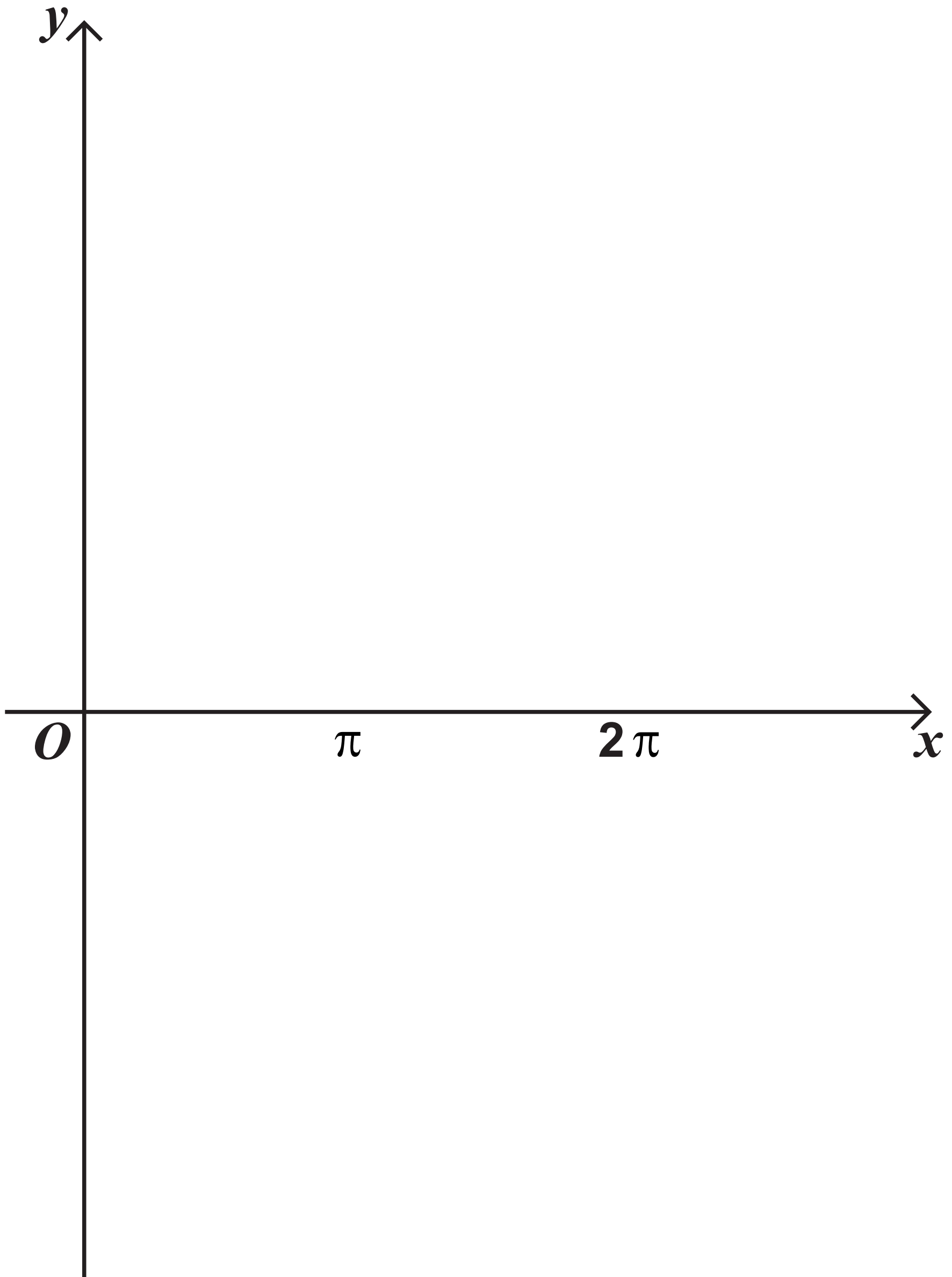


7 Sketch the graph of

$$y = \cot \left( x - \frac{\pi}{2} \right)$$

for  $0 \leq x \leq 2\pi$  opposite.  
[3 marks]





**[Turn over]**



8 The lines  $L_1$  and  $L_2$  are parallel.

$L_1$  has equation

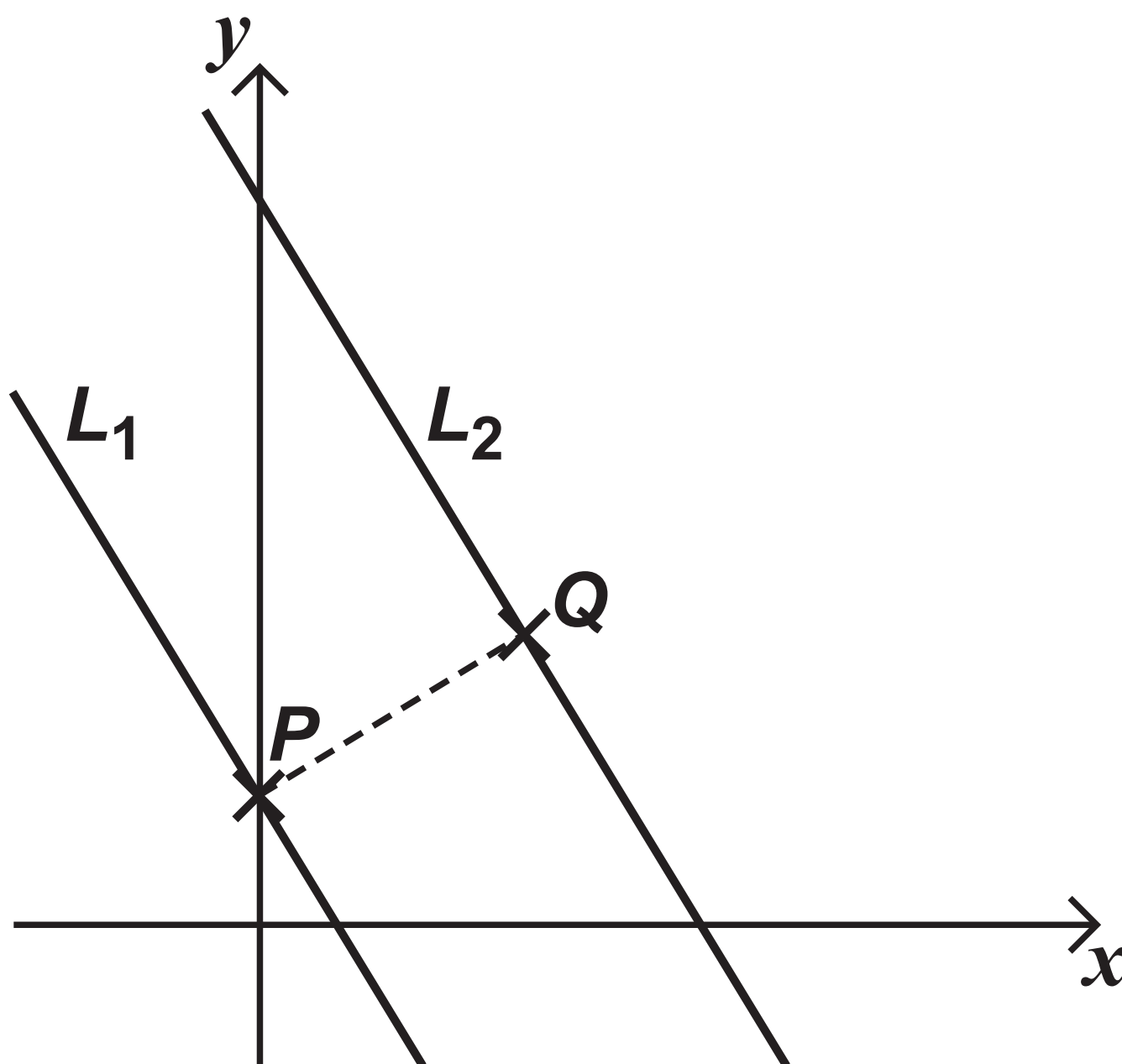
$$5x + 3y = 15$$

and  $L_2$  has equation

$$5x + 3y = 83$$

$L_1$  intersects the  $y$ -axis at the point  $P$ .

The point  $Q$  is the point on  $L_2$  closest to  $P$ , as shown in the diagram opposite.



**8 (a) (i) Find the coordinates of Q.  
[5 marks]**

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8 (a) (ii) Hence show that  $PQ = k\sqrt{34}$ ,  
where  $k$  is an integer to be found.  
[2 marks]

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**8(b) (ii) Find the equation of C. [2 marks]**

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- 9 The first three terms of an arithmetic sequence are given by

$$2x + 5$$

$$5x + 1$$

$$6x + 7$$

- 9 (a) Show that  $x = 5$  is the only value which gives an arithmetic sequence. [3 marks]

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**9(b) (i) Write down the value of the first term of the sequence. [1 mark]**

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**9(b) (ii) Find the value of the common difference of the sequence. [1 mark]**

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9 (c) The sum of the first  $N$  terms of the arithmetic sequence is  $S_N$  where

$$S_N < 100\,000$$

$$S_{N+1} > 100\,000$$

Find the value of  $N$ . [4 marks]

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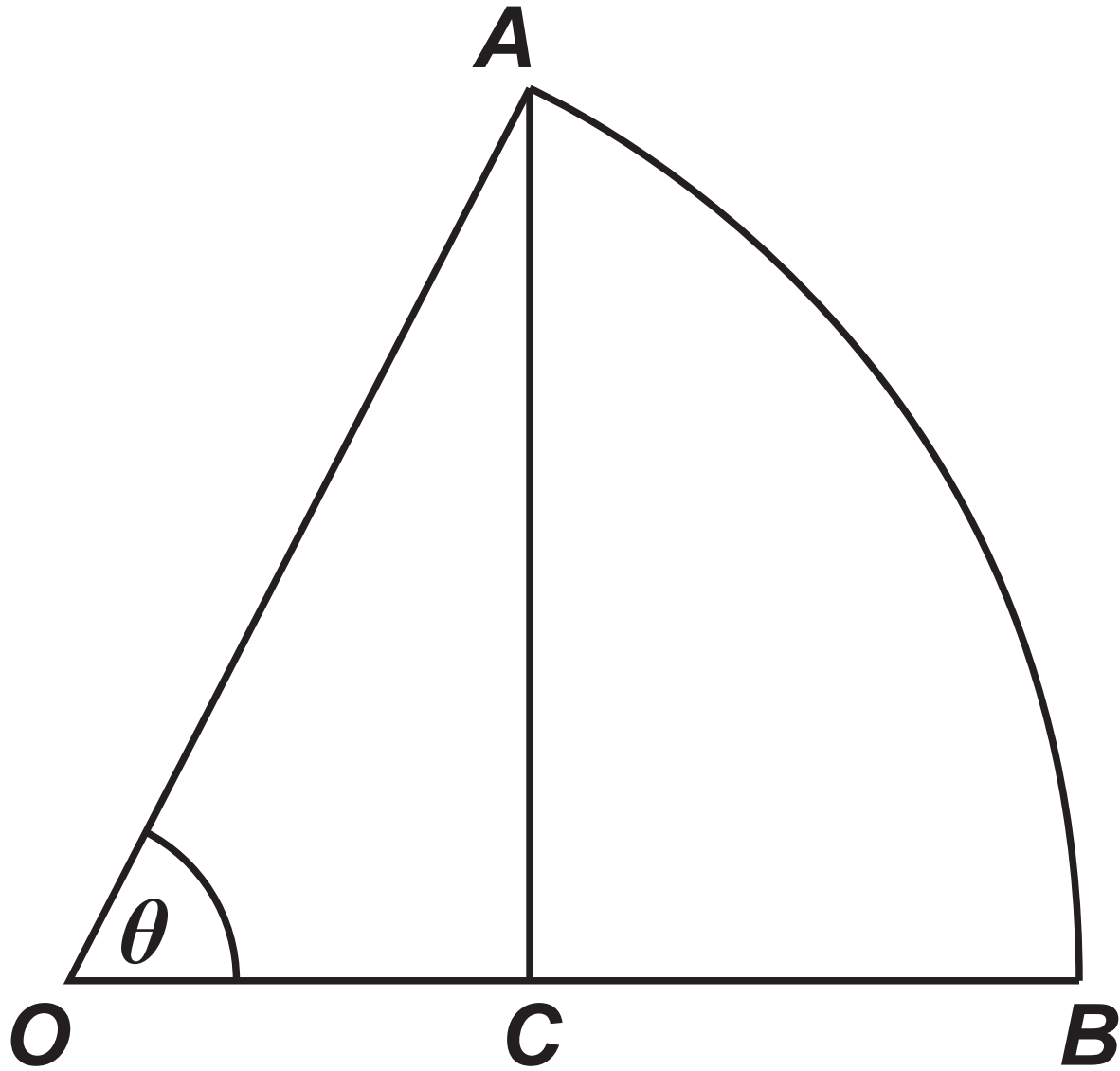








- 10 The diagram shows a sector of a circle  $OAB$ .



The point  $C$  lies on  $OB$  such that  $AC$  is perpendicular to  $OB$ .

Angle  $AOB$  is  $\theta$  radians.



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10(b) Use a suitable **CHANGE OF SIGN** to show that a solution to the equation

$$\theta = \sin 2\theta$$

lies in the interval given

by  $\theta \in \left[ \frac{\pi}{5}, \frac{2\pi}{5} \right]$  [2 marks]

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**10 (c)** The Newton-Raphson method is used to find an approximate solution to the equation

$$\theta = \sin 2\theta$$

**10 (c) (i)** Using  $\theta_1 = \frac{\pi}{5}$  as a first approximation for  $\theta$  apply the Newton-Raphson method twice to find the value of  $\theta_3$

**Give your answer to three decimal places. [3 marks]**

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**10 (c) (ii) Explain how a more accurate approximation for  $\theta$  can be found using the Newton-Raphson method. [1 mark]**

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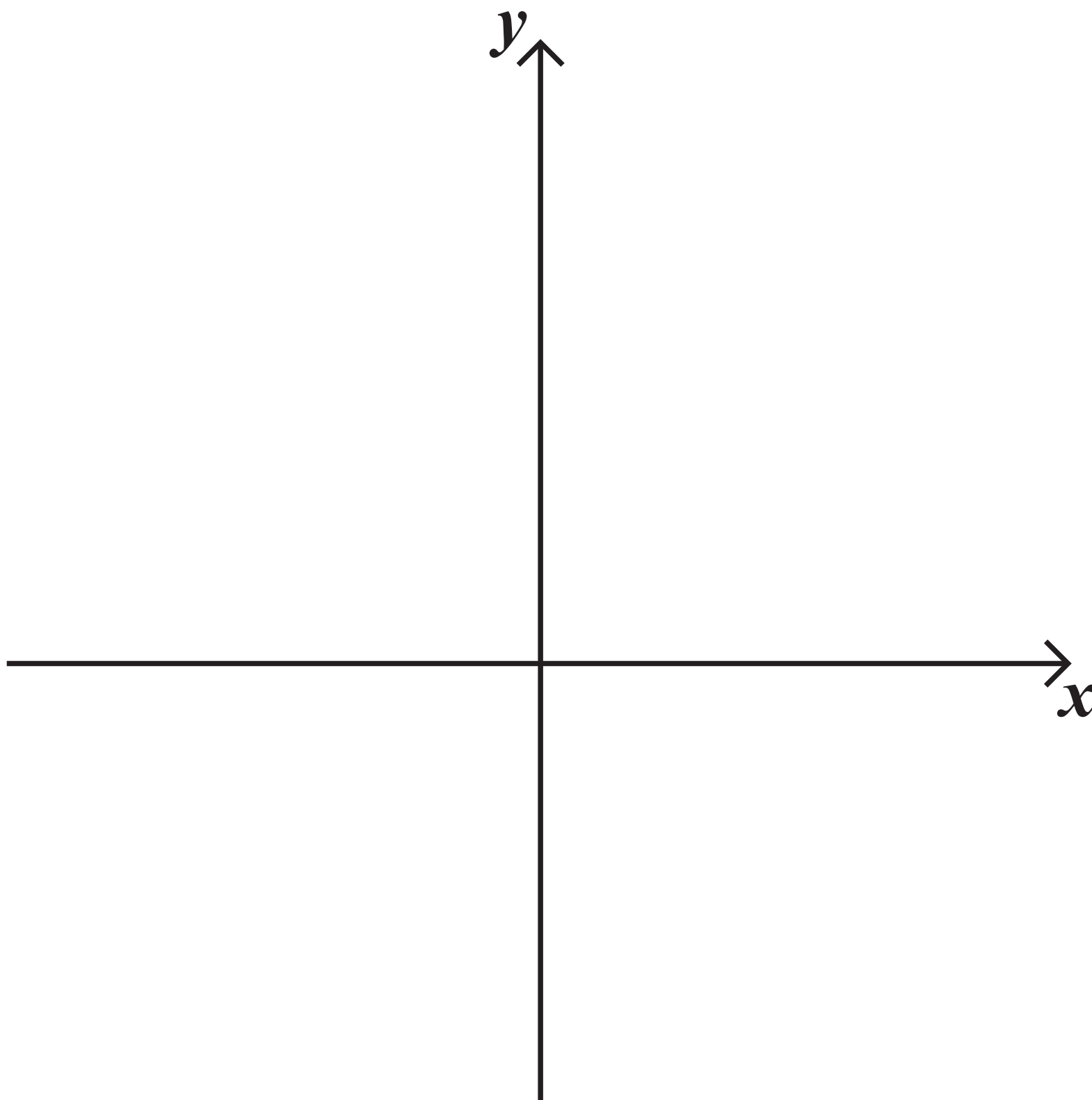
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**[Turn over]**



11 (b) The graph of  $y = p(x)$  meets the  $x$ -axis at exactly two points.

11 (b) (i) Sketch a possible graph of  $y = p(x)$  [3 marks]



**11 (b) (ii) Given  $p(x)$  can be written as**

$$p(x) = (x + 2)(x^2 + bx + 4)$$

**find the value of  $b$ .**

**Fully justify your answer.**

**[4 marks]**

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**12 (a)** A geometric sequence has first term 1 and common ratio  $\frac{1}{2}$

**12 (a) (i)** Find the sum to infinity of the sequence. [2 marks]

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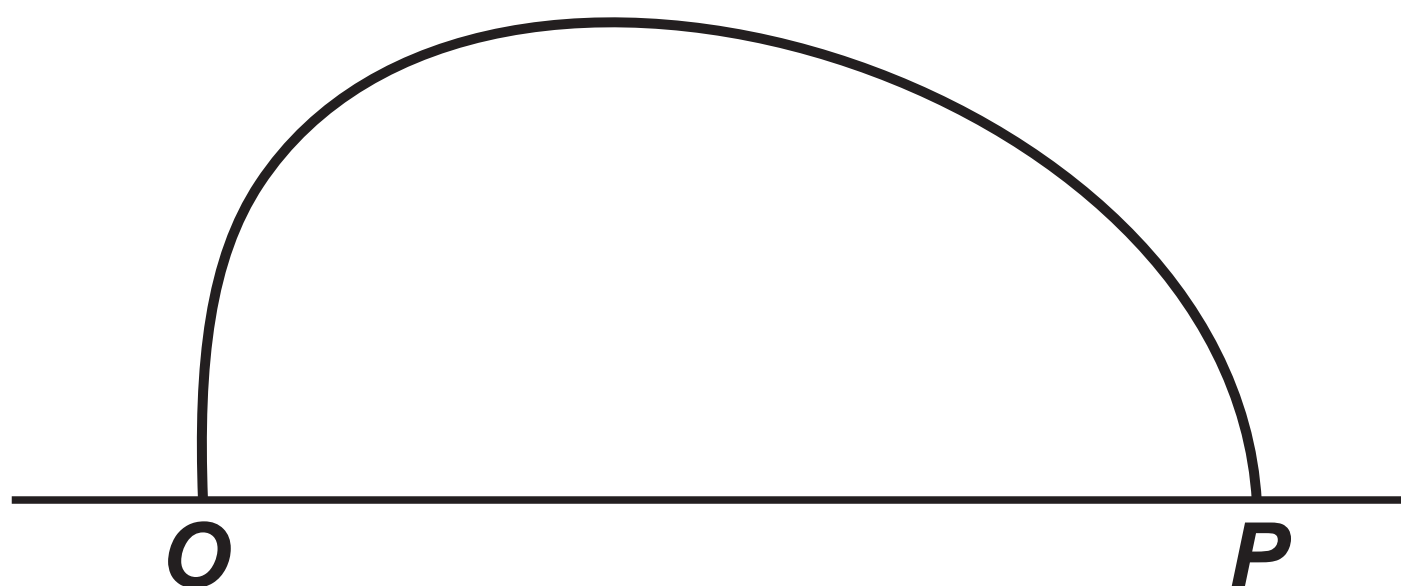


13

**FIGURE 2** shows the approximate shape of the vertical cross section of the entrance to a cave. The cave has a horizontal floor.

The entrance to the cave joins the floor at the points  $O$  and  $P$ .

**FIGURE 2**



Garry models the shape of the cross section of the entrance to the cave using the equation

$$x^2 + y^2 = a\sqrt{x} - y$$

where  $a$  is a constant, and  $x$  and  $y$  are the horizontal and vertical distances respectively, in metres, measured from  $O$ .





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- 13 (b) Show that the maximum height of the cave above  $OP$  is approximately 10.5 metres.  
[6 marks]

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13 (c)

**Suggest one limitation of the model Garry has used. [1 mark]**

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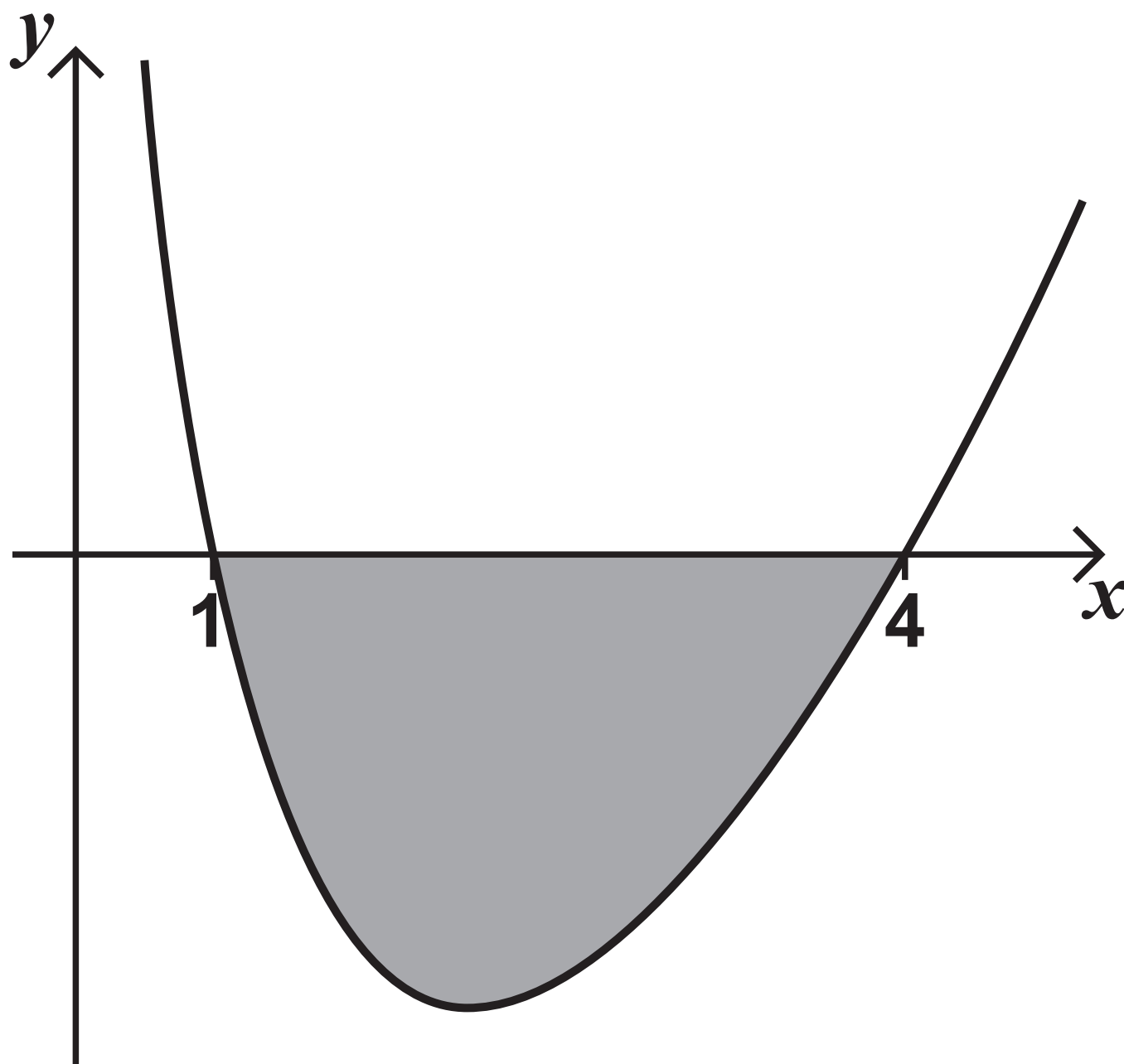
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14 The region bounded by the curve

$$y = (2x - 8) \ln x$$

and the  $x$ -axis is shaded in the diagram below.



14 (a) Use the trapezium rule with 5 ORDINATES to find an estimate for the area of the shaded region.

Give your answer correct to three significant figures.  
[3 marks]







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14(b) Show that the exact area is given by

$$32 \ln 2 - \frac{33}{2}$$

Fully justify your answer.  
[6 marks]

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**[Turn over]**

**15 (a) Given that**

$$y = \operatorname{cosec} \theta$$

**15 (a) (i) Express  $y$  in terms of  $\sin \theta$ .  
[1 mark]**

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**15 (a) (iii) Show that**

$$\frac{\sqrt{y^2 - 1}}{y} = \cos \theta \quad \text{for } 0 < \theta < \frac{\pi}{2}$$

**[3 marks]**

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**15(b) (i) Use the substitution**

$$**x = 2 \operatorname{cosec} u**$$

**to show that**

$$\int \frac{1}{x^2 \sqrt{x^2 - 4}} dx \quad \text{for } x > 2$$

**can be written as**

$$**k \int \sin u \, du**$$

**where  $k$  is a constant to be found.  
[6 marks]**

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**15 (b) (ii) Hence, show**

$$\int \frac{1}{x^2 \sqrt{x^2 - 4}} dx = \frac{\sqrt{x^2 - 4}}{4x} + c$$

**for  $x > 2$**

**where  $c$  is a constant. [3 marks]**

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**END OF QUESTIONS**



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