AQA

A-LEVEL
MATHEMATICS

7357/1 Paper 1
Report on the Examination

7357
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Version: 1.0

Overall

Good responses were seen across the whole paper, with notable improvements in work on sequences and series, small angle approximations, curve sketching, the factor theorem and calculus. Work on proof, trigonometry and questions that require explanation still proved to be challenging for many students. The general standard of solutions showed structural improvement and algebraic skills were noted to have improved. It was pleasing to see that more students were able to use a calculator efficiently and appropriately to solve particular questions.

Question 1

This question proved to be a very successful starter with over 80% of students choosing the correct answer. The incorrect answer that was most often chosen was $\frac{y}{x} = \tan\theta$, indicating that students recognised this to be a true statement but did not realise that it was not a Cartesian equation due to the presence of θ. No option was left unchosen.

Question 2

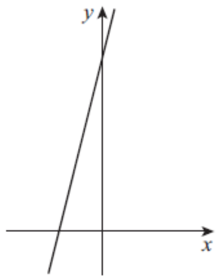
This question proved to be a reasonably successful starter with two-thirds of students choosing the correct answer. There were two incorrect answers almost equally chosen by students, which were –1 and 1. These students had not understood the definition of the word 'period' in the context of sequences. They had picked –1 as it appeared in the formula or picked 1, as they had realised that the period of a sequence must be positive. No option was left unchosen.

Question 3

This question proved to be a very successful starter with 80% of students choosing the correct answer. The incorrect answer that was most often chosen was $y = \frac{1}{2}\log_4 x$, indicating that students knew that a stretch parallel to the y-axis results in an equation of the form $y = k\log_4 x$ but chose the wrong value of k. No option was left unchosen.

Question 4

This question proved to be the least successful multiple-choice question with less than 60% of students choosing the correct answer. The incorrect answer that was most often chosen was:



No option was left unchosen.

Question 5

This question was written with the expectation that students would use the appropriate function on their calculator to obtain the gradient at $x = 0$. Only a handful of students used this method. The vast majority of students correctly used the chain rule to obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = 4(x-2)^3$ before evaluating this at $x = 0$. Some incorrect evaluations seen at this point were –8 and –16. A significant number of students chose to use the binomial theorem to expand $(x-2)^4$ but unfortunately made errors and then scored only one mark for substituting $x = 0$ into their incorrectly expanded and subsequently incorrect derivative. A few students slipped up by only obtaining the correct gradient of the tangent but not then finding the correct equation of the tangent. Almost 60% of students scored all three marks.

Question 6

In this question, students were more successful in answering part (a) than part (b), however responses to both parts showed improvement on what had been seen in the past. In part (a), nearly 80% of students obtained both marks. When only one mark was scored the typical mistake was a sign error. In part (b) it was pleasing to see that the vast majority of students knew the relevant angle approximations. Although the second mark was only awarded for using the approximation for cosine inside the square root sign, over three quarters of students were able to do this successfully. The final two marks proved more challenging with the following errors quite often seen:

- $\sqrt{1-\frac{x^2}{2}} = 1-\frac{x}{\sqrt{2}}$
- $\sqrt{1-\frac{x^2}{2}} = 1-\frac{x}{4}$
- Obtaining $1 + 4x - \frac{1}{4}x^2$ but then multiplying by 4 to give the final answer as $4 + 16x - x^2$

Some students mixed up using x and θ throughout and this was penalised unless they correctly and consistently reverted to using x in their final answer.

Question 7

Around two-thirds of students scored one mark, almost all for correctly sketching the middle section of the graph. A small proportion of students failed to score any marks as they had sketched the graph of a tangent function. The second mark was often lost for only having two sections of the graph. To score the final mark the asymptotes needed to be indicated, usually by dotted lines and the curve had to approach these asymptotes. Students lost the final mark most often for not indicating the asymptotes.

Question 8

Part (a)(i) proved to be successful, with most students attempting to find the equation of *PQ* and then using simultaneous equations. Around 60% of students scored at least three marks. Common errors seen were:

- obtaining an incorrect y-coordinate for *P*
- using $\frac{5}{3}$ or $-\frac{3}{5}$ as the gradient of *PQ*
- rearranging the equation of L_2 incorrectly
- making errors when solving simultaneous equations, which could have easily been solved on the calculator.

In part (a)(ii) those students who had scored five marks in the first part were generally able to score both marks here. Other students who had lost marks earlier did score the first mark for displaying their knowledge of using the formula for the distance between two points.

Part (b)(i) proved very challenging, with less than 20% of students scoring both marks. There were however many ways to solve this problem, some of which are indicated in the mark scheme. The most successful method used by students was to use (a , –17) as the midpoint of a line segment from L_1 to L_2.

In part (b)(ii), over half of the students were able to score one mark for knowing that the equation of the circle contained an expression of the form $\left(x \pm a\right)^2 + (y \pm 17)^2$, which was awarded even if the value of a had not been found.

Question 9

It was pleasing to see increased confidence with arithmetic sequences. In part (a) whilst over 80% of students scored the first mark, only 10% were able to score all three marks. The key part of this question required students to show that $x = 5$ was the **only** value that resulted in an arithmetic sequence. Therefore students who substituted $x = 5$ to obtain 15, 26, 37 and then indicated that the common difference was 11 were only awarded one mark since they had not shown it was the only value. Testing further values was fruitless as clearly it is impossible to test all other values. However, almost half of students scored two marks by setting up an appropriate linear equation in x and solving it to get $x = 5$. The final mark was often lost as students did not conclude with an appropriate statement. The easiest way to do this was just to repeat the phrasing from the question.

Both(b)(i) and (b)(ii) were done very well with almost all students scoring both marks.

In part (c) more than three quarters of students were able to use the appropriate formula for the sum of an arithmetic series and set up an appropriate equation or inequality. This was a significant improvement on previous examinations. Around 40% of students were then able to solve the quadratic equation and identify the correct solution of $N = 133$. There were some very good trial and improvement solutions seen, which scored full marks if they obtained the correct final solution from fully correct working.

Question 10

Part (a) proved very challenging, with only 20% of students scoring all four marks. Almost all students were able to get started by using the correct formula for the area of a sector. Around half of students were then able to form a suitable equation involving the area of a triangle (in its most general form) and the area of a sector. Many students then struggled to use trigonometry correctly to obtain expressions for the height and base of the triangle. Almost all students who were able to do this correctly went on to use the appropriate double angle identity and completed the proof.

Use of a sign change in part (b) continues to be a challenge for students. In order to score the first mark, the stated equation must be rewritten as $\theta - \sin 2\theta = 0$ or $\sin 2\theta - \theta = 0$. If this was not explicitly stated then students scored 0, as in the past. The vast majority of students were able to substitute the values $\frac{\pi}{5}$ and $\frac{2\pi}{5}$ for θ into $\theta - \sin 2\theta$ or $\sin 2\theta - \theta$ and check for a change of sign. The second mark was only awarded for a fully correct conclusion which had to refer explicitly to the root being between $\frac{\pi}{5}$ and $\frac{2\pi}{5}$. It is not sufficient to say ‘between these values’.
It was expected that many students would score well in part (c)(i) as the Newton-Raphson formula is given in the formulae booklet. It was therefore disappointing to find that less than a third of students scored all three marks. The main error was to think that the derivative of $\sin 2\theta - \theta$ was $2\cos 2\theta$, thus thinking that the derivative of θ was 0, not 1. These students could only then score one mark for identifying the correct derivative of $\sin 2\theta$.

Part (c)(ii) proved successful with over 60% of students scoring the mark for knowing that the approximation would be more accurate if the process was repeated more times.

Part (c)(iii) proved far less successful, with around a third of students scoring a mark for stating a general reason as to why the process might not converge. The other mark was awarded for referring specifically to $1 - 2\cos 2\frac{\pi}{6} = 0$, usually then concluding with reference to it being on a stationary point, to score both marks. These marks were independent of each other.

Question 11

A good improvement was shown here with students clearly knowing and being able to use the factor theorem correctly. As such, around 85% of students scored two marks as they demonstrated that by substituting $x = -2$ into the stated polynomial it resulted in 0. The final mark was often then lost as no full conclusion, referring to b, was then stated. In this case an appropriate conclusion would have been: 'therefore $(x + 2)$ is a factor for all values of b.'

Sketching curves has improved and over 70% of students scored at least two marks in part (b)(i) for knowing the orientation and that the curve would touch the x-axis. Where students lost the final mark, this was due to failing on one of the points listed below:

- a correctly orientated cubic graph
- single root labelled at -2
- y-intercept labelled at 8
- repeated root on the positive x-axis.

The final part (b)(ii) proved most challenging to students, with just over a third scoring at least three marks. Most students went down the route of identifying that a perfect square would be a factor, but then did not continue their argument correctly, with only a few using the discriminant appropriately.

Question 12

It was pleasing to see greater confidence with geometric series than seen in the past. Part (a)(i) was done very well, with over 90% of students showing they knew how to use the sum to infinity formula and scoring two marks. Part (a)(ii) was less successful with around 50% of students scoring both marks. The preferred approach here was to generate the first few terms and again use the sum to infinity formula, with very few simply halving their answer to (a)(i). When students made errors generating the sequence they often thought the first term was 1.

Part (b) was much more discriminating, with around 40% of students able to set up a correct equation at the start. Having set up the equation a common error was to identify both a and r as $\cos\theta$ thus incorrectly writing $\frac{\cos\theta}{1-\cos\theta} = 2 - \sqrt{2}$. However, some completely correct solutions were seen where the sum was adjusted by removing the first term and then beginning with $\frac{\cos\theta}{1-\cos\theta} = 1 - \sqrt{2}$.

Around one quarter of students were able to score the third and fourth marks. Those that failed to do so often manipulated their equation incorrectly, did not use exact values or forgot to find the angle, stopping once r had been found.

Question 13

Part (a) proved slightly unusual in that students generally scored 0 or 2, with only a handful scoring one mark. Those that scored 0 often tried to use complicated simultaneous equations or implicit differentiation, neither of which were required. The students who scored one mark had substituted $y = 0$ and $x = 16$ correctly but then made an error concluding that the value of a was one of the following ± 64, 16, 4, 2.

In part (b) nearly 60% of students scored at least three marks, showing good knowledge of implicit differentiation. These marks were awarded even if the value of a had not been found or an incorrect value of a was used. When students dropped the accuracy mark it was usually for a sign error, an incorrect fractional power of x or using $-y\frac{\mathrm{d}y}{\mathrm{d}x}$ for the derivative of y.

Relatively few students were able to use $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and correctly solve the resulting equation.
Around a quarter of students obtained a value for x and substituted it to obtain a value for y.
The final mark was only awarded for students who had obtained the y value of 10.51 or better and then made a concluding statement relating to the maximum height being approximately 10.5 metres. This mark was most often lost for not stating the units or for not referring to maximum height.

Students' explanations in part (c) failed to refer to a limitation of the model. An explanation referring to the cave entrance not being a smooth curve would have scored the mark: this demonstrates how the mathematical model is limited in fitting the reality of the cave.

Question 14

There was a disappointing response to part (a), with only a third of students scoring all three marks. A significant number of students used five strips rather than five ordinates and were then only awarded one mark if they obtained the answer of 5.42 or –5.42. Of the students who did attempt to use five ordinates the following errors occurred regularly:

- incorrect evaluation of y-values
- incorrect brackets in the trapezium rule formula
- incorrect value of h used.

Note that full marks were given for obtaining either 5.28 or –5.28, because understanding the interpretation of the negative value was assessed in part (b).

Around 60% of students began part (b) by attempting to set up integration by parts correctly. However, the success rate soon tailed off with errors in integration, errors in signs and errors in mixing up the terms in the formula. Around one third of students demonstrated they knew how to substitute limits for the fourth mark. Note it is important to show full working, with limits clearly substituted into the integrated expression, to score the method mark. The manipulation then required proved to be beyond most students, although around 20% scored at least five marks for obtaining either $\frac{33}{2} - 32\ln 2$ or $\frac{33}{2} - 16\ln 4$.

Many students got into a muddle by trying to match the printed answer without justification and changing signs and terms to fit. The explanation mark was only scored by 3% of students who correctly referred to the 'area being below the x-axis' to justify the change of sign.

Question 15

Unsurprisingly this question proved to be the most challenging. However, it was pleasing to see almost all students achieving some success and scoring an odd mark or two in various parts.

Part (a)(i) was done well with over 90% of students identifying $(\sin\theta)^{-1}$ or $\frac{1}{\sin\theta}$.

In part (a)(ii), more than half of students scored two marks for applying the quotient or chain rule correctly. The final mark needed some clear explanation using identities to achieve the printed answer and had to include $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ at some point; only a third of students managed to satisfy both criteria.

Part (a)(iii) started successfully with around 60% of students substituting $y = \operatorname{cosec}\theta$ or obtaining y^2 or $\frac{1}{y^2}$ in terms of $\cos\theta$. Students then went down one of two routes, using appropriate identities involving $\sin\theta$ and $\cos\theta$ or replacing $\operatorname{cosec}^2\theta - 1$ with $\cot^2\theta$. Indeed, it was noted that confidence with these trigonometrical identities had improved. Around a third of students successfully managed to negotiate one of these routes fully correctly.

Part (b)(i) was very challenging, although nearly 60% of students were awarded the first mark for adapting the result from part (a)(ii) and obtaining $\frac{\mathrm{d}x}{\mathrm{d}u} = -2\operatorname{cosec}u\cot u$. Some students failed to score this mark for one of the following reasons:

- missing the negative sign
- missing the factor of 2
- getting muddled with letters and writing $\frac{\mathrm{d}u}{\mathrm{d}x}$ or using θ.

The success rate then declined sharply with each mark with around one third of students being awarded three marks for being able to substitute everything correctly to obtain an integrand

$$\frac{-2\operatorname{cosec}u\cot u}{4\operatorname{cosec}^2 u\sqrt{4\operatorname{cosec}^2 u - 4}}$$

or equivalent.

Lots of mistakes were seen from this point, meaning that only 12% of students scored all six marks. Typical errors seen were:

- incorrect use of a trigonometric identity with $\sec\theta$ often seen
- square rooting $4\operatorname{cosec}^2 u - 4$ incorrectly to obtain $2\operatorname{cosec}u - 2$
- muddling up letters – using a mixture of θ and u
- cancelling terms incorrectly.

The last part relied on joining the previous parts together and this was successfully done by only 8% of students. However, just over a quarter of students scored the first mark for knowing that

$-\int \sin u \ \mathrm{d}u = \cos u$. Where students did continue from this they were often let down by their algebraic manipulation skills and failed to score further marks.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.