A

Surname ______________________________

Other Names ______________________________

Centre Number ______________________________

Candidate Number ______________________________

Candidate Signature ______________________________

I declare this is my own work.

A-level
FURTHER MATHEMATICS

Paper 1

7367/1

Time allowed: 2 hours

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

[Turn over]

- **You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.**
- **You should have a scientific calculator that meets the requirements of the specification.**

INSTRUCTIONS

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Answer ALL questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do NOT write on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO

SECTION A

Answer ALL questions in the spaces provided.

1 The displacement of a particle from its equilibrium position is x metres at time t seconds.

The motion of the particle obeys the differential equation

$$\frac{d^2x}{dt^2} = -9x$$

Calculate the period of its motion in seconds.

Circle your answer. [1 mark]

$\frac{\pi}{9}$

$\frac{2\pi}{9}$

$\frac{\pi}{3}$

$\frac{2\pi}{3}$

2 **Simplify**

$$\frac{\cos\left(\frac{6\pi}{13}\right)+\mathrm{i}\sin\left(\frac{6\pi}{13}\right)}{\cos\left(\frac{2\pi}{13}\right)-\mathrm{i}\sin\left(\frac{2\pi}{13}\right)}$$

Tick (✓) ONE box. [1 mark]

☐ $\cos\left(\frac{8\pi}{13}\right)+\mathrm{i}\sin\left(\frac{8\pi}{13}\right)$

☐ $\cos\left(\frac{8\pi}{13}\right)-\mathrm{i}\sin\left(\frac{8\pi}{13}\right)$

☐ $\cos\left(\frac{4\pi}{13}\right)+\mathrm{i}\sin\left(\frac{4\pi}{13}\right)$

☐ $\cos\left(\frac{4\pi}{13}\right)-\mathrm{i}\sin\left(\frac{4\pi}{13}\right)$

3 Given that $y = \operatorname{sech} x$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$

Tick (✓) ONE box. [1 mark]

- [ ] $\operatorname{sech} x \tanh x$
- [ ] $-\operatorname{sech} x \tanh x$
- [ ] $\operatorname{cosech} x \coth x$
- [ ] $-\operatorname{cosech} x \coth x$

4 The vector $\mathbf{v}$ is an eigenvector of the matrix $\mathbf{N}$ with corresponding eigenvalue 4

The vector $\mathbf{v}$ is also an eigenvector of the matrix $\mathbf{M}$ with corresponding eigenvalue 3

Given that

$$\mathbf{N}\mathbf{M}^2\mathbf{v} = \lambda\mathbf{v}$$

find the value of λ

Circle your answer. [1 mark]

10

24

36

144

5 It is given that $z = -\frac{3}{2} + \mathrm{i}\frac{\sqrt{11}}{2}$ is a root of the equation

$$z^4 - 3z^3 - 5z^2 + kz + 40 = 0$$

where k is a real number.

5 (a) Find the other three roots. [5 marks]

5 (b) **Given that $x \in \mathbb{R}$, solve**

$$x^4 - 3x^3 - 5x^2 + kx + 40 < 0$$

[1 mark]

[Turn over]

6 (a) Given that $|x| < 1$, prove that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

[4 marks]

6 (b) **Solve the equation**

$$20\,\mathrm{sech}^2 x - 11\tanh x = 16$$

Give your answer in logarithmic form.
[4 marks]

7 **The matrix $\mathbf{M}$ is defined as**

$$\mathbf{M} = \begin{bmatrix} 1 & 7 & -3 \\ 3 & 6 & k+1 \\ 1 & 3 & 2 \end{bmatrix}$$

where k is a constant.

7 (a) (i) Given that $\mathbf{M}$ is a non-singular matrix, find $\mathbf{M}^{-1}$ in terms of k [5 marks]

7 (a) (ii) State any restrictions on the value of k
[1 mark]

[Turn over]

7 (b) **Using your answer to part (a)(i), solve**

$$x + 7y - 3z = 6$$

$$3x + 6y + 6z = 3$$

$$x + 3y + 2z = 1$$

[3 marks]

[Turn over]

8 (a) **The complex number w is such that**

$$\arg(w + 2\mathrm{i}) = \tan^{-1}\frac{1}{2}$$

It is given that $w = x + \mathrm{i}y$, where x and y are real and $x > 0$

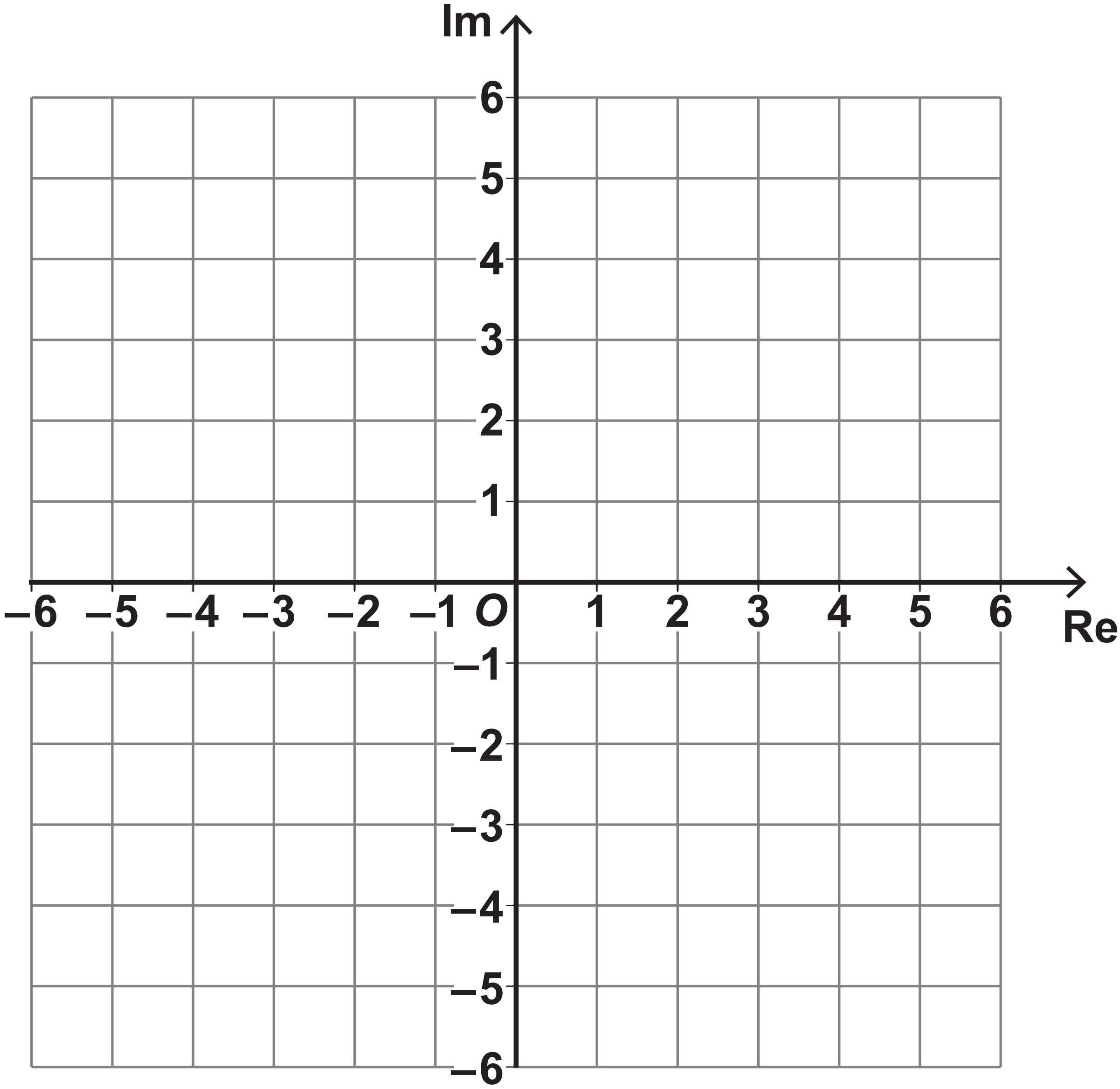
Find an equation for y in terms of x [2 marks]

8 (b) The complex number z satisfies both

$$-\frac{\pi}{2} \leq \arg(z + 2\mathrm{i}) \leq \tan^{-1}\frac{1}{2} \qquad \text{AND} \qquad |z - 2 + 3\mathrm{i}| \leq 2$$

The region R is the locus of z

Sketch the region R on the Argand diagram below. [4 marks]



8 (c) **z_1 is the point in *R* at which $|z|$ is minimum.**

8 (c) (i) Calculate the exact value of $|z_1|$ [3 marks]

8 (c) (ii) Express z_1 in the form $a + \mathrm{i}b$, where a and b are real. [2 marks]

[Turn over]

9 Roberto is solving this mathematics problem:

The curve C_1 has polar equation

$r^2 = 9 \sin 2\theta$

for all possible values of θ

Find the area enclosed by C_1

Roberto's solution is as follows:

$$A = \frac{1}{2}\int_{-\pi}^{\pi} 9 \sin 2\theta \, \mathrm{d}\theta$$

$$= \left[-\frac{9}{4}\cos 2\theta\right]_{-\pi}^{\pi}$$

$$= 0$$

9 (a) Sketch the curve C_1 [2 marks]

[Turn over]

9 (b) Explain what Roberto has done wrong.
[2 marks]

9 (c) Find the area enclosed by C_1 [2 marks]

[Turn over]

9 (d) ***P* and *Q* are distinct points on C_1 for which r is a maximum.**
***P* is above the initial line.**

Find the polar coordinates of *P* and *Q*
[2 marks]

9 (e) The matrix $\mathbf{M} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ represents the transformation T

T maps C_1 onto a curve C_2

9 (e) (i) T maps P onto the point P'

Find the polar coordinates of P' [4 marks]

9 (e) (ii) Find the area enclosed by C_2

Fully justify your answer. [2 marks]

[Turn over]

10 **In this question all measurements are in centimetres.**

A small, thin laser pen is set up with one end at *A*(7, 2, −3) and the other end at *B*(9, −3, −2)

A laser beam travels from *A* to *B* and continues in a straight line towards a large thin sheet of glass.

The sheet of glass lies within a plane Π_1 which is modelled by the equation

$$4x + py + 5z = 9$$

where p is an integer.

10 (a) **The laser beam hits Π_1 at an acute angle α, where $\sin\alpha = \dfrac{\sqrt{15}}{75}$**

Find the value of p [6 marks]

10 (b) **A second large sheet of glass lies on the other side of Π_1**

This second sheet lies within a plane Π_2 which is modelled by the equation

$$4x + py + 5z = -5$$

Calculate the distance between the sheets of glass. [2 marks]

10 (c) The point $A(7, 2, -3)$ is reflected in Π_1

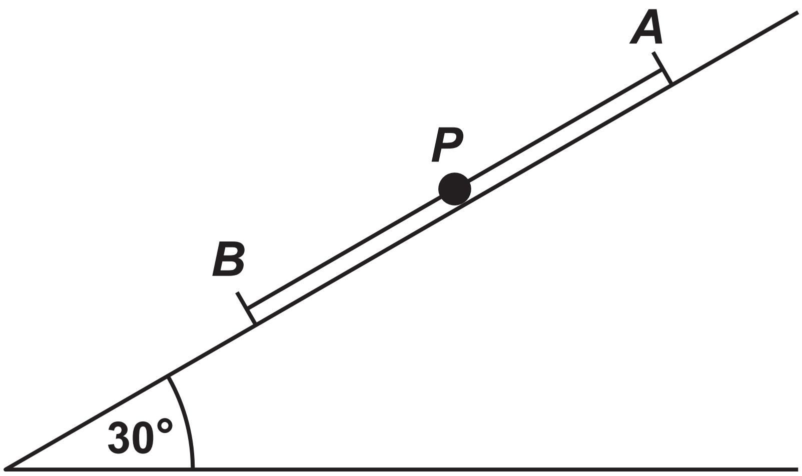
Find the coordinates of the image of A after reflection in Π_1 [4 marks]

11 **IN THIS QUESTION USE g AS 10 m s^{-2}**

A smooth plane is inclined at 30° to the horizontal.
The fixed points *A* and *B* are 3.6 metres apart on the line of greatest slope of the plane, with *A* higher than *B*

A particle *P* of mass 0.32 kg is attached to one end of each of two light elastic strings.
The other ends of these strings are attached to the points *A* and *B* respectively.

The particle *P* moves on a straight line that passes through *A* and *B*



The natural length of the string *AP* is 1.4 metres.
When the extension of the string *AP* is e_A metres, the tension in the string *AP* is $7e_A$ newtons.
The natural length of the string *BP* is 1 metre.
When the extension of the string *BP* is e_B metres, the tension in the string *BP* is $9e_B$ newtons.

The particle P is held at the point between A and B which is 0.2 metres from its equilibrium position and lower than its equilibrium position. The particle P is then released from rest.

At time t seconds after P is released, its displacement towards B from its equilibrium position is x metres.

11 (a) **Show that during the subsequent motion the object satisfies the equation**

$$\ddot{x} + 50x = 0$$

Fully justify your answer. [5 marks]

11 (b) **The experiment is repeated in a large tank of oil. During the motion the oil causes a resistive force of kv newtons to act on the particle, where v m s^{-1} is the speed of the particle.**

The oil causes critical damping to occur.

11 (b) (i) **Show that $k = \dfrac{16\sqrt{2}}{5}$ [3 marks]**

[Turn over]

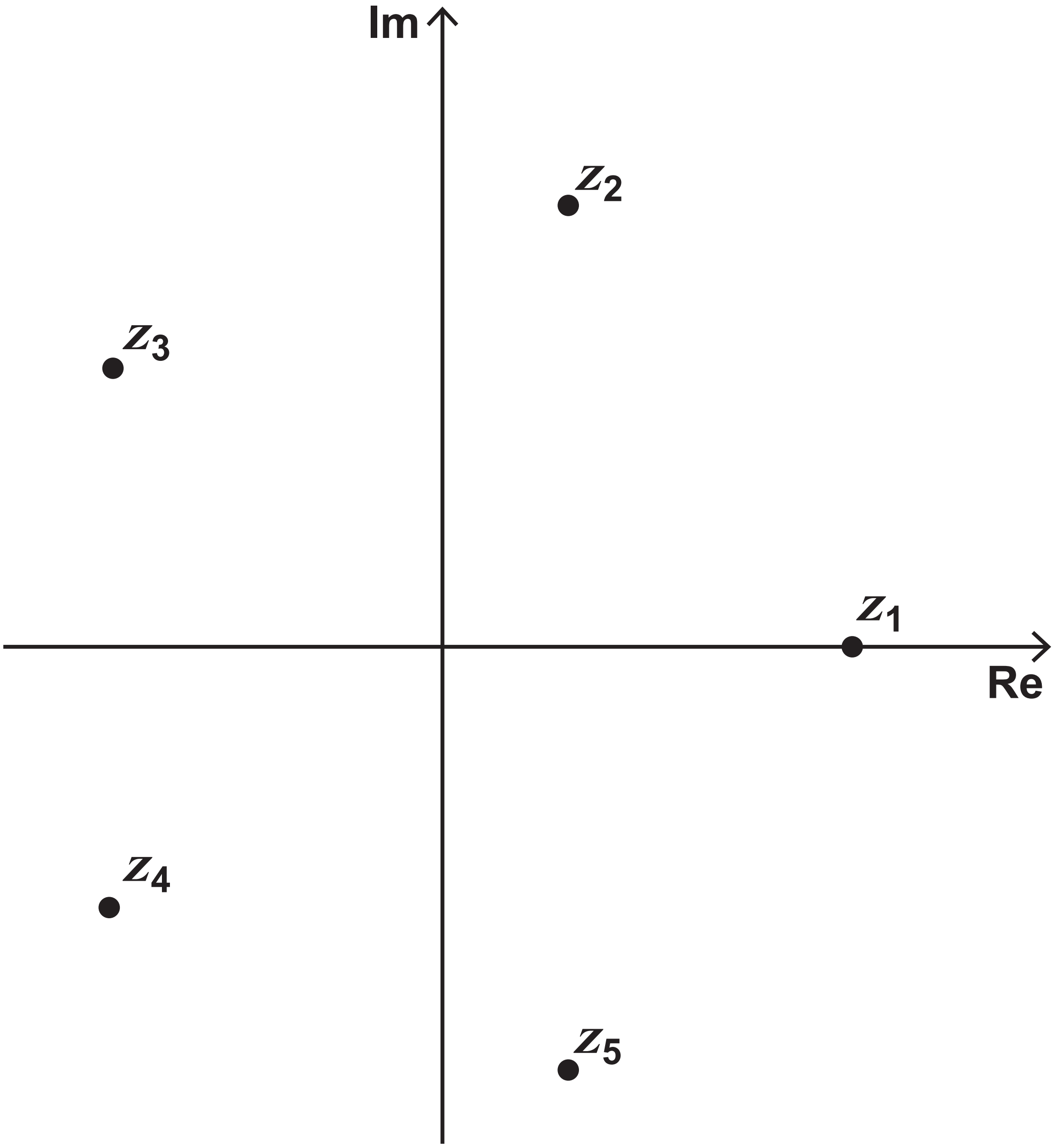
11 (b) (ii) Find x in terms of t, giving your answer in exact form. [6 marks]

[Turn over]

11 (b) (iii) **Calculate the maximum speed of the particle.**
[5 marks]

[Turn over]

12 **The Argand diagram shows the solutions to the equation $z^5 = 1$**



12 (a) Solve the equation

$z^5 = 1$

giving your answers in the form
$z = \cos\theta + \mathrm{i}\sin\theta$, where $0 \leq \theta < 2\pi$
[2 marks]

[Turn over]

12 (b) Explain why the points on an Argand diagram which represent the solutions found in part (a) are the vertices of a regular pentagon.
[2 marks]

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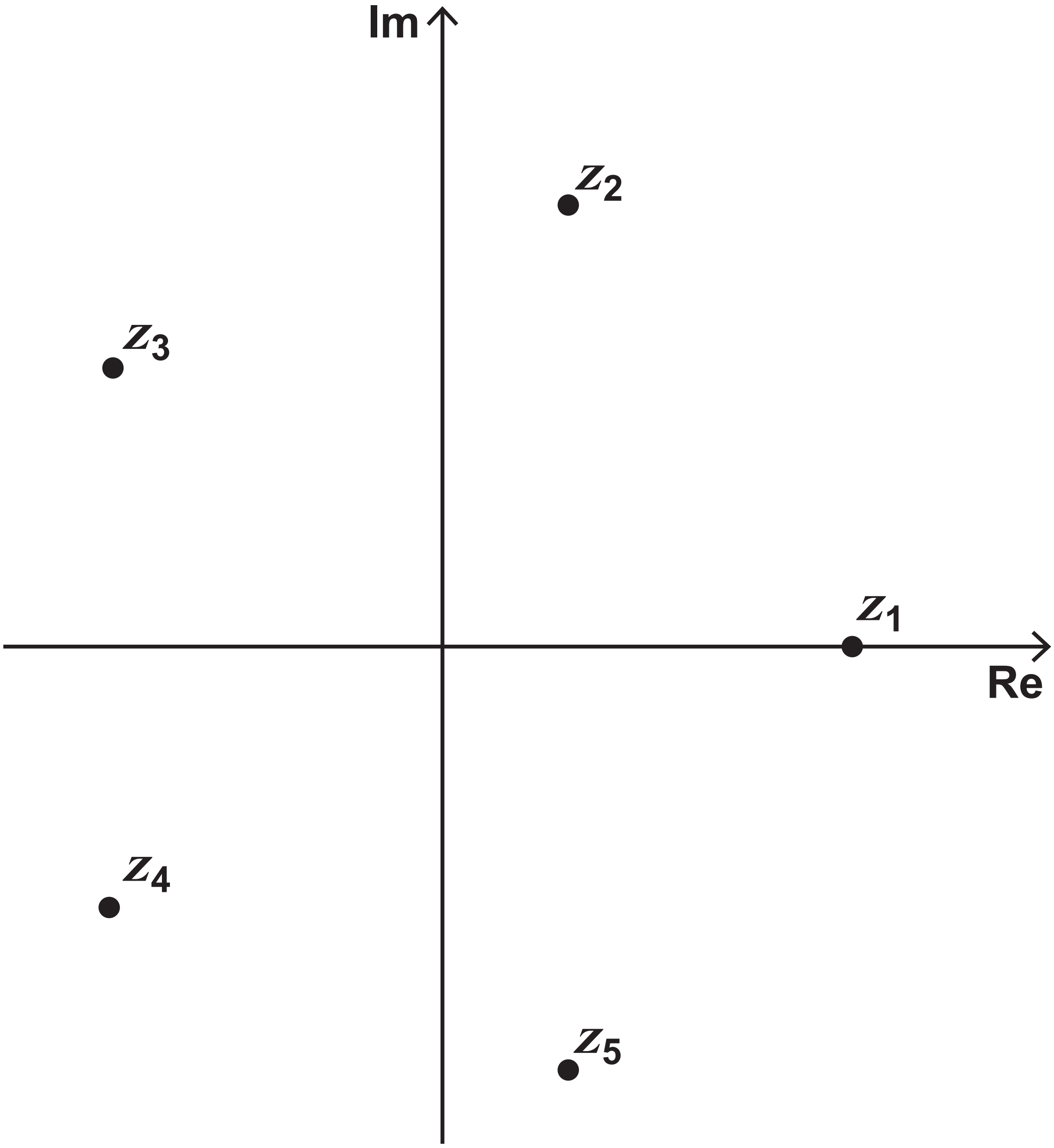
[Turn over]

12 (c) **Show that if $c = \cos\theta$, where $z = \cos\theta + \mathrm{i}\sin\theta$ is a solution to the equation $z^5 = 1$, then c satisfies the equation**

$$16c^5 - 20c^3 + 5c - 1 = 0$$

[5 marks]

12 (d) **The Argand diagram on page 44 is repeated below.**



Explain, with reference to the Argand diagram, why the expression

$$16c^5 - 20c^3 + 5c - 1$$

has a repeated quadratic factor. **[3 marks]**

[Turn over]

12 (e) ***O*** **is the centre of a regular pentagon *ABCDE* such that $OA = OB = OC = OD = OE = 1$ unit. The distance from *O* to *AB* is h**

By solving the equation $16c^5 - 20c^3 + 5c - 1 = 0$, show that

$$h = \frac{\sqrt{5} + 1}{4}$$

[5 marks]

END OF QUESTIONS

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Write the question numbers in the left-hand margin.

Additional page, if required.
Write the question numbers in the left-hand margin.

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For Examiner's Use	
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