



Surname _____

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I declare this is my own work.

**A-level
FURTHER MATHEMATICS
Paper 2
7367/2**

Time allowed: 2 hours

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

[Turn over]



- **You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.**
- **You should have a scientific calculator that meets the requirements of the specification.**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).**
- **Do NOT write on blank pages.**
- **Show all necessary working; otherwise marks for method may be lost.**

- **Do all rough work in this book. Cross through any work that you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO



Answer ALL questions in the spaces provided.

1 Find the imaginary part of

$$\frac{5 + i}{1 - i}$$

Circle your answer. [1 mark]

−3

−2

2

3



- 2 Find the mean value of the function $f(x) = 10x^4$ between $x = 0$ and $x = a$

Circle your answer. [1 mark]

$$10a^3$$

$$40a^3$$

$$2a^4$$

$$4a^5$$

[Turn over]



- 3 The roots of the equation $x^2 - px - 6 = 0$ are α and β
Find $\alpha^2 + \beta^2$ in terms of p
Circle your answer. [1 mark]

$$p^2 - 6$$

$$p^2 + 6$$

$$p^2 - 12$$

$$p^2 + 12$$

- 4 Which of the following graphs intersects the graph of $y = \sinh x$ at exactly one point?

Circle your answer. [1 mark]

$$y = \operatorname{cosech} x$$

$$y = \cosh x$$

$$y = \coth x$$

$$y = \operatorname{sech} x$$

[Turn over]



5 Prove by induction that, for all integers $n \geq 1$,

$$\sum_{r=1}^n r^3 = \left\{ \frac{1}{2} n(n+1) \right\}^2$$

[4 marks]



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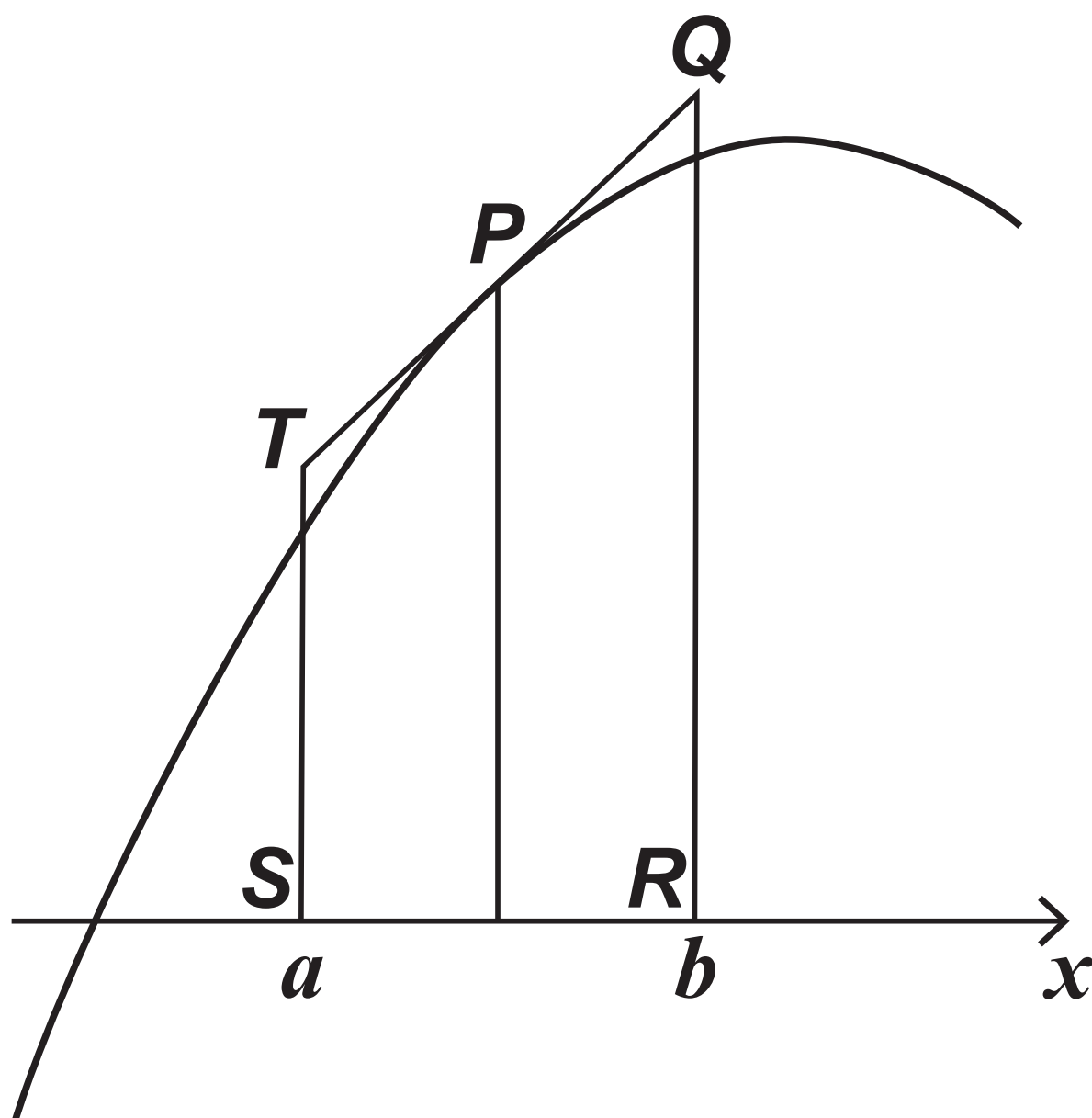
6 The diagram below shows part of the graph of $y = f(x)$

The line TPQ is a tangent to the graph of $y = f(x)$ at the

point $P\left(\frac{a+b}{2}, f\left(\frac{a+b}{2}\right)\right)$

The points $S(a, 0)$ and T lie on the line $x = a$

The points Q and $R(b, 0)$ lie on the line $x = b$



Sharon uses the mid-ordinate rule with one strip to estimate the

value of the integral $\int_a^b f(x) \, dx$

By considering the area of the trapezium $QRST$, state, giving reasons, whether you would expect Sharon's estimate to be an under-estimate or an over-estimate. [3 marks]

[Turn over]





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[Turn over]



7 The function f is defined by

$$f(x) = \frac{ax - 5}{2x + b} \quad x \in \mathbb{R}, x \neq \frac{9}{2}$$

where a and b are integers.

The graph of $y = f(x)$ has
asymptotes $x = \frac{9}{2}$ and $y = 3$

7 (a) Find the value of a and the value of b [2 marks]

[Turn over]



7(b) Solve the inequality

$$f(x) \leq x + 2$$

Fully justify your answer.
[6 marks]

[illegible]

[Turn over]



20

[illegible]

[Turn over]



8 (a) The function f is defined as
 $f(x) = \sec x$

8 (a) (i) Show that $f^{(4)}(0) = 5$
[4 marks]





8(a) (ii) Hence find the first three non-zero terms of the Maclaurin series for $f(x) = \sec x$ [2 marks]

[Turn over]



8(b) Prove that

$$\lim_{x \rightarrow 0} \left(\frac{\sec x - \cosh x}{x^4} \right) = \frac{1}{6}$$

[4 marks]

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- 9 (a) A curve passes through the point (5, 12.3) and satisfies the differential equation

$$\frac{dy}{dx} = (x^2 - 9)^{\frac{1}{2}} + \frac{2xy}{x^2 - 9} \quad x > 3$$

Use Euler's step by step method once, and then the midpoint formula

$$y_{r+1} = y_{r-1} + 2hf(x_r, y_r), \quad x_{r+1} = x_r + h$$

once, each with a step length of 0.1, to estimate the value of y when $x = 5.2$

Give your answer to six significant figures. [4 marks]

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9(b) (i) Find the general solution of the differential equation

$$\frac{dy}{dx} = (x^2 - 9)^{\frac{1}{2}} + \frac{2xy}{x^2 - 9} \quad (x > 3)$$

[6 marks]

[Turn over]



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9(b) (ii) Given that y satisfies the differential equation in part (b)(i) and that $y = 12.3$ when $x = 5$, find the value of y when $x = 5.2$

Give your answer to six significant figures. [3 marks]

[Turn over]





**9 (c) Comment on the accuracy of your
answer to part (a). [1 mark]**

[Turn over]

10 The curve C_1 has equation

$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$

The curve C_2 has equation

$$x^2 - 25y^2 - 6x - 200y - 416 = 0$$

10 (a) Find a sequence of transformations that maps the graph of C_1 onto the graph of C_2 [4 marks]





10(b) Find the equations of the asymptotes to C_2

Give your answers in the form $ax + by + c = 0$ where a , b and c are integers. [3 marks]

[Turn over]



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[Turn over]



11 (a)

Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{M} = \begin{bmatrix} \frac{5}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{13}{2} \end{bmatrix}$$

[5 marks]





- 11(b) (i) Describe how the directions of the invariant lines of the transformation represented by \mathbf{M} are related to each other.**

**Fully justify your answer.
[2 marks]**

[Turn over]



11 (b) (ii) Describe fully the transformation represented by M [2 marks]

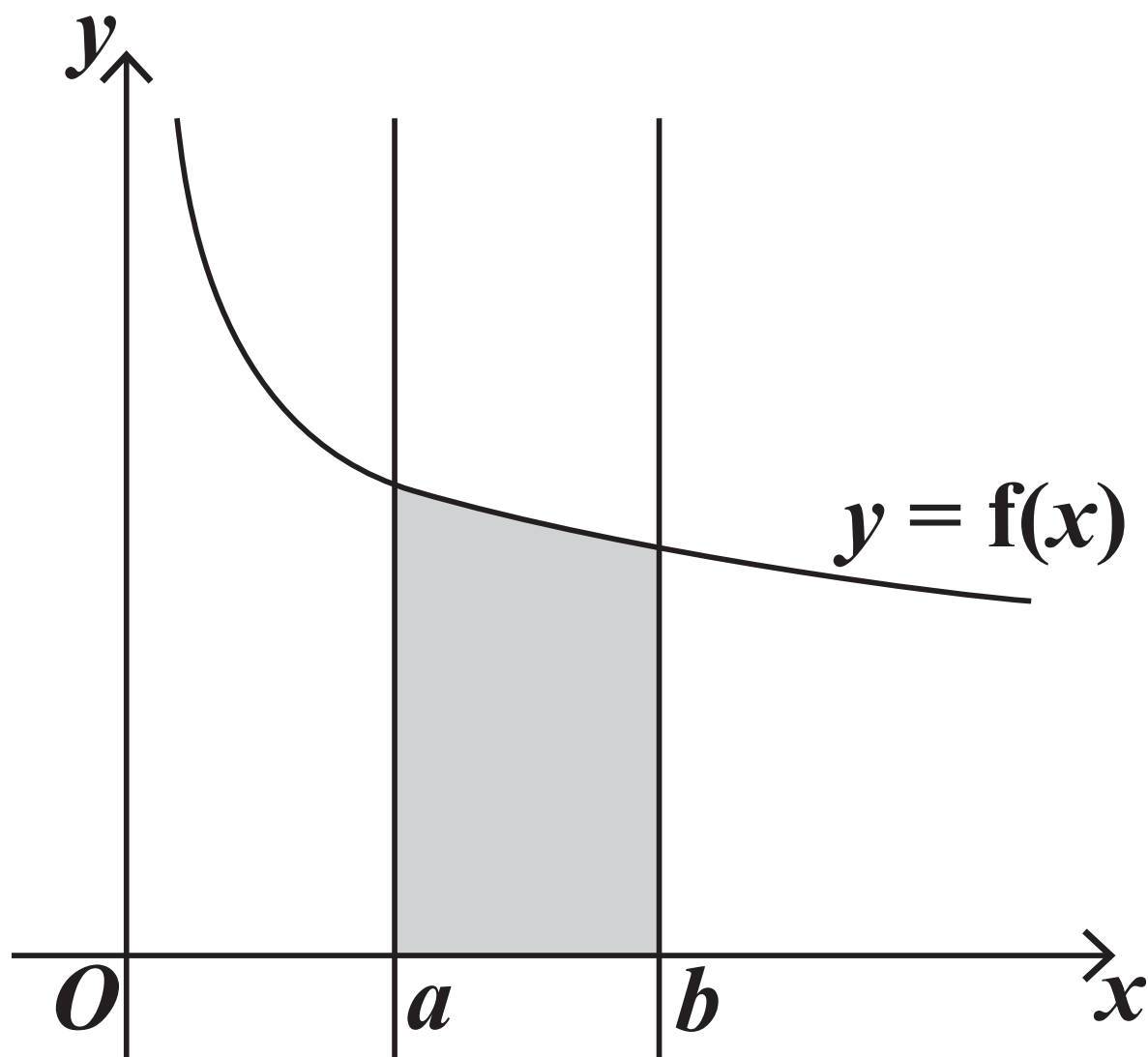
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12

The shaded region shown in the diagram below is bounded by the x -axis, the curve $y = f(x)$, and the lines $x = a$ and $x = b$



The shaded region is rotated through 2π radians about the x -axis to form a solid.

12 (a) **Show that the volume of this solid is**

$$\pi \int_a^b (f(x))^2 dx$$

[4 marks]

[Turn over]





12(b) In the case where $a = 1$,
 $b = 2$ and

$$f(x) = \frac{x + 3}{(x + 1)\sqrt{x}}$$

show that the volume of the
solid is

$$\pi \left(\ln \left(\frac{2^m}{3^n} \right) - \frac{2}{3} \right)$$

where m and n are integers.
[7 marks]

[Turn over]



52

[illegible]

[Turn over]



13 (a)

The matrix **A** represents a reflection in the line $y = mx$, where m is a constant.

Show that

$$\mathbf{A} = \left(\frac{1}{m^2 + 1} \right) \begin{bmatrix} 1 - m^2 & 2m \\ 2m & m^2 - 1 \end{bmatrix}$$

You may use the result in the formulae booklet. [5 marks]

13(b) The matrix **B** is defined

$$\text{as } \mathbf{B} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Show that $(\mathbf{BA})^2 = k\mathbf{I}$

where **I** is the 2×2 identity matrix
and k is an integer. [3 marks]

[Turn over]

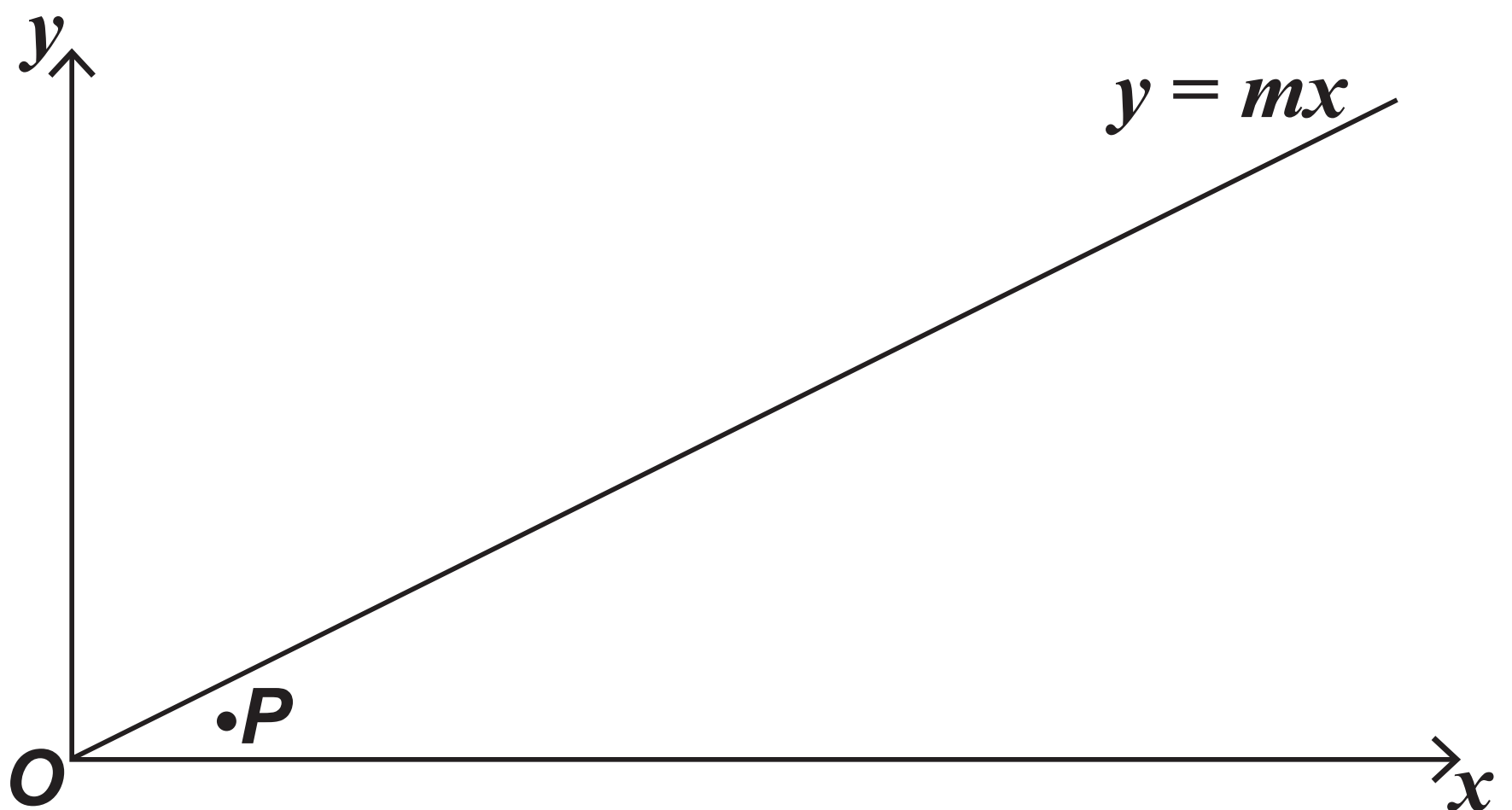


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13 (c) (i) The diagram below shows a point P and the line $y = mx$

Draw four lines on the diagram to demonstrate the result proved in part (b).

Label as P' the image of P under the transformation represented by $(\mathbf{BA})^2$ [2 marks]



[Turn over]



**13 (c) (ii) Explain how your completed diagram on page 59 shows the result proved in part (b).
[2 marks]**

13(d) The matrix **C** is defined

$$\text{as } \mathbf{C} = \begin{bmatrix} \frac{12}{5} & \frac{9}{5} \\ \frac{9}{5} & -\frac{12}{5} \end{bmatrix}$$

Find the value of m such that
C = BA

Fully justify your answer.
[4 marks]

[Turn over]



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14

On an isolated island some rabbits have been accidentally introduced.

In order to eliminate them, conservationists have introduced some birds of prey.

At time t years ($t \geq 0$) there are x rabbits and y birds of prey.

At time $t = 0$ there are 1755 rabbits and 30 birds of prey.

When $t > 0$ it is assumed that:

- the rabbits will reproduce at a rate of $a\%$ per year
- each bird of prey will kill, on average, b rabbits per year
- the death rate of the birds of prey is c birds per year
- the number of birds of prey will increase at a rate of $d\%$ of the rabbit population per year.

This system is represented by the coupled differential equations:

$$\frac{dx}{dt} = 0.4x - 13y \quad (1)$$

$$\frac{dy}{dt} = 0.01x - 1.95 \quad (2)$$

[Turn over]



14 (a) **State the value of a , the value of b , the value of c and the value of d [2 marks]**

14(b) Solve the coupled differential equations to find both x and y in terms of t [9 marks]

[Turn over]



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- 14 (c)** **Given that x and y are both positive for $0 \leq t \leq 5$, use your answer to part (b) to show that the conservationists' plan will succeed. [3 marks]**

[Turn over]



END OF QUESTIONS



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