A-LEVEL
MATHEMATICS

7357/3 Paper 3
Report on the Examination

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General

This was the fifth examination for this specification with the previous two series in 2020 and 2021 being the exceptional November series. It was pleasing to see that most students attempted all questions and gained part marks wherever possible, even in challenging questions. Students performed better in the multiple-choice questions on average compared to the previous series. Students would benefit from additional practice in interpreting solutions to problems in context.

Generally, students scored better in the statistics section than the pure maths section.

Overview of Entry

This paper provided the opportunity for all students to demonstrate their knowledge and skills in both pure and statistics sections, with the full range of available marks being scored.

Topics that were done well included:

Section A (Pure):
- Functions
- Indefinite integration
- Logarithms
- Parametric equations

Section B (Statistics):
- Binomial distribution
- Normal distribution
- Statistical sampling
- Use of a Venn diagram in probability

Topics which students found challenging included:

Section A (Pure):
- Mathematical proof
- Rates of change
- Use of mathematical notation

Section B (Statistics):
- Data presentation and interpretation
- Independent events
- Statistical hypothesis testing

Comments on individual questions

Section A (Pure)

Question 1 (multiple-choice)

Most scored the mark for this question. The most common incorrect response was $|x| < \frac{1}{4}$.

Question 2 (multiple-choice)

This question proved to be the most challenging multiple-choice question in the paper. Very few students selected the correct answer. Students were able to spot that a definite integral was required, but $x^2 - 5x$ was most often chosen, incorrectly, as the integrand. No option was left unchosen.

Question 3 (multiple choice)

Most scored the mark for this question. The most common incorrect response was $x = 1$.

Question 4

Nearly 80% of students scored full marks. It was pleasing to see that most were able to get at least one mark for correctly integrating either x^2 or $x^{\frac{1}{2}}$ correctly. In general, when the final mark was lost it was for omitting $+ c$ or simplifying $\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}$ incorrectly to $\dfrac{3x^{\frac{3}{2}}}{2}$. However, there were a few who differentiated the expression instead of integrating it.

Question 5

Most students attempted both parts of this question with about three-quarters of them scoring full marks in part (a). Of those who lost the second mark, this was usually because their graph had an inconsistent amplitude or did not pass through 90, 180, 270 and 360 on the x-axis. It was not uncommon to see graphs of sin x, sin $3x$ or sin $4x$ being drawn as a final answer.

About 60% of students scored the mark in part (b) as they recognised that the maximum and minimum values for the graph were 1 and –1. A common incorrect method was to give the values of x as the answers. It was unfortunate that some students were able to mark the ± 1 on the y-axis of their graph but could not deduce that these were the possible values of A.

Question 6

Students performed well in parts (a) and (b)(i). In part (a), a few lost the final mark for not stating the positive value of the length, leaving their final answer as a negative value. On a few occasions, students worked out $x(0) - x(9.5)$ to get the final correct answer. A common incorrect method was to set $y = 0$ to get $t = 90/7$ and then substitute this value instead of 9.5 into the expression for x.

In part (b)(i) the vast majority of students were able to apply the chain rule to find dy/dx from the parametric equations. Some students lost the final mark because of errors in manipulating their expression with incorrect use of brackets or sign errors being the most common mistakes.

Most students found part (b)(ii) very challenging with a significant proportion struggling to get the final mark for correctly comparing the width and length with $\frac{1}{3}$ or 3 followed by a correct concluding statement. When students found the value of t correctly, they then substituted into the expression for y to find the value of half of the width but some did not deduce that the actual width of the surfboard was double their y-value.

Question 7

Many students made a good start in this question involving logarithms. In part (a)(i) it was evident that students could correctly recall the formula to calculate the gradient between two points. Once they found this, which is the value of b, they used $y = mx + c$ or an equivalent form to find the value of a. Some formed simultaneous equations in a and b, but with varying degrees of success, although a calculator could be used to solve the equations. The most common misconception was to write, for example, $y = \log_{10} 4.49$ rather than $y = 4.49$. Although about one-third of students scored both marks in part (a)(ii), most were able to score at least a mark for using one of the laws of logarithms correctly, including those who started from $T = \mathrm{K}\, d^{\mathrm{n}}$. It is important to note that as this is a 'Show that' question, students could not score the final mark if they started from $T = \mathrm{K}\, d^{n}$

It was common to see unsuccessful attempts at converting logs to powers of 10 especially in the manipulation of the constant –0.7 where students often incorrectly stated that K = –0.7 rather than $10^{-0.7}$ A few students found the values of K and n separately, but did not obtain the equation for T in the form of $\mathrm{K}\, d^{n}$.

Not many scored full marks in part (b). Of those who managed to get the fully correct equation for T in the previous part, the majority were able to score well in the last part. Two-thirds of the students were able to get at least one mark for substituting 60 000 for T in their equation. Some students did not know how to solve the unknown with a fractional power which resulted in the wrong value of d: this could most easily be done using the equation solver feature of a calculator. Many also lost the final mark for not stating the correct unit for distance, typically making no reference to 'millions'.

Question 8

This question was not well-answered with about one-tenth and one-fifth of students scoring full marks in parts (a) and (b) respectively. Students were able to differentiate V with respect to h, but not many could use the rate of change (8 cm^3/s) to find the actual volume at time $t = 3$. As a result, they couldn't progress further to obtain h or h^2 correctly. It was pleasing to see that some students recognised the need to rearrange the formula for V to make h the subject to obtain $h = \sqrt[3]{\frac{12V}{\pi}}$ and then substitute the value of V. Manipulation involving exact values was weak and many struggled to show how to reach to the given answer of $6\sqrt[3]{6\pi}$, although this final stage of the solution was worth only 1 mark.

Two-thirds of students could not start part (b): they did not recognise the need to use the chain rule. However, those who did went on to score a good proportion of the marks available. The final mark was lost without stating the correct units, cm/s. There were a few students who stated that $h = \sqrt[3]{\frac{96t}{\pi}}$, differentiated this with respect to t to obtain a correct derivative for h and then substituted $t = 3$ to obtain the correct answer.

Question 9

Generally, students found this structured proof question very challenging. They could not follow the rigorous steps required and it was evident that many did not understand the logical sequence of the proof. The question was designed to draw on key ideas from the proof of the irrationality of $\sqrt{2}$ and it seemed that many students were not familiar with this standard proof.

In part (a), although students began by rearranging the equation to make a^2 the subject, they were not always successful in factorising $4b + 2$ with $2(2b+2)$ being common, or a sign error arising when rearranging the equation. Students who decided to prove that $4b + 2$ is even without factorising first, often stated that $4b$ is even, but then did not comment on the number 2 being even. It was common to see students concluding that a was even without first stating that a^2 was even, hence losing the final mark.

Nearly 80% of students did not score any marks in part (b). It was unfortunate that students did not follow on from the previous part as indicated by the word ‘hence’. Very few students recognised that they should replace a (even number) with $2k$. However, a common error was to use this substitution incorrectly, for instance starting with $a = 2k$ to incorrectly obtain $a^2 = 2k^2$
The most common approach which received no credit was to write $2b + 1 = \frac{a^2}{2}$ and state that because a is even, therefore $\frac{a^2}{2}$ must be even too.

In part (c), about 5% of students gained the mark. Although they recognised there is no solution to the equation, most did not refer back to the initial condition of a and b being integers and use this to comment on the solutions. The vast majority of students instead referred to whether or not a and b were odd or even, positive or negative, real numbers, rational or irrational, or to the number of solutions. Of those that realised that they needed to comment on a and b as integers, most were unable to recognise that although one or other of a or b could be an integer, there were no solutions where both a and b are integers.

Question 10

This question discriminated well between students of varying abilities. In part (a) about 70% of students were able to state the correct value which is not in the domain of the given function.

In part (b) some students were able to provide the general definition of a many-to-one function although some confused the roles of x- and y-values. However, only a few were able to justify why the given graph shows that the function is many-to-one. The most common response was the use of the horizontal line test with an explanation. It was pleasing to see that some students were able to give examples of two values of x which give the same value of y. Although the reference to curves having maximum and minimum points was made correctly by some students, it was also common to state incorrectly that stationary points indicate a function is many-to-one which did not gain any credit. The incorrect response that the function contained an x^2 term which showed that it was a many-to-one function because $(+a)^2 = (-a)^2$ appeared frequently too.

About three-quarters of students scored full marks in parts (c)(i) and (c)(ii). In part (c)(i), some students did not recognise that *P* and *Q* are the points of intersection of $y = \mathrm{f}(x)$ and $y = x$ and assumed *P* and *Q* were stationary points and started differentiating which did not gain any credit. There were some basic arithmetic slips when rearranging. In part (c)(ii) a few students gave their answer in decimals instead of exact values as required in the question. Students typically used their calculators to solve the equation so should have obtained exact values if they set the correct output mode.

In part (d) about 20% of students scored full marks. However, many were able to gain some of the marks available. Students often applied the quotient (or product) rule well, but on some occasions made slips with the mathematical operations. Most who lost the final mark did not write a conclusion or made errors in their working out such as missing brackets

Two-thirds of students scored nothing in part (e) and only 5% of students scored both marks. Although they could identify the range of the function, very few were able to express this accurately using set notation. This is an area for further development.

Section B (Statistics)

Question 11 (multiple-choice)

This was the most successful of all the multiple-choice questions on the paper with approximately 80% of students selecting the correct response. The most frequently chosen incorrect response was 0.35 due to misunderstanding normal distribution notation.

Question 12 (multiple-choice)
This question proved to be challenging with 60% of students choosing an incorrect answer, most commonly ‘negatively’ skewed.

Question 13
Understanding of the Large Data Set (LDS), although showing some improvement, remains an area for further development. Only about 20% of students gained both marks. Students were able to gain one of the two marks available as they could give a reason that the LDS did not represent all cars in the UK. Very few students recalled that the LDS only shows data for the year 2002 and 2016 and nothing in between. Incorrect statements referring to electric vehicles or climate change were common.

Question 14

Students still do not perform well on questions that require giving reasons or explanations in context. In part (a), just under one-quarter of students were able to state the two assumptions for the binomial distribution in context. The most common correct assumptions were that the probability of getting a complaint call is fixed and there are only two outcomes of either complaint or non-complaint. It is unfortunate that although many students were able to recall the assumptions for a binomial distribution model to be valid, they were unable to put them in context.

The use of a calculator to find binomial probabilities was good with a large proportion of students scoring the marks in parts (b)(i), (b)(ii) and (b)(iii). In part (c), students were able to state the formula for the variance of a binomial distribution from the formulae booklet. However, they often formed the incorrect equation $10p(1-p)=1.5$ which resulted in the loss of the final two marks. However, those who obtained the correct equation were often very successful in gaining all three marks, although there were sometimes errors when rearranging $10p - 10p^2 = 2.25$.

Question 15

Students performed well in this question. In part (a), they understood what was meant by the population, but some were not able to give a precise description of it in context. Common incorrect descriptions of the population were 'adults,' 'the town,' 'the electoral register,' 'the adult population which votes' and 'adults in the town on social media.' It was also common to see the selection of every 100th adult being referred to as the population, giving incorrect answers such as the population of town divided by 100 or 1/100 in electoral register or every 100th adult.

Over 70% of students stated the correct method of sampling in part (b). The most common wrong answer was stratified sampling.

Students were still able to gain the mark in part (b)(ii) even if they answered (b)(i) incorrectly. The most common correct answers seen were reference to a lack of bias or that the method is either easy or inexpensive. However, reference to it being "random" gained no credit. A few students did not read the question properly so instead gave a disadvantage rather than an advantage.

Question 16

Students showed good understanding of using the Venn diagram with about 70% scoring full marks in part (a). Most correctly placed 17 where C, D and T overlap. However, some were not able to make good progress from that point as they made errors by writing 48, 21 and 32 directly into regions in the diagram without considering the overlap with the central region $C \cap D \cap T$. A few students lost the final mark due to the omission of 56, the number of households without pets.

It was pleasing to see that students were able to score well in parts (b)(i) and (b)(ii). It is also worth noting that some students went on to unnecessarily simplify or convert their correct fraction or decimal further which at times led to incorrect final answers. On this occasion such errors were not penalised.

The conditional probability in part (b)(iii) caught out many students, with some using the wrong denominator of 240 instead of 195. The easiest way to answer this question is to cover up the circle for tortoises and find the required probability by considering only the remaining regions.

Students were generally aware of the independence rule required in part (c) but were not able to work out which two probabilities to multiply and which probability to compare with. The most common successful approach to determine whether the events were independent was to calculate $P(C) \times P(T)$ and compare with $P(C \cap T)$ to gain the full mark. However, those who used conditional probability often couldn't compare correctly to score the final mark, for example $P(C)$ should be compared with $P(C/T)$.

Question 17

Most students were able to gain some marks in this question. Good use of a calculator was evident where students successfully used either the ‘Inverse Normal’ to find the critical value or the ‘Normal Cumulative Distribution Function’ to find the probability. However, only about 20% of students scored all six marks. Students did not always use the correct notation for the population mean when stating their hypotheses. A few students were confused between the sample mean and the population mean, writing 36.2 instead of 34 in their hypotheses. Students commonly carried out the hypothesis test using a probability method and were much more successful than those who tried to use a critical region or convert the test statistic to a z-score and then compare with a critical region.

Once a student has made an appropriate comparison eg comparing the probability with the significance level, we expect to see a simple statement, in this case ‘Reject H_0’. Not many could conclude correctly in context using the correct wording. In this case the conclusion should begin with “There is sufficient evidence to suggest…” and end with a clear description of the manager’s claim, taken from the question: “…that the mean working hours per week of the company’s employees has increased.” Students should be encouraged to learn this standard phrasing and not to create their own version of the conclusion.

Question 18

Many unsuccessful attempts were seen in part (a) with only 1% gaining full marks. Some students were able to obtain the correct values of $1.78 \pm 2 \times 0.23$ but could not go further by stating they are approximately equal to the values of 1.33 and 2.22 given in the question. It was very common for students to write $1.32 < 1.33$ and $2.24 > 2.22$ instead, therefore losing the remaining two marks. Of those who were able to proceed, they typically did not state that height is a continuous variable or that 95% of heights lie within two standard deviations of the mean in a normal distribution.

However, students were able to score better in the next four parts.

Students found it difficult to give explanations in context with only about 15% students scoring both marks in part (d). Students needed to be more detailed in their comparisons. For example, comments about the mean needed to explain that summer athletes were taller, on average, than those in winter. Similarly, comments about the standard deviation needed to say winter heights were more spread out than those in summer. Various key words were accepted, but many students compared standard deviations using the word ‘range,’ which was not accepted.

Question 19

Less than 20% of students were able to perform the hypothesis test using a binomial distribution fully correctly but most were able to gain some marks in the process of carrying out the test.
A number of students used incorrect notation for the proportion when stating their hypotheses. Those who wrote P(X) or X for the proportion did not gain the first mark. Many wrote down the distribution that they intended to use and this was largely correctly done. In the calculation involving the binomial distribution, it was common to see P($X >$ 18) or P($X =$ 18) being calculated instead of the correct P($X \geq$ 18). Many students were able to compare their probability with 0.1 before concluding. Those who found the correct critical value of 19 could often not clearly formulate a critical region and thus found it difficult to make a clear, correct comparison with the test statistic.

Like question 17, after an appropriate comparison we expect to see a simple statement ‘Do not reject H_0.’ (The statement “Accept H_0” is permitted too.) Not many students could conclude correctly in context. The conclusion should begin with “There is not sufficient evidence to suggest…”and end with a clear description of the claim, taken from the question: “…that there has been an increase in the proportion of customers registered for an internet banking account.” Students should be encouraged to learn this standard phrasing and not to create their own version of the conclusion.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.